Effect of Gravitational Force of Earth on Shock Waves Moving In Non- Uniform Region of Sea Water

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Abstract: *The CCW method has been used to study the effect of gravitational force of earth on shock wave moving in non-uniform region. Assuming the initial density distributional region law as* $\rho_0 = \rho'(1 + xr)$, *Where* ρ', *is the density is the medium, x is constant and r is propagation distance. The flow of variables behind strong shock waves propagating in non-uniform region of water. Considering the effect of gravity of earth. Neglecting the effect of overtaking disturbances, the analytical relations for shock strength (strength of tsunami), shock velocity, particle velocity, pressure and angles are obtained for freely propagation of strong diverging shock. The dependence of shock strength on gravitational force is studied. Finally, the variables are computed and discussed with the help of tables.*

Keywords: Shock waves, Shock strength, gravitational force on earth

1. Introduction

The study of blast wave propagation is now-a-days the target of scientist and medical doctors who are using treatment as tools. Due to its medical and technical applications, many techniques have been used to study the propagation of shock waves in water/liquids/solid/gases by researchers. The first compendium of shock wave profiles was published by Los Alamos National Laboratory (1982).

Thomas[1] used "Energy hypotheses for spherical waves. This hypothesis was successfully applied by **Bhutani**[2] to cylindrical blast waves in magnetogasdynamics. A strong shock wave in a medium with exponentially varying density by **Singh and Mishra**[3]. Propagation of spherical shock waves in water is given by **Singh and Madan.**[4]. **Vishwakarma et al.**[5] considered earth's gravitation and time dependant energy release. The analysis is completely analytical and is used for diverging shocks only. The diverging shock propagation in uniform and non-uniform gas has been studied by **Yadav**[6], **Yadav and Tripathi**[7] considering the effect of overtaking disturbances. These results are in good agreement with experimental results. **Tsytovich et al.**[8] gives on comments Plasma Phys. **Malchanov et al.**[9] Seismo-electromagnetics and related phenomena. History and latest results. **Yadav and Gangwar**[10] have used **Chester**[11]**- Chisnell**[12]**-Whitham**[13] method to study the propagation of strong spherical shock in non-uniform medium neglecting the effect of overtaking disturbances. Being an important phenomenon, it is essential to take into account the effect of overtaking disturbances in the motion of shock waves. The effect of overtaking disturbances, on the propagation of shock waves in water has been studied by **Yadav et al.**[14] .Recently **Yadav et al.**[15] studied the motion of strong shock waves in a highly viscous medium in presence of overtaking disturbances and discussed the flow variables of the perturbed medium excluding temperature and entropy variation, very important variables of the medium.

The purpose of present paper is to analysis the flow variables behind strong shock wave propagating in non - uniform region of water considering the effect of gravity of earth. The Chester-Chisnell-Whitham theory is used to obtain the analytical relations for shock velocity and shock strength for freely propagation of shock. The dependence of shock strength on gravitational force is studied. The shock velocity; the pressure and fluid velocity behind the shock are discussed with the help of tables and graphs.

2. Basic Equations

The basic equations governing the flow behind the shock propagating in water in presence of earth gravity are given by:

$$\frac{\partial \rho}{\partial r} + \frac{\partial}{\partial r}(\rho u) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v) + \frac{\alpha \rho u}{r} + \frac{\cot\theta}{r}\rho v = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + \frac{1}{\rho}\frac{\partial P}{\partial r} + g\cos\theta = 0 \qquad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{1}{\rho r}\frac{\partial P}{\partial \theta} - g\sin\theta = 0 \qquad (3)$$

where, $u(r;t)$ $P(r;t)$, (r,t) and $v(r,t)$ respectively the particle velocity , pressure, density and transversal fluid velocity for the flow . r is the radial coordinate and is the angle made by a shock radius with vertical direction .

3. Boundary Conditions

The jump conditions are given by:

$$u = \frac{2a_0 M}{(n+1)}, P = \frac{2a_0^2 M^2 \rho_0}{(n+1)}, \rho = \left(\frac{n+1}{n-1}\right)\rho_0$$

$$(4)$$

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$$a = S M a_0 \left(\frac{n-1}{n+1}\right) \rho_0, \quad S = \sqrt{\frac{2n}{n+1}}$$

Where, $M = U/a_0$ is called the Mach number, a_0, P, and n denote the undisturbed the value of density, local sound velocity behind the shock strength, pressure the density and the specific heat index of water respectively.

4. Characteristic Equations

The characteristic form of system of equations (1) - (3) is easily obtained by forming a linear combination of two equations (1) and (2) in only direction in the (r,t) plane is;

$$dp + a\rho du + \frac{a\rho}{(u+a)}\left(\frac{\alpha a u}{r} + g\cos\theta\right) dr = 0 \qquad (5)$$

Equation (5) represents characteristic form of diverging shock waves .Using boundary conditions (4) in equation (5), we get -

$$M = \left[\frac{k - \dfrac{B\, g\, \cos\theta \rho' \left[\frac{r^{A+1}}{(A+1)} + \frac{2\, x\, r^{A+2}}{(A+2)(S+2)}\right]}{n}}{\dfrac{r^A[K - \rho' r(1 + xr/2)g\cos\theta]}{(1+xr)^{S/(2+S)}}}\right]^{1/2} \qquad (6)$$

Where, K is constant of integration.

Equation (6) represents shock strength of diverging shock waves propagating in non- uniform medium.

The shock velocity of can be written as;

$$U = a_0 \left[\frac{k - \dfrac{B\, g\, \cos\theta \rho' \left[\frac{r^{A+1}}{(A+1)} + \frac{2\, x\, r^{A+2}}{(A+2)(S+2)}\right]}{n}}{\dfrac{r^A[K - \rho' r(1 + xr/2)g\cos\theta]}{(1+xr)^{S/(2+S)}}}\right]^{1/2} \qquad (7)$$

The particle velocity and pressure just behind the diverging shock wave will be;

$$u = \frac{2a_0}{(n+1)} \left[\frac{k - \dfrac{B\, g\, \cos\theta \rho' \left[\frac{r^{A+1}}{(A+1)} + \frac{2\, x\, r^{A+2}}{(A+2)(S+2)}\right]}{n}}{\dfrac{r^A[K - \rho' r(1 + xr/2)g\cos\theta]}{(1+xr)^{S/(2+S)}}}\right]^{1/2} \qquad (8)$$

$$P = \frac{2a_0^2 \rho_0}{(n+1)} \left[\frac{k - \dfrac{B\, g\, \cos\theta \rho' \left[\frac{r^{A+1}}{(A+1)} + \frac{2\, x\, r^{A+2}}{(A+2)(S+2)}\right]}{n}}{\dfrac{r^A[K - \rho' r(1 + xr/2)g\cos\theta]}{(1+xr)^{S/(2+S)}}}\right]^{1/2} \qquad (9)$$

5. Results and Discussion

Variation of shock strength with gravitational force per unit mass, propagation distance, specific heat index of water and angle.

Expression (6) represents the shock strength of spherical shock wave propagating freely in non-uniform region of water. It is found that shock strength is a function of propagation distance r, specific heat index of water n, density ρ, angle cos θ and force per unit mass i.e. Gravity.

Initially taking M=10 at r=2.2 for n=7.5, x=1.22, $\rho' = 1$ and $\alpha=2$, we have computed shock strength using equation (6) and obtained results are presenting in following tables (1),(2),(3)and(4).

Table 1: The variation of shock strength with gravitational force per unit mass i.e. Gravity	
Gravity (g)	*Shock strength (M)*
9.62	4.1079
9.64	4.3096
9.66	4.5448
9.68	4.8241
9.70	5.1632
9.72	5.5869
9.74	6.1371
9.76	6.8927
9.78	8.0240
9.80	10.000
9.82	14.984

Table 2: The variation of shock Strength with propagation distance	
Propagation distance (r)	*Shock strength (M)*
2.10	2.3715
2.11	2.4761
2.12	2.5996
2.13	2.7481
2.14	2.9311
2.15	3.1635
2.16	3.4719
2.17	3.9071
2.18	4.5864
2.19	5.8678
2.20	10.000

Table 3: The variation of shock strength with specific heat index of water	
Specific heat index of water (n)	*Shock strength (M)*
7.3	10.0296
7.4	10.014
7.5	10.000
7.6	9.9857
7.7	9.9718
7.8	9.9583
7.9	9.9452
8.0	9.9323
8.1	9.9198
8.2	9.9076
8.3	9.8956
8.4	9.8840
8.5	9.8726

Table 4: The variation of shock strength with angle		
Angle θ	*cos θ*	*Shock strength (M)*
0^0		
15^0	1.0000	10.0000
20^0	0.9659	3.1539
25^0	0.9397	2.4437
30^0	0.9063	2.0014
35^0	0.8660	1.7040
40^0	0.8192	1.4926
45^0	0.7660	1.3353
50^0	0.7070	1.2150
55^0	0.6428	1.1208
60^0	0.5736	1.0453
65^0	0.5000	0.9840
70^0	0.4230	0.9336
	0.3420	0.8913

It is found that changing value of g increases9.62 to 9.82, the shock strength takes the values from 4.1079 to 14.9842. It shows that as 'g' increases, shock strength M increases and changing value of r from 2.10 to 2.20, the shock strength takes the values from 2.3715 to 10.0000. It shows that as propagation distance (r) increases, shock strength (M) increases and changing the value of n from 7.3 to 8.5, the shock strength takes the values from10.0296 to 9.8726 which show that as specific heat index of water

(n) increases, shock strength (M) decreases. The variation of shock strength with angle cos θ is computed. It is found that changing the values from 0^0 to 70^0, the shock strength takes the values from 10.0000 to 0.8913.It shows that as increases, shock strength (M) decreases.

Variation of shock velocity with gravitational force per unit mass, propagation distance, specific heat index of water and angle:

Expression (7) represents the shock velocity of spherical shock wave propagating freely in non-uniform region of water. It is found that shock velocity is a function of propagation distance r, specific heat index of water n, density ρ, angle cos θ and force per unit mass i.e. Gravity.

Initially taking M=10 at r=2.2 for n=7.5, x=1.22, ρ'= 1 and α=2, we have computed shock strength using equation (7) and obtained results are presenting in following tables (5),(6),(7) and (8).

Table 5: *The variation of shock velocity with gravitational force per unit mass i.e. Gravity.*		Table 6: *The variation of shock velocity with propagation distance*	
Gravity (g)	*Shock velocity (U)*	*Propagation distance (r)*	*Shock velocity (U)*
9.62	6.1837	2.10	6.6516
9.64	6.1796	2.11	6.6007
9.66	6.1756	2.12	6.5498
9.68	6.1715	2.13	6.4991
9.70	6.1674	2.14	6.4485
9.72	6.1633	2.15	6.3980
9.74	6.1592	2.16	6.3476
9.76	6.1551	2.17	6.2973
9.78	6.1510	2.18	6.2471
9.80	6.1469	2.19	6.1970
9.82	6.1428	2.20	6.1469

Table 7: The variation of shock velocity with specific heat index of water.	
Specific heat index of water (n)	*Shock velocity (U)*
7.3	6.0824
7.4	6.1147
7.5	6.1469
7.6	6.1790
7.7	6.2108
7.8	6.2426
7.9	6.2742
8.0	6.3056
8.1	6.3369
8.2	6.3680
8.3	6.3990
8.4	6.4299
8.5	6.4606

Table 8: The variation of shock velocity with angle.		
Angle θ	*cos θ*	*Shock velocity (U)*
0^0		
15^0		
20^0	1.0000	6.1469
25^0	0.9659	6.2151
30^0	0.9397	6.2669
35^0	0.9063	6.3323
40^0	0.8660	6.4104
45^0	0.8192	6.5000
50^0	0.7660	6.6002
55^0	0.7070	6.7097
60^0	0.6428	6.8268
65^0	0.5736	6.9509
70^0	0.5000	7.0804
	0.4230	7.2134
	0.3420	7.3508

It is found that changing value of g increases9.62 to 9.82, the shock velocity takes the values from 6.1837 to 6.1428. It shows that as 'g' increases, shock velocity U decreases and changing value of r from 2.10 to 2.20, the shock velocity takes the values from 6.6516 to 6.1469 .It shows that as propagation distance (r) increases, shock velocity (U) decreases and changing the value of n from 7.3 to 8.5, the shock velocity takes the values from 6.0824 to 6.4606 which shows that as specific heat index of water (n) increases, shock velocity (U) increases. The variation of shock velocity with angle cos is computed. It is found that changing the values from 0^0 to 70^0, the shock velocity takes the values from 6.1469 to 7.3508.It shows that as increases, shock velocity (U) increases.

Variation of particle velocity with gravitational force per unit mass, propagation distance, specific heat index of water and angle:

Expression (8) represents the particle velocity of spherical shock wave propagating freely in non-uniform region of water. It is found that particle velocity is a function of propagation distance r, specific heat index of water n, density ρ, angle cos θ and force per unit mass i.e. Gravity.

Initially taking M=10 at r=2.2 for n=7.5, x=1.22, ρ'= 1 and α=2 we have computed particle velocity using equation (8) and obtained results are presenting in following tables (9), (10), (11) and (12).

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Table 9: The variation of particle velocity with gravitational force per unit mass i.e. Gravity.

Gravity (g)	Particle velocity (u)
9.62	1.4550
9.64	1.4540
9.66	1.4530
9.68	1.4521
9.70	1.4511
9.72	1.4502
9.74	1.4492
9.76	1.4482
9.78	1.4473
9.80	1.4463
9.82	1.4453

Table 10: The variation of particle velocity with propagation distance.

Propagation distance (r)	Particle velocity (u)
2.10	1.5650
2.11	1.5531
2.12	1.5411
2.13	1.5292
2.14	1.5173
2.15	1.5054
2.16	1.4935
2.17	1.4817
2.18	1.4699
2.19	1.4581
2.20	1.4463

Table 11: The variation of particle velocity with specific heat index of water

Specific heat index of water (n)	Particle velocity (u)
7.3	1.4656
7.4	1.4559
7.5	1.4463
7.6	1.4369
7.7	1.4277
7.8	1.4187
7.9	1.4099
8.0	1.4012
8.1	1.3927
8.2	1.3843
8.3	1.3761
8.4	1.3680
8.5	1.3601

Table 12: The variation of particle velocity with angle

Angle θ	cos θ	Particle velocity (u)
0^0	1.0000	1.4463
15^0	0.9659	1.4623
20^0	0.9397	1.4745
25^0	0.9063	1.4899
30^0	0.8660	1.5083
35^0	0.8192	1.5294
40^0	0.7660	1.5530
45^0	0.7070	1.5787
50^0	0.6428	1.6063
55^0	0.5736	1.6355
60^0	0.5000	1.6659
65^0	0.4230	1.6972
70^0	0.3420	1.7296

It is found that changing value of g increases9.62 to 9.82, the particle velocity takes the values from 1.4550 to 1.4453. It shows that as 'g' increases, particle velocity u decreases and changing value of r from 2.10 to 2.20, the particle velocity takes the values from 1.5650 to 1.4463.It shows that as propagation distance (r) increases, particle velocity (u) decreases and changing the value of n from 7.3 to 8.5, the particle velocity takes the values from1.4656 to 1.3601 which shows that as specific heat index of water (n) increases, particle velocity (u) decreases. The variation of particle velocity with angle cos is computed. It is found that changing the values from 0^0 to 70^0, the particle velocity takes the values from 1.4463 to 1.7296.It shows that as increases, particle velocity (u) increases.

Variation of pressure with gravitational force per unit mass, propagation distance, specific heat index of water and angle:

Expression (9) represents the pressure of spherical shock wave propagating freely in non-uniform region of water. It is found that pressure is a function of propagation distance r, specific heat index of water n, density ρ, angle cos θ and force per unit mass i.e. Gravity.

Initially taking M=10 at r=2.2 for n=7.5, x=1.22, ρ'= 1 and α=2 we have computed pressure using equation (9) and obtained results are presenting in following tables (13), (14), (15) and (16).

Table 13: The variation of pressure with gravitational force per unit mass i.e. Gravity

Gravity (g)	Pressure (P)
9.62	33.1464
9.64	33.1027
9.66	33.0590
9.68	33.0153
9.70	32.9716
9.72	32.9280
9.74	32.8843
9.76	32.8406
9.78	32.7969
9.80	32.7532
9.82	32.7095

Table 14: The variation of pressure with propagation distance

Propagation distance (r)	Pressure (P)
2.10	37.0821
2.11	36.6412
2.12	36.2022
2.13	35.7651
2.14	35.3297
2.15	34.8961
2.16	34.4643
2.17	34.0341
2.18	33.6055
2.19	33.1786
2.20	32.7532

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Table 15: The variation of pressure with specific heat index of water.

Specific heat index of water (n)	Pressure (P)
7.3	32.8418
7.4	32.7969
7.5	32.7532
7.6	32.7106
7.7	32.6692
7.8	32.6287
7.9	32.5894
8.0	32.5510
8.1	32.5135
8.2	32.4770
8.3	32.4413
8.4	32.4065
8.5	32.3725

Table 16: The variation of pressure with angle

Angle θ	cos θ	Pressure (P)
0^0		
15^0		
20^0	1.0000	32.7532
25^0	0.9659	33.4831
30^0	0.9397	34.0440
35^0	0.9063	34.7589
40^0	0.8660	35.6216
45^0	0.8192	36.6234
50^0	0.7660	37.7622
55^0	0.7070	39.0251
60^0	0.6428	40.3994
65^0	0.5736	41.8807
70^0	0.5000	43.4561
	0.4230	45.1044
	0.3420	46.8382

It is found that changing value of g increases 9.62 to 9.82, the pressure takes the values from 33.1464 to 32.7095. It shows that as 'g' increases, pressure P decreases and changing value of r from 2.10 to 2.20, the pressure takes the values from 37.0821 to 32.7532 .It shows that as propagation distance (r) increases, pressure (P) decreases and changing the value of n from 7.3 to 8.5, the pressure takes the values from32.8418 to 32.3725 which shows that as specific heat index of water (n) increases, pressure (P) decreases. The variation of pressure with angle cos is computed. It is found that changing the values from 0^0 to 70^0, the pressure takes the values from 32.7532 to 46.8382.It shows that as increases, shock pressure (P) increases.

6. Conclusions

It is concluded that shock strength (i.e. strength of tsunami) is depends on distance moves by tsunami, specific heat index of water 'n', density of water and earth gravity. It is found that:

1. Shock strength increases as gravity of earth increases.
2. Shock velocity decreases with increase in gravity.
3. The pressure and particle velocity behind the shock front decreases with the increases in gravitational force.
4. Strength of tsunami increases as it moves from it sources.
5. Shock velocity decreases as shock advances.
6. The pressure and particle velocity both decreases with propagation distance.
7. Strength of tsunami decreases as specific heat index of water increases.
8. The pressure and particle velocity decreases as specific heat index of water increases.
9. Strength of tsunami decreases as angle θ increases.
10. Shock velocity, the pressure and particle velocity increase with angle θ

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