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Flow in Convergent and Divergent Tubes of Varying Cross Section with Permeable Wall

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Abstract: ***In this paper, we study the low Reynolds number steady flow in a tube of varying cross section with permeable wall. The fluid is assumed to be incompressible and Newtonian. The wall assumed to be rigid and permeable. The wall permeability is assumed to be a function of axial distance and obeys Starling's Law. we are interested to analyze the effects of Reynolds number and permeability on flow characteristics when the initial flux in the tube is prescribed. The effect of variable permeability of the wall on various parameters on flow characteristics is discussed.***

Keywords: Reynolds number, Incompressible fluid, Newtonian fluid, Starling's Law

1. Introduction

Flow in tubes of varying cross-section is a good area for research work due to its importance in physiological and engineering flow problems. In particular, it plays a significant role in understanding the flow in blood vessels. Most of these studies have considered the tube walls to be impermeable. J.S.V.R. Krishna Prasad and Peeyush Chandra have investigated flow through tubes of uniform cross-section and permeable wall. It has wide range of application in engineering problems such as water purification, desalinization, gel filtration, cooling of jet engines and nuclear reactors etc.

The fluid exchange across the tube wall is accounted either by prescribing normal fluid velocity at the wall which is equivalent to prescribing flow flux at different cross-sections of the tube or through starling's law which states that normal velocity of the fluid at the wall is proportional to the pressure difference across the vessel wall.

In this paper, we study low Reynolds number flow in convergent and divergent tubes of varying cross section with permeable wall. Further, we assume that wall permeability K is a function of axial distance. An initial value problem is formulated where flux and mean pressure at the initial cross section have been prescribed. We are interested to study the effects of Re and K on flow characteristics.

2. Formulation of the Problem

Consider steady flow of a Newtonian incompressible fluid in an axisymmetric tube of varying cross-section with permeable wall [16] and [17]. Using cylindrical polar coordinates (X, R, θ) where R = O is the axis of symmetry for the tube, the equations of motion and continuity are given as

$$UU_X + V\,U_R = -\frac{1}{\rho}P_X + \nu[U_{XX} + (RU_R)_R\;/R] \quad (1)$$

$$UU_X + V\,V_R = -\frac{1}{\rho}P_X + \nu[\,V_{XX} + \frac{1}{R}(RV_R)_R - \frac{V}{R^2}] \quad (2)$$

$$U_X + (RV)_R / R = 0 \quad (3)$$

Where (U, V) are the fluid velocity components in (X, R) directions respectively, P is the pressure, ν is the kinematic coefficient of viscosity and ρ is the constant fluid density. We consider tube of slowly varying cross-section, and hence, the radius of the tube R = a (X) is given as :

$$a(x) = a_0 S(\varepsilon X / a_0)$$
$$\varepsilon = a_0 / L << 1,\ S(O) = 1, \quad (4)$$

Where ε is the wall variation parameter, a_0 is the tube radius at the initial cross-section, L is the characteristic length and $S(\varepsilon X / a_0)$ is an arbitrary function of X. The fluid exchange across the permeable wall is given by Starling's law and the net external pressure acting on the surface of the wall is assumed to be constant. This gives the normal fluid velocity at the tube wall as :

$$V - a_X U = K(P - P_{ext}) \text{ at } R = a(X). \quad (5)$$

The tangential velocity of the fluid at the wall is zero, hence,

$$U + a_X V = 0 \text{ at } R = a(X) \quad (6)$$

The axisymmetry of the flow implies

$$U_R = 0 \;\; V=0 \text{ at } R = 0. \quad (7)$$

Further, we prescribe the mean pressure P_{mean} i. e.

$$P_{mean} = \frac{1}{\pi a^2(x)}\int_0^{a(x)} 2\pi R\, P\, dR \quad (8)$$

$$\text{And the flux Q, } Q = \int_0^{a(x)} 2\pi R\, U\, dR \quad (9)$$

At the initial cross-section (X = 0) as P_{in} and Q_0 respectively, which gives

$$\left.\begin{array}{l} P_{mean} = P_{in} \\ Q = Q_0 \end{array}\right\} \text{ at } X = 0. \quad (10)$$

The wall permeability is assumed to be a function of axial distance

K(X) = m (1+nX)

Where m and n are real constants less than 1. It may be noted that when n=0, our case reduces to constant permeability as given by [16] and [17].

3. Analysis and Method of Solution

Using the non-dimensional quantities,

$$x = \varepsilon X / a_0 \quad , \quad r = R / a_0 \quad , \quad u = 2\pi\, a_0^2\, U / Q_0 \quad ,$$

$$v = 2\pi\, a_0^2\, V / \varepsilon Q_0 \quad , \quad (p, p_{ext}) = 2\pi\, a_0^3 \varepsilon (P, P_{ext}) / \nu\rho Q_0 ,$$

$$k = \nu\rho K / \varepsilon^2 a_0 \quad , \quad q = Q / Q_0 \; ,$$

and the perturbation technique in terms of parameter ε with

$$(u,v,p,q) = (u^{(0)}, v^{(0)}, p^{(0)}, q^{(0)}) + \varepsilon(u^{(1)}, v^{(1)}, p^{(1)}, q^{(1)}) + O(\varepsilon^2)$$

, we get

$$u^{(0)} = \frac{1}{4} p_x^{(0)} (r^2 - S^2) \qquad (11)$$

$$v^{(0)} = -\frac{1}{16} r[p_{xx}^{(0)} (r^2 - 2S^2) - 4SS_x p_x^{(0)}] \qquad (12)$$

first order velocity components as follows :

$$u^{(1)} = \frac{1}{4} p_x^{(1)} (r^2 - S^2)$$

$$+\frac{R_e}{2304} p_x^{(0)} [p_{xx}^{(0)} (2r^6 - 9r^4S^2 + 36r^2S^4 - 29S^6) + 72S^3S_x p_x^{(0)} (r^2 - S^2)] \qquad (13)$$

$$v^{(1)} = -\frac{1}{16} r[p_{xx}^{(1)} (r^2 - 2S^2) - 4S\, S_x p_x^{(1)}]$$

$$-\frac{R_r}{9216} r[p_{xx}^{(0)^2} + p_x^{(0)} p_{xxx}^{(0)})(r^6 - 6r^4S^2 + 36r^2S^4 - 58S^6)$$

$$+12SS_x p_x^{(0)} p_{xx}^{(0)} (24r^2S^2 - r^4 - 41S^4)$$

$$+72S^2 p_x^{(0)^2} \{S_{xx}(r^2 - 2S^2) + S_x^2 (3r^2 - 10S^2)\}]. \qquad (14)$$

4. Flow Rate and Wall Shear Stress

The non-dimensional volumetric flow rate (q) Wall shear stress is given by:

$$q = \int_0^{S(x)} r\, u\, dr$$

$$q = -\frac{S^4}{16} [p_x^{(0)} + \varepsilon\{p_x^{(1)} + \frac{R_e}{16} S^3 p_x^{(0)} (12k(p^{(0)} - P_{ext}) - S_x p_x^{(0)})\}] + O(\varepsilon^2). \qquad (15)$$

The wall shear stress in non-dimensional form is given as

$$T_w = 2\pi\, a_0^3 \tau_w / \rho\nu Q_0 \qquad (16)$$

$$T_w = \frac{S}{2} p_x^{(0)} + \varepsilon[p_x^{(1)} + \frac{R_e}{24} S p_x^{(0)} \{16k(p^{(0)} - p_{ext}) - S^2 S_x p_x^{(0)}\}] + O(\varepsilon^2). \qquad (17)$$

5. Calculation of Pressure

Here, the expressions for various flow variables are given in terms of $p^{(0)}, p^{(1)}$ and their derivatives. These flow variables can be determined once $p^{(0)}$ and $p^{(1)}$ are evaluated. The equation governing pressure is obtained through Starling's law.

Thus, using conditions expression for $v^{(0)}$ and $v^{(1)}$, we get the following differential equations for $p^{(0)}$ and $p^{(1)}$,

$$p_{xx}^{(0)} + 4\frac{S_x}{S} p_x^{(0)} - 16\frac{k}{S^3}(p^{(0)} - p_{ext}) = 0 \qquad (18)$$

$$p_{xx}^{(1)} + 4\frac{S_x}{S} p_x^{(1)} - \frac{16k}{S^3} p^{(1)}$$

$$= -\frac{R_e}{64} S^2 [3S^2 (p_x^{(0)^2} + p_x^{(0)} p_{xxx}^{(0)}) + 40SS_x p_x^{(0)} p_{xx}^{(0)} + 8p_x^{(0)^2} (SS_{xx} + 7S_x^2)]. \qquad (19)$$

$$p^{(0)} = p_{in} \; , \; p_x^{(0)} = -16 \qquad (20)$$

$$p^{(1)} = 0 \, , p_x^{(1)} = 4R_e [3k(p^{(0)} - p_{ext}) + 4S_x] \qquad (21)$$

The differential eqns. (21) and (22) with initial conditions (23) and (24) form two point initial value problems for $p^{(0)}$ and $p^{(1)}$. for a given tube geometry, these equations can be solved and the mean pressure drop Δp at a given cross-section:

$$\Delta p = p_{mean}^{(0)} - p_{mean}^{(x)} = p_{in} - p^{(0)}(x) - \varepsilon p^{(1)}(x) + O(\varepsilon^2) \qquad (22)$$

Can be calculated.

6. Numerical Solution

In general, analytical solutions of the equations (18), (19) are not feasible and equations have to be solved numerically for a given S(x). however, in a particular case of S(x) $= e^{mx}$,Convergent and Divergent tubes .It is possible to find analytic solution for $p^{(0)}$ analytically. But in this case also, it becomes very tedious to solve for $p^{(0)}$ analytically. In view of this, fourth order R-K Method is used to evaluate $p^{(0)}$ and $p^{(1)}$ numerically. Hence, we evaluate the expressions flow rate (Q) and wall shear stress $|T_W|$.

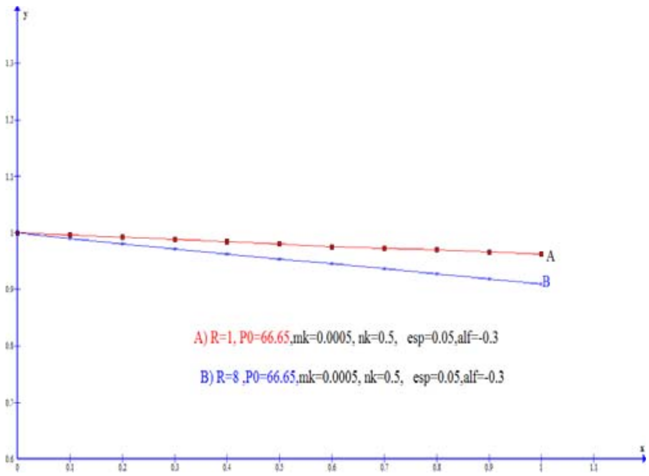

Figure 1: Flow rate Q vs axial distance X for convergent tube with uniformly increasing permeability

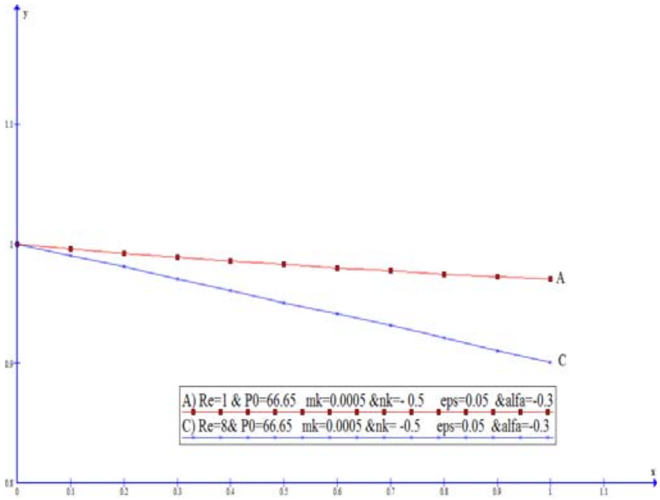

Figure 2: Flow rate Q vs axial distance X for convergent tube with uniformly decreasing permeability.

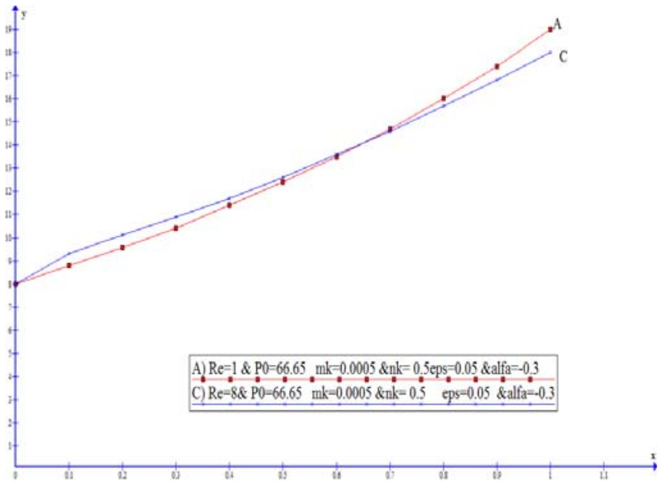

Figure 3: Wall shear stress Tw Vs axial X distance for convergent tube with uniformly increasing permeability.

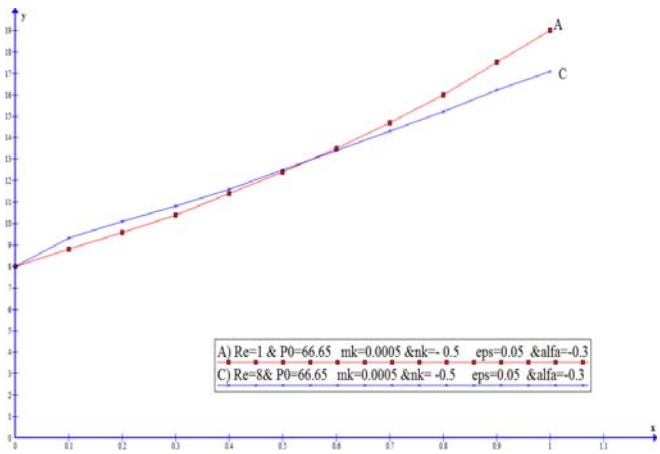

Figure 4: Wall shear stress Tw Vs axial X distance for convergent tube with uniformly decreasing permeability.

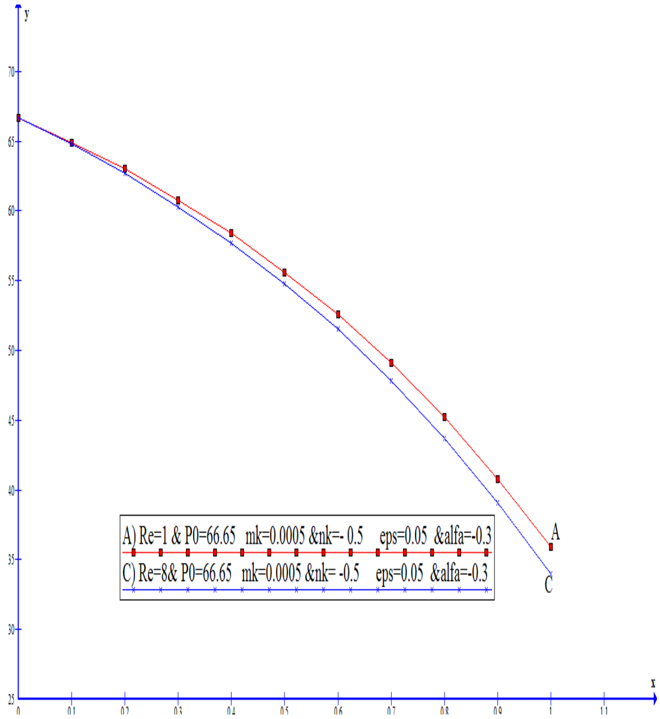

Figure 5: Pressure P Vs axial X distance for convergent tube with uniformly increasing permeability.

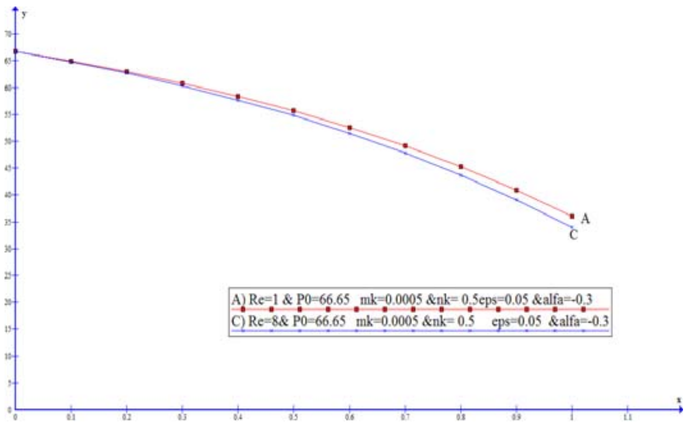

Figure 6: Pressure P Vs axial X distance for convergent tube with uniformly decreasing permeability.

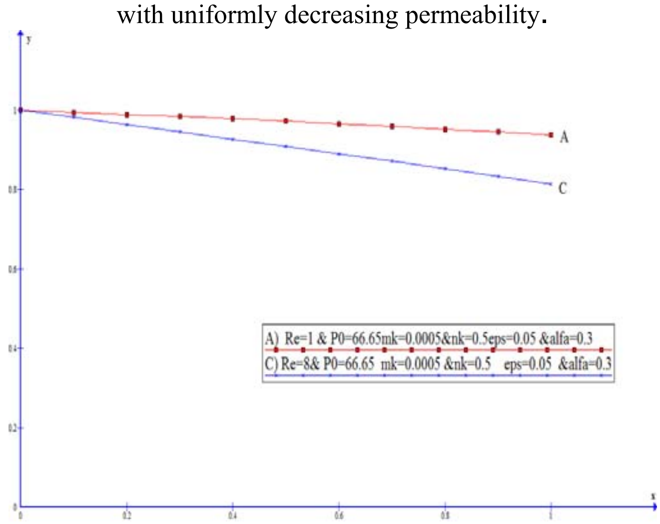

Figure 7: Flow rate Q vs axial distance X for divergent tube with uniformly increasing permeability.

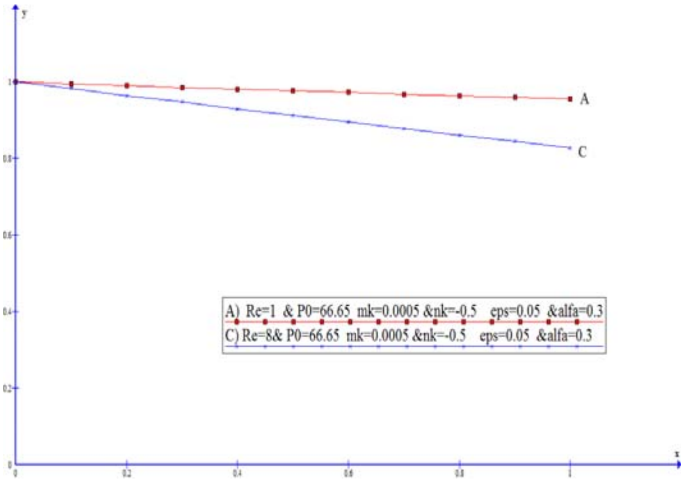

Figure 8: Flow rate Q vs axial distance X for divergent tube with uniformly decreasing permeability.

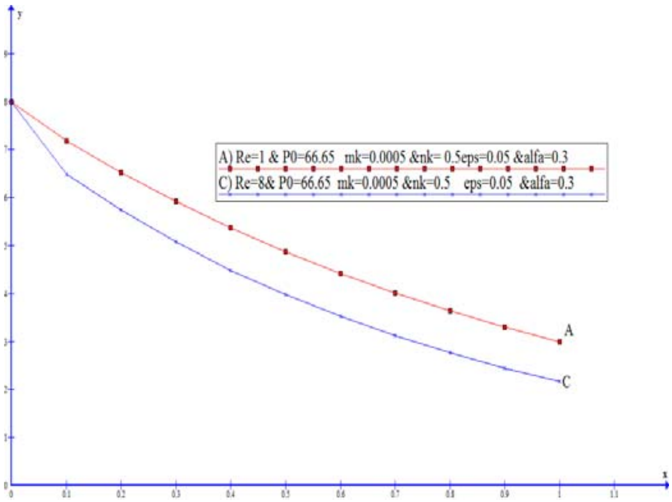

Figure 9: Wall shear stress Tw Vs axial X distance for divergent tube with uniformly increasing permeability.

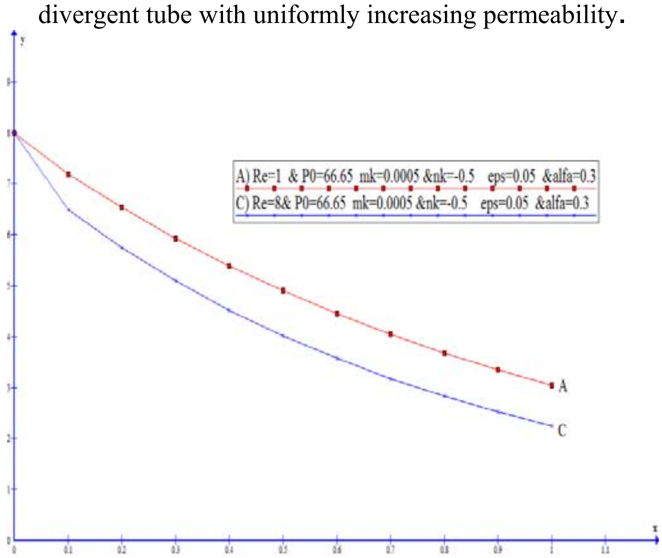

Figure 10: Wall shear stress Tw Vs axial X distance for divergent tube with uniformly decreasing permeability.

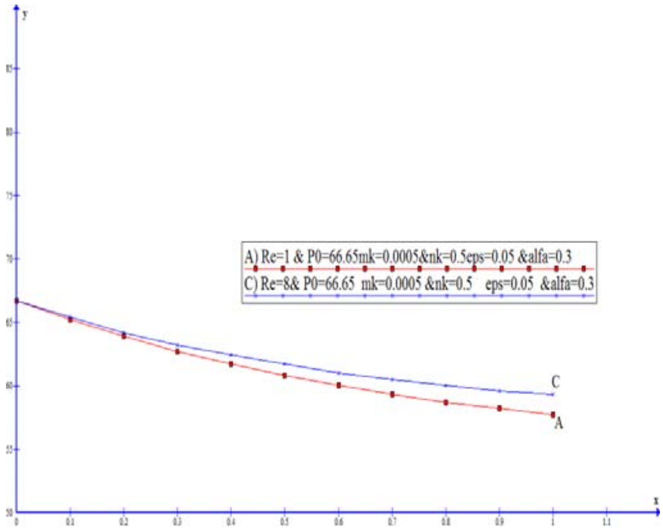

Figure 11: Pressure P Vs axial X distance for divergent tube with uniformly increasing permeability.

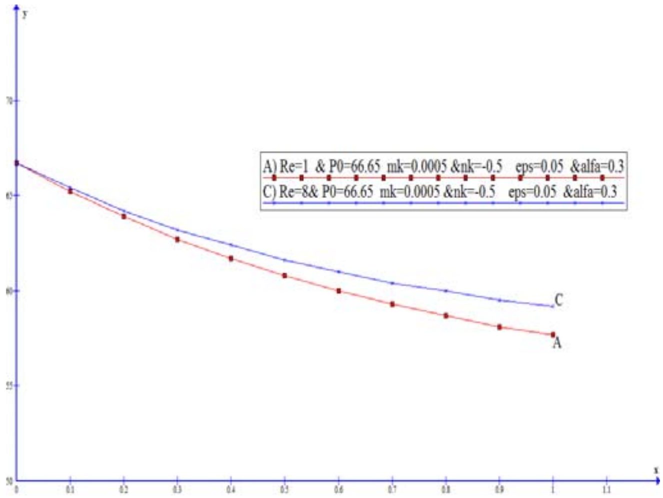

Figure 12: Pressure P Vs axial X distance for divergent tube with uniformly decreasing permeability.

7. Conclusion

Using numerical values of $P^{(0)}$ and $P^{(1)}$ and their derivatives, value of flow rate (Q) and wall shear stress (ITwI) are calculated. We have taken ϵ=0.05 for numerical calculation. The numerical solutions obtain by fourth order using R-k method.

In this paper, we have considered effect of wall permeability (Kp) and Reynolds number Re on wall shear stress, pressure and flow flux for convergent and divergent tube. The flow flux decreases in both convergent and divergent tube as permeability increases or decreases. The pressure drop decreases as permeability increases or decreases. However, wall shear stress ITwI increases in convergent tube and decreases in divergent tube as axial distance increases and pressure drop become decreases.

For convergent tube for small value of Re wall shear stress increases as Re increases wall shear stress oscillates .but in divergent tube as Re increases wall shear stress decreases.

References

[1] Berman, A.S. (1953), "Laminar flow in channels with porous walls", J. Appl. Phys., 24,p.71
[2] Berman, A.S. (1956), "Concerning laminar flow in channels with porous walls", J.Appl.Phys., 27,p.1557
[3] Bestman, A.R. (1981), "Low Reynolds number non-Newtonian flow in slowly varying asymmetrical tubes", Acta Machanik,44,p.107
[4] Chandra, Peeyush and Radhakrishnamacharya, G. (1983), "Viscous flow through a permeable tube of varying crossection with reference to proximal renal tubule", Proc. I International Conference on Physiol. Fluid Dyn., ed. Nigam, S.P. and Meghasingh, p.113
[5] Colgan, T. and Terril, R.M. (1989), "An investigation of some axisymmetric flows through circular pipes of slowly changing radius", Int. J. Engng. Sci. 27, p.433
[6] Manton, M.J. (1971), "Low Reynolds number flow in slowly varying axisymmetric tubes", J.Fluids Mech.,49,p.401
[7] Qualie, J.P. and Levy, E.K. (1975), "Laminar flow in a porous tube with suction", J.heat Transfer, 97,p.66
[8] Radhakrishnamacharya, G. (1978), "Pulsatile flow of a dusty fluid through a constricted channel", Z.A.M.P., 29,p.217
[9] Verma, P.D. and Sacheti, N.C. (1975), "On two dimensional flow of power law fluids through ducts with naturally permeable walls", Z.A.M.M.,55,p.475
[10] Terril, R.M. and Shreshta, G.M. (1965), "Laminar flow through parallel and uniformly porous walls of different permeability", Z.A.M.P., 16,p.470
[11] White, F.M. (1962), "Laminar flow in uniformly porous pipe", J. Appl. Mech., 29, p.203
[12] Friedman, M. and Gillis, J. (1967), "Viscous flow in a pipe with absorbing walls", J. Appl. Mech., 34,p.819
[13] Radhakrishnamacharya, G. ,Peeyush Chandra and Kaimel, M.R. (1981), "A hydrodynamical study of the flow in renal tubule", Bull. Math Biol., 43,p.151
[14] J.S.V.R. Krishna Prasad and Peeyush Chandra (1990), "Low Reynolds number flow in channel with varying cross section and permeable boundaries", Biomechanics, p. 39-45, Ed, K.B. Sahay and B.K. Saxena, Wiley Eastern, New Delhi
[15] J.S.V.R. Krishna Prasad(1990), "Flow in channel of varying cross section with permeable boundaries", Proc. Nat. Acad. Sci. India, 60 (A), III, p.317-326
[16] J.S.V.R. Krishna Prasad and Peeyush Chandra (1992), "Low Reynolds number flow in tubes of varying cross section with absorbing walls", J.Math. Phy. Sci., 26, No. 1, p 19-36
[17] J.S.V.R. Krishna Prasad and Peeyush Chandra (1992), "Flow through circular tubes of non-uniform cross section with permeable walls", physicological fluid dynamics III, Ed. N.V.C.Swamy and MeghaSngh, Narosa Publications, p.165-170

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