

Denoising by Wavelet Thresholding Applied to DHPIV

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Abstract: Digital Holographic Particle Image Velocimetry is a promising technique for flow field measurement. However, the low resolution of the sensors still remains poor and needs to be compensated by image processing techniques. The study is organized as follows. We present the two in-line holograms algorithm which permits, at the same time, to retrieve the absorption and the phase shift images. The use of the phase shift information to particle localization permits to increase the accuracy along the depth direction. A wavelet thresholding method is employed to remove noise in the reconstructed absorption and phase shift by using a simple subband adaptive threshold. The proposed methods are illustrated with computer simulation.

Keywords: Digital Holography, Particle image velocimetry PIV, Wavelet Denoising, Wavelet Thresholding

1. Introduction

In recent years digital holography DH technique has been demonstrated to be a useful method in different fields of optics. The idea of digital holography was first proposed by Goodman and Laurence [1], and the fundamental theory was established by Yaroslavskii and Merzlyakov [2].

Digital holography is nowadays used in Particle Image Velocimetry. The application of digital in-line holography to particle measurement was first reported by Adams et al [3]. The typical digital in-line holographic configuration contains only collimating optics and a CCD camera which will record the interference intensity pattern between the collimated on-axis reference light and the forward scattering light from the particles. The setup is quite simple and practical.

We consider the problem of extracting particle positions from recorded in-line holograms for PIV applications. A new approach introduced in this paper is to use the phase shift information to extract particles positions at a given plane with a good accuracy along the depth direction. The reconstruction algorithm presented here uses two in-line holograms [4]. The particles positions are obtained from absorption and phase shift information. In this work, a wavelet thresholding method is employed to remove noise from the reconstructed absorption and phase shift. Discrete Wavelet Transform (DWT) is a very useful tool in time-frequency analysis because of its excellent localization both in time and frequency [5]. In the recent years there has been a fair amount of research on wavelet thresholding. It has been an attractive and efficient tool in image denoising [6, 7, and 8].

The paper is organized as follows. Section 2 introduces the concept of two in-line holograms algorithm to extract particles positions. Section 3 describes the principles of wavelet transforms and the concept of wavelet thresholding. Finally numerical results and discussions are given in section 4.

2. Digital Holography

In a typical setup of in-line particle holography, a coherent plane wave propagates through the particle field. The light, scattered by particles, serves as the object wave and the unscattered light serves as the reference wave. In-line holograms record the Fresnel field diffraction pattern of particles. Figure1 shows the hardware components of the digital in-line holography system. The whole setup is quite simple, requiring only one collimated laser beam for generating both object and reference wave. In-line holograms of particles are directly recorded by a CCD camera and then transferred to a computer for numerical reconstruction and particle extraction.

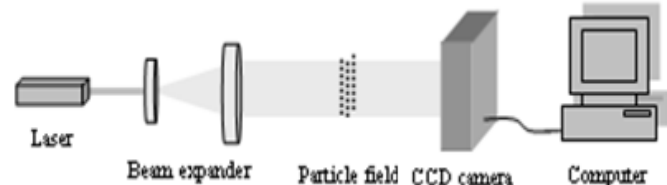


Figure 1: The typical hardware component of DHPIV

2.1. The Recording Stage

Let us consider the formation of an in-line hologram. The object in plane $s(x, y)$ has a pure absorption of $a(x, y)$ and phase shift $\phi(x, y)$

$$s(x, y) = a(x, y) + j\phi(x, y)$$

When the object is illuminated by a reference plane wave, the field emerging from the object is [9,10]

$$u_0(x, y) = 1 - s(x, y)$$

The complex amplitude of the wave in the hologram plane can be calculated by

$$u_z(x, y) = u_0(x, y) * h_z(x, y)$$

Where $*$ denotes the 2-D convolution operation and h_z is the convolution kernel function defined as

$$h_z(x, y) = -\frac{j}{\lambda z} \cdot \exp(j2\pi z / \lambda) \cdot \exp(j\pi(x^2 + y^2) / \lambda z)$$

Where λ is the wavelength of coherent light wave, $k=2$ and z is the distance between the object- plane and the hologram- plane. The exponential term $\exp(jkz)$ is a constant at a given (x, y) plane, and it will only add a constant phase shift to each pixel; thus we will not take this term into account in the following discussion.

The intensity distribution in the hologram plane at distance- z (relative to the hologram plane) can be written as

$$I_z(x, y) = u_z(x, y) \cdot u_z^*(x, y)$$

$$I_z = 1 - s^* * h_z^* - s * h_z$$

Where $*$ denotes the complex conjugate. We note here that we assume $\|s(x, y)\| \ll 1$. These assumptions can be easily satisfied. Indeed, being given that the size of the object is not too large, the absorption and phase shift will be quite small.

2.2. The reconstruction stage

The reconstruction stage permits the determination of intensity and of phase distribution of the object at any arbitrary plane of the investigated volume. From the two in- line holograms $I_z(x, y)$ and $I_{2z}(x, y)$, the object absorption $a(x, y)$ can be calculated by [4]

$$a(x, y) = [(I_{2z} + 1) / 2] - [I_z * \text{Re}(h_z)]$$

Where:

$$I_z = 1 - s^* * h_z^* - s * h_z$$

$$I_{2z} = 1 - s^* * h_{2z}^* - s * h_{2z}$$

$$1 - (s^* + s) = I_z * h_z^* - I_z * h_z - I_{2z}$$

The phase shift $\phi(x, y)$ can be calculated as follow:

$$I_z = 1 - s^* * h_z^* - s * h_z$$

Where $s = a + j\phi$

Then

$$\phi * h_z - \phi * h_z^* = j(1 - a * h_z^* - a * h_z - I_z)$$

If $T = j(1 - a * h_z^* - a * h_z - I_z)$, then

$$T * h_z = \phi * h_{2z} - \phi$$

$$T * h_{3z} = \phi * h_{4z} - \phi * h_{2z}$$

$$T * h_{5z} = \phi * h_{6z} - \phi * h_{4z}$$

$$\sum_1^M T * h_{(2n-1)z} = \phi * h_{2nz} - \phi$$

Finally:

$$\phi = -\sum_{n=1}^M j(1 - a * h_z^* - a * h_z - I_z) * h_{(2n+1)z}$$

with $\phi * h_{2nz} \approx 0$, when the distance $2nz$ becomes too large.

3. Wavelet Transform Denoising

3.1. Discrete Wavelet Transform

Discrete Wavelet Transform (DWT) is a very useful tool in time-frequency analysis because of its excellent localization both in time and frequency. It has been very successful in several areas such as image compression, communication and denoising. A brief description of the principles of wavelet transform [5, 11, and 12] is given here for completeness.

Mallat's algorithm is considered the most important algorithm to calculate DWT coefficients. This algorithm consists of convolutions with the filters defining the scaling and wavelet function and down sampling.

The problem is stated briefly as follows. Given a signal s , we are expected to generate a set of wavelet coefficients and scaling coefficients from a pair of mirror FIR filters. According to wavelet transform, a signal s can be approximated by the smooth version projection onto multiple scaling subspaces and the detailed version projection to the wavelet subspaces, as in

$$s(x) = \sum_k c(j_0, k) \phi_{j,k}(x) + \sum_k \sum_{j=1}^{j_0} d(j, k) \psi_{j,k}(x)$$

$$\phi_{j,k}(x) = 2^{-j/2} \phi(2^{-j} x - k), (j, k) \in Z^2$$

$$\psi_{j,k}(x) = 2^{-j/2} \psi(2^{-j} x - k), (j, k) \in Z^2$$

$\phi_{j,k}(x)$ and $\psi_{j,k}(x)$ are respectively the family associated with scaling function $\phi(x)$ and wavelet function $\psi(x)$ for dyadic scales $a=2^j$ and dyadic translations $b=ka$. They form an orthogonal basis of the signal space. The computation of one scale includes two sets of coefficients: scaling coefficients $c(j_0, k)$ that are the coarse representation of signal where j_0 is a desired scale and wavelet coefficients $d(j, k)$ that are the detailed representation of the signal.

Mallat's algorithm states that the coefficients $c(j_0, k)$ and $d(j, k)$ can be calculated by cascade FIR with a pair of filters: low-pass filter $H(\omega)$ derived from the scaling function and high-pass filter $G(\omega)$ derived from wavelet functions. They are computed from specific wavelet construction methods and they have a symmetric property which makes,

$$g(n) = [-1]^{n+1} h(L + 1 - n)$$

Here $\{h_n\}$ and $\{g_n\}$ are the sequences defining the filters H and G , L is the length of the filters. Filters constructed in this way are called quadrature mirror filters.

The computation constitutes of two steps recursively: initially the original signal $s(n)$ with length N ($N=2^j$) is considered as the scale 0. In the first step, the signal is both low-pass filtered by $H(\omega)$ and high-pass filtered by $G(\omega)$. This first step generated a set of coefficients $c(1, k)$ and $d(1, k)$. In the second step, the coefficients are down-sampled by decimation operator. In the next scale computation, $d(1, k)$ is kept as the final result but $c(1, k)$ is used as the data and decomposed recursively as above procedure until it reaches the desired scale j_0 .

3.2. Wavelet Thresholding

Most of the wavelet denoising techniques are based on the wavelet coefficient shrinkage procedure for discrete wavelet transform [13]. The two more popular thresholding schemes are the so-called soft and hard-thresholding. In both cases, the threshold level is proportional to the noise standard deviation. Hard thresholding consists in putting to zero wavelet coefficients of amplitude smaller than threshold. Soft-thresholding additionally reduces the amplitude of the other coefficients by the threshold quantity. The soft thresholding rule is normally chosen, because it has been shown to achieve near-optimal minimax rate, and the optimal soft thresholding estimator yields a smaller risk than the optimal hard thresholding estimator.

Let W and W^{-1} denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then $y = Ws$ represents the matrix of wavelet coefficients of s having four subbands (LL , LH , HL and HH) [14, 15]. The subbands HH_j , HL_j , LH_j are called details, where j is the scale varying from 1 to J and J is the total number of decompositions. The subband LL_j is the low-resolution residue. The wavelet thresholding denoising method processes each coefficient of y from the detail subbands with a soft threshold function to obtain y^* . The denoised estimate is inverse transformed to $s^* = W^{-1}y^*$. The procedure removes noise by thresholding only the wavelet coefficients of the detail subbands, while keeping the low-resolution coefficients unaltered. We describe here the method for computing the threshold value T_n , which is adaptive to different subband characteristics [16]

$$T_n = \beta \frac{\sigma^2}{\sigma_y}$$

Where, the scale parameter β is computed once for each scale using the following equation

$$\beta = \sqrt{\log(L_k / J)}$$

L_k is the length of the subband at k^{th} scale and J is the total number of decompositions. σ_y is the standard deviation of the subband under consideration. σ is the noise variance, which is estimated from the subband HH_1 , using the formula [14,17]

$$\sigma^2 = \left[\frac{\text{median}(|y_{ij}|)}{0.6745} \right]^2 \quad y_{ij} \in \text{subband}HH_1$$

4. Numerical Results

A number of computer simulations were carried out in which synthetic digital holograms were generated. The investigated volume (Fig.2) is represented by three planes P_1 , P_2 and P_3 which contain tracer particles randomly distributed. The planes (P_1 , P_2 and P_3) separation distance is $\Delta z = 250 \mu m$. The wavelength of illuminating light is $\lambda = 0.63 \mu m$. The particle size is in the range of $20 \mu m$. The number of particles is about 80 by plane. The recording hologram distance (between object plane and hologram plane) is $z = 50 \text{ mm}$. The two holograms I_z and I_{2z} generated by simulation are shown in Fig.3. The size of the holograms is a 256×256 pixel.

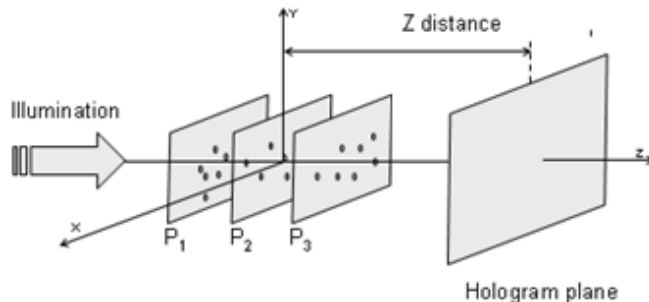
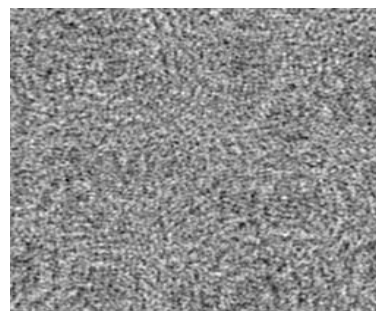
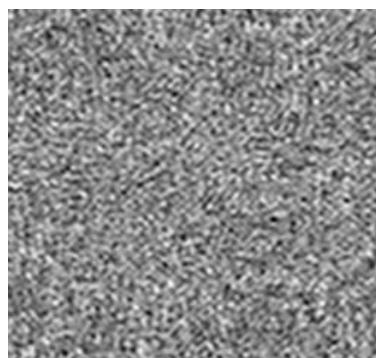


Figure 2: Synthetic volume seeded with tracer particle

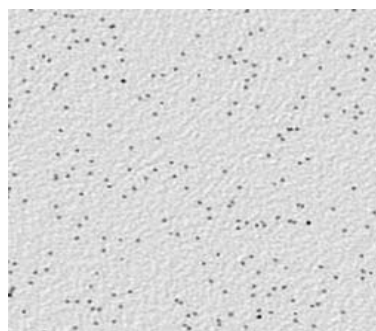


(a)

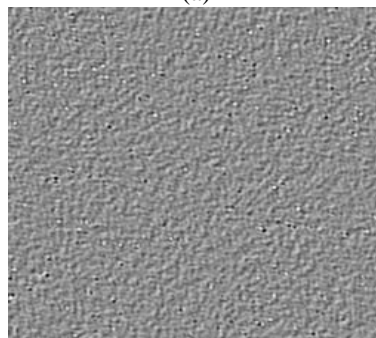


(b)

Figure 3: (a) The hologram I_z and (b) the hologram I_{2z}



(a)



(b)

Figure 4: The reconstructed object (P_2 plane): (a) the absorption and (b) the phase shift

In Fig.4 we present the absorption $a(x, y)$ and phase shift $\phi(x, y)$ of plane P_2 obtained by the two in-line holograms reconstruction algorithm.

The wavelet transform employs Daubachies wavelet with four vanishing moments at four scales of decomposition. After the DWT decomposition, the absorption and phase shift images are denoized by shrinkage the coefficients from the detail Subbands with a Soft-thresholding. The procedure removes noise by thresholding only the wavelet coefficients of the detail subbands HL, LH and HH, while keeping the low- resolution coefficients LL unaltered. The threshold value is adaptive to different subband characteristics. The denoised absorption and phase shift images are shown Fig.5. The wavelet thresholding method, give a good result.

The velocity vector field of each plane is computed by applying PIV (Particle Image Velocimetry) algorithm [18]. The determined velocity vector field of the reconstructed plane P_2 is show Fig.7. The PSNR is equal to 28.65 dB.

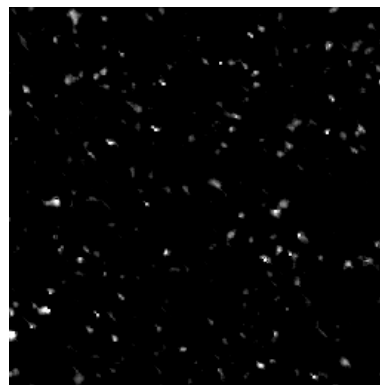
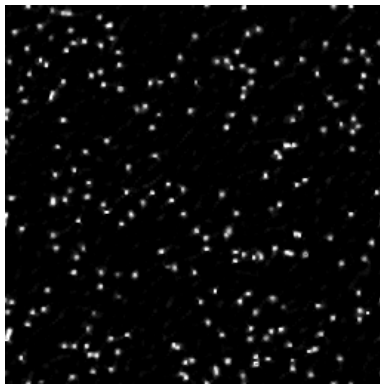


Figure 5: (a) the denoised absorption image and (b) the denoised phase shift image



Figure 6: The reconstructed plane (P_2): particle image

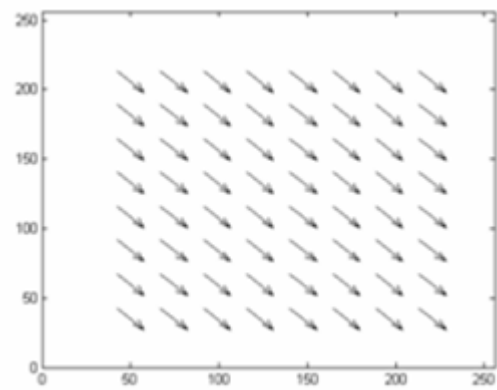


Figure 7: The plane (P_2) velocity field

5. Conclusion

Digital Holographic Particle Image Velocimetry is a promising technique for flow field measurement. However, the low resolution of the sensors still remains poor and needs to be compensated by image processing techniques. In this article, we had presented the two in-line holograms algorithm. This algorithm permits, at the same time, to retrieve the absorption, the phase shift images. The use of the phase shift information to particle localization permits to increase the accuracy along the depth direction. A wavelet thresholding method is employed to remove noise in the reconstructed absorption and phase shift by using a simple subband adaptive threshold. In the computer simulation, we had demonstrated that this method can be applied to flow field measurement. The obtained results are good.

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