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Maximization of the Fishermen's Profits Exploiting a Fish Population in Several Fishery Zones

Kanza CHOUAYAKH[1, 2], Chakib EL BEKKALI[1], Youssef EL FOUTAYENI[2], Mohamed KHALADI[3, 4], Mostafa RACHIK[2]

[1]Computing, Imaging and Digital Analysis Laboratory, University Sidi Mohamed Ben Abdellah Fez, Morocco

[2]Analysis, Modeling and Simulation Laboratory, University Hassan II. Faculty of Sciences Ben M'sik, PO Box: 7955 Sidi Othman. Casablanca, Morocco

[3]Mathematical Populations Dynamics Laboratory, University Cadi Ayyad Marrakech, Morocco

[4]UMI UMMISCO, IRD - UPMC, France

Abstract: ***In this paper, we make a mathematical study of a bio-economic model of fishing for multi-site, exploiting by several fishermen, except one of them which is defined as not exploitable free fishing zone. This mathematical analysis allows maintaining an ecological balance by implementation of sustainable development. The objective of our work is to maximize the profit of fishermen according to the efforts of fishing at biological equilibrium. This model contains three parts, a biological part connecting captures with the sites of biomasses, an exploitation part connecting captures with efforts of fishing and finally an economic part connecting efforts of fishing with profits.***

Keywords: Bio-economic model, Multi-site fishery, Migration, Aggregation method, Biological equilibrium, Generalized Nash Equilibrium, Maximization of fishermen's profits.

1. Introduction

The overexploitation is mostly caused by overfishing as a consequence of economic incentive for fishermen to maximize their profit instead of investing in the conservation of their exploited fish stocks. Therefore, the modeling of the commercial exploitations of renewable resource representing a challenging task, because it involves the nonlinear interaction of biological, economic, social components and much uncertainty. Many problems of mathematical models have been developed to describe the dynamics of fisheries, see H. S. Gordon [11], C. W. Clark [3], [4], P. Auger et al. [2]. In recent years, many researchers have studied fishing activity on different spatial zones connected by migrations, considered as an artificial pelagic multisite fishery on fish aggregating (FADs) or on artificial habitats (AHs). This model includes two time scales; a fast one associated to quick movements between the fishing zones, on the contrary a slow one corresponding to the growth of the fish population. We take this two time scales to construct a reduced model by using the aggregation methods. In this context, there is the work of R. Mchich et al. [15] where they optimize the spatial distribution of the fishing effort and the identification of an efficient set of management measures, which corresponds in one hand to set an appropriate system of tax and/or subsidies, and on the other hand to control the displacement of the fleets between the fishing zones, in order to increase the total activity. Among other works, there is also the work of P. Auger et al. [16] where they present a stock-effort of a dynamical model of fishery subdivided into fishing zones, where they obtain either a stable equilibrium or a stable limit cycle which involves large cyclic variations of the total fish stock and fishing effort and finally, they show that there exists an optimal number of fishing zones that maximizes the total catch at equilibrium.

In this paper, we study fishing activity on several zones connected with a free stock by the migrations. The purpose of this paper is to consider a bio-economic equilibrium model of a fish population exploited by several fishing fleets represented by their fishing efforts $(E_i)_{i=1,\cdots,n}$, where n is the number of the fishing fleets. The objective is to find the fishing effort E_i^* which maximize each fisherman's profit, at biological equilibrium, without any consultation between the fishing fleets but, all of them have to respect two constraints. The first one is the sustainable management of the resources; the second one is the preservation of the biodiversity. Each fishing fleets strives to maximize its profit by choosing a fishing effort strategy. With all these considerations, the problem leads to a generalized Nash Equilibrium Problem. It is very interesting to note that the fishing effort $E^* = (E_1^*,\cdots,E_n^*)$ will depends on: (a) the catchability coefficients q_i, (b) the costs of fishing c_i and (c) the price of fish population p. It allows us to discuss the trends of individual fishing effort in terms of competitiveness.

This paper is organized as follow. In the next section, we present the model which consists in a system of two ordinary differential equations governing the N number of fishing zones and free stock. After that, we present a bio-economic equilibrium model which describes the dynamics of fish population and exploited by several fishermen seeking to maximize their profits. And finally, we prove that the resolution of bio-economic equilibrium model is equivalent to solve a Generalized Nash Equilibrium Problem GNEP. In section 3, we show that the last problem GNEP has a unique solution and we compute this solution. In Section 4, we give discussion and we conclude with some potential perspectives.

2. The Bio-Economic Equilibrium Model

The description of the bio-economic equilibrium model is divided into two parts: The first one is the mathematical model and hypotheses; the second one is the bio-economic equilibrium model of fishery.

2.1 The mathematical model and hypotheses

We consider a model which describes the dynamics of a fish population of densities B_i located at fishing zone $i\epsilon[1,N]$, and exploited by a fishing fleet represented by their fishing effort E_i on each area i (that is population B_i, situated in the i^{th} zone, is harvested by fishing fleet E_i). The fish density of the free stock (unattached to FADs) at time t is noted $B_s(t)$ and the fish density on FAD i at time t is denoted $B_i(t)$ with i ∈ [1,N]. We consider the coast as a linear chain of N sites (either FADs or AHs) with migration of fish between the N sites and a free stock (Fig. 1). Let k_i be the fish carrying capacity for FAD i and k_s of the free stock. We assume that the coastal area has a global fish carrying capacity

$K = k_s + \sum_{i=1}^{N} k_i$, which is constant. We take $k_s = \alpha K$ thus $\sum_{i=1}^{N} k_i = (1-\alpha)K$. The fish population is assumed to follow a logistic growth with an intrinsic growth rate r_i on FAD i ∈ [1, N], and r_s on a free zone.

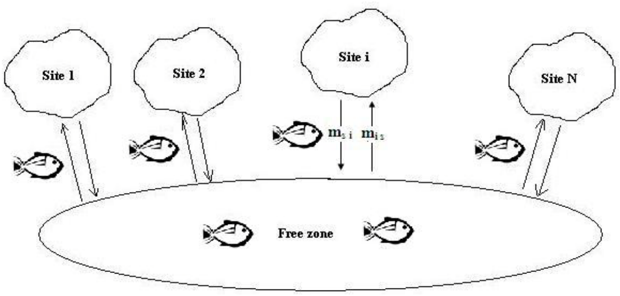

Figure 1: Representation of the multi-site fishery model. Fish move between site and the free stock site.

We suppose that two processes occur at two different time scales.

At the fast time scale, the total stock is constant. Thus, the fast part of the model only describes the displacement of fish between the N zones and the only free stock.

At the slow time scale, the total fish stock isn't constant; the evolution of the stock in each of the N zones is represented by the stock-effort Schaefer model (see M. B. Schaefer [18]). Thus, we assume that fish movements and boat displacements occur at a fast time scale τ, whereas fish's growth and the dynamics of the fishery occur at a slow time scale $t = \varepsilon\tau$, $\varepsilon \ll 1$, being a small dimension less parameter. We assume that the fish migration rate m_{ij} depend on the carrying capacity. If the carrying capacity of a patch i is high, fish are more likely to stay on this patch. If the carrying capacity is low, fish are rapidly leaving the patch. According to these assumptions, we choose $m_{ij} = \frac{m_0}{k_j}$. According to previous assumptions, the complete system reads as follows

$$\begin{cases} \frac{dB_s}{dt} = R(\sum_{i=1}^{N} m_{si}B_i - \sum_{i=1}^{N} m_{is}B_s) + r_s B_s\left(1-\frac{B_s}{k_s}\right) \\ \frac{dB_i}{dt} = R(m_{is}B_s - m_{si}B_i) + r_i B_i\left(1-\frac{B_i}{k_i}\right),\ i\epsilon[1,N] \end{cases} \quad (1)$$

If we set $R = \frac{1}{\varepsilon}$ and $\tau = \frac{1}{\varepsilon}$ with $\varepsilon \ll 1$, then Eq. (1) can be rewritten in the following form

$$\begin{cases} \frac{dB_s}{d\tau} = (\sum_{i=1}^{N} m_{si}B_i - \sum_{i=1}^{N} m_{is}B_s) + \varepsilon\, r_s B_s\left(1-\frac{B_s}{k_s}\right) \\ \frac{dB_i}{d\tau} = m_{is}B_s - m_{si}B_i + \varepsilon\, r_i B_i\left(1-\frac{B_i}{k_i}\right),\ i\epsilon[1,N] \end{cases} \quad (2)$$

where τ represents the fast time scale with respect to t. Eq. (2) is now in the form that allows its reduction by means of the so-called aggregation methods (see P. Auger et al. [1], R. Mchich et al. [14]).

Now, we apply aggregation methods to obtain a reduced system: the total fish stock $B(t) = B_s + \sum_{i=1}^{N} B_i$.

The fast model is obtained by neglecting the slow dynamics, leading to equations (2)

$$\begin{cases} \frac{dB_s}{d\tau} = (\sum_{i=1}^{N} m_{si}B_i - \sum_{i=1}^{N} m_{is}B_s) \\ \frac{dB_i}{d\tau} = m_{is}B_s - m_{si}B_i \end{cases} \quad (3)$$

Fast equilibria are the solutions of the following system

$$\begin{matrix} \sum_{i=1}^{N} m_{si}B_i - \sum_{i=1}^{N} m_{is}B_s = 0 \\ m_{is}B_s - m_{si}B_i = 0 \end{matrix} \quad (4)$$

A simple calculation leads to the following result

$$m_{is}B_s - m_{si}B_i = 0 \quad (5)$$

The fast model is conservative.

At the fast time, the total fish density $B(t) = B_s + \sum_{i=1}^{N} B_i$ remains constant.

A simple calculation leads to the following result

$$\begin{cases} B_s^* = \frac{k_s}{K}B \\ B_i^* = \frac{k_i}{K}B \end{cases}$$

Now, coming back to the complete initial Eq. (1), we substitute the fast equilibria (Eq. (3)) and add the two fish stock equations. The state variables are replaced in terms of the fast equilibria as follows

$$\begin{cases} B_s = \frac{k_s}{K}B \\ B_i = \frac{k_i}{K}B \end{cases} \quad (6)$$

After some algebra, from system (1) one obtains the following equation

$$\frac{dB}{dt} = rB\left(1-\frac{B}{K}\right) \quad (7)$$

where $r = r_s \frac{k_s}{K} + \sum_{i=1}^{N} r_i \frac{k_i}{K}$

2.1.1 Expression of biomass by introducing harvest

In this part, we introduce the harvest of fishing by reducing the rate of fish population growth by the amount $H = FB$, where F is the fishing mortality applied to a population, it is composed by a catchability coefficient q and a fishing effort E term: $F = qE$. The first has been defined as the mortality generated by a unit of fishing effort and its dynamics which have already been explained. Under these assumptions, biomass changes through time can be expressed as

$$\dot{B}(t) = rB\left(1 - \frac{B}{K}\right) - H \qquad (8)$$

Besides, the total fishing mortality suffered by an exploited population F_{total} is the sum of the mortalities generated by each fishing fleet F_i, each of them is the product of the individual catchability q_i and the individual activity or effort E_i.

That is to say $F_{total} = \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} q_i E_i$. Note that the fishing effort E is one of the easiest concepts to understand and the most difficult to define and quantify in fishery science, but in general, we can say that fishing effort is defined as the product of a fishing activity and a fishing power. The fishing effort exerted by a fleet is the sum of these products over all fishing units in the fleet. The fishing activity is in units of time. The fishing power is the ability of a fishing unit to catch fish and it is a complex function depending on vessel, gear and crew. However, since measures of fishing power may not be available, activity (such as hours or days fished) has often been used as a substitute for effort.

It is interesting to note that according to the literature, the effort depends on several variables, namely for example: number of hours spent fishing; search time; number of hours since the last fishing; number of days spent fishing; number of operations; number of sorties flown; ship, technology, fishing gear, crew, etc. However, in this paper, the fishing effort is treated as a unidimensional variable which includes a combination of all these factors.

2.2 The bio-economic equilibrium model of fishery

The description of the bio-economic equilibrium model of fishery is divided into four parts: The total revenue, the total cost, the profits and the bio-economic equilibrium model optimization.

2.2.1 Total revenue

The total revenue of a fishery, noted R_i, is proportional to catch H_i. We can calculate the total revenue to fisherman i using the following formula:

$$R_i = pH_i$$

where p is the price of the fish population. In the following analysis, p is constant across time and quantity.

The total revenue to fisherman i, at biological equilibrium, can be represented as

$$R_i = \frac{pK}{r} q_i^2 E_i^2 + pKq_i\left(1 - \frac{1}{r}\sum_{j=1, j\neq i}^{n} q_j E_j\right) E_i \qquad (9)$$

where q_i is the catchability coefficient of fisherman i and E_i is the fishing effort strategy of fisherman i.

2.2.2 Total cost

The total cost determined by $Cost = FC + VC$ where FC is a Fixed Cost and VC is a Variable Cost which is determined by "fishing effort" VC= labor+ equipment+ fuel+ depreciation, etc. So, the total cost of fishing effort, noted C, is proportional to the fishing effort. In this work we will keep to the simplest hypothesis: that is the total costs to fisherman i is proportional to fishing effort E_i, expressed mathematically as

$$C_i = c_i E_i \qquad (10)$$

where c_i is the harvesting costs per fishing effort employed by fisherman i .

2.2.3 Profits

The model establishes that the profit π derived from fishing are a function of total sustainable revenues (R) and total costs (C).

The profit of fisherman i is the difference between the total revenue of the fishery R and the total fishing cost C as following:

$$\pi_i = R_i - C_i \qquad (11)$$

It follows from (9) and (10) that

$$\pi_i(E) = -\frac{pK}{r} q_i^2 E_i^2 + pKq_i\left(1 - \frac{c_i}{pKq_i} - \frac{1}{r}\sum_{j=1, j\neq i}^{n} q_j E_j\right) E_i \qquad (12)$$

2.2.4 Bio-economic equilibrium model optimization

The objective of each fisherman (player) is individual profit maximization by respecting two constraints:

(a) The first one is the preservation of biodiversity of fish population, expressed mathematically as $B = K\left(1 - \frac{1}{r}\sum_{j=1}^{n} q_j E_j\right) \geq B_{min} > 0$, where B_{min} is a constant.

(b) The second one is that the effort strategies solution searched $(E_1^*, \cdots, E_n^*)$ must satisfy the following conditions: for all $i = 1, \cdots, n$

$$\pi_i(E_1^*, \cdots, E_i^*, \cdots, E_n^*) \geq \pi_i(E_1^*, \cdots, E_i, \cdots, E_n^*) \quad (13)$$

with the conditions that $(E_i)_{i=1,\cdots,n} \geq 0$.

With all these considerations, our bio-economic equilibrium model can be translated into the following mathematical problem:

Each fisherman i must solve problem (P_i)

$$\begin{cases} \max \pi_i(E) = -\frac{pK}{r} q_i^2 E_i^2 + pKq_i \left(1 - \frac{c_i}{pKq_i} - \frac{1}{r} \sum_{j=1, j\neq i}^{n} q_j E_j \right) E_i \\ \textit{Subject to} \\ \frac{1}{r} q_i E_i < -\frac{1}{r} \sum_{j=1, j\neq i}^{n} q_j E_j + 1 - \frac{B_{min}}{K} \\ E_i \geq 0 \\ \left(E_j\right)_{j=1,\cdots,n\ ; j\neq i} \textit{ are given} \end{cases} \quad (14)$$

3. Computing the Generalized Nash Equilibrium

The Generalized Nash Equilibrium Problem (GNEP) is an extension of the Nash Equilibrium Problem (NEP), which each fisherman's strategy (fishing effort) set is dependent on the rival fishermen strategies. Mathematically, $(E_1^*, \cdots, E_n^*)$ is called Generalized Nash equilibrium point, if and only if, E_i^* is a solution of the problem (P_i) for $\left(E_j^*\right)_{j=1,\cdots,n; j\neq i}$ are given.

The optimality conditions of Karush-Kuhn-Tucker for (P_i) are that if E_i^* is a solution of the problem (P_i) then there exist constants $u_i \epsilon\, \mathbb{R}^+, v\, \epsilon\, \mathbb{R}^+ and\ \lambda_i \epsilon\, \mathbb{R}^+$ such that the following relations hold

for all $i = 1, \cdots, n$

$$\begin{cases} 2\frac{pKq_i^2}{r} E_i^* + pKq_i \left(\frac{c_i - pKq_i}{pKq_i} + \frac{1}{r} \sum_{j=1; j\neq i}^{n} q_j E_j^* \right) - u_i + \frac{q_i}{r} \lambda_i = 0 \\ \frac{1}{r} \sum_{j=1}^{n} q_j E_j^* + v = 1 - \frac{B_{min}}{K} \\ u_i E_i^* = \lambda_i v = 0 \\ u_i, E_i^*, \lambda_i, v \geq 0 \end{cases} \quad (15)$$

To maintain ecological balance of fish population, it is natural to assume that the biomass remain strictly positive, that is $B \geq B_{min} > 0$; therefore $v = \frac{B}{K} \geq \frac{B_{min}}{K} > 0$. As the product $\lambda_i v = 0$, so $\lambda_i = 0$ for all $i = 1, \cdots, n$

The problem (15) reduces to the following expression for all $i = 1, \cdots, n$

$$\begin{cases} 2q_i E_i^* + \left(r\frac{c_i - pKq_i}{pKq_i} + \sum_{j=1; j\neq i}^{n} q_j E_j^* \right) - \frac{r}{pKq_i} u_i = 0 \\ \frac{1}{r} \sum_{j=1}^{n} q_j E_j^* + v = 1 - \frac{B_{min}}{K} \\ u_i E_i^* = 0 \\ u_i, E_i^*, v \geq 0 \end{cases} \quad (16)$$

If we set $\bar{E}_i = q_i E_i^*$, $\bar{u}_i = \frac{r}{pKq_i} u_i$ and $\bar{v} = rv$ then (16) can be rewritten in the following form

$$\begin{cases} \bar{u} = A\bar{E} + b \\ \bar{v} = r - \sum_{i=1}^{n} \bar{E}_i \\ \bar{u}^T \bar{E} = 0 \\ \bar{E}, \bar{v}, \bar{u} \geq 0 \end{cases} \quad (17)$$

where
$A = \left(a_{ij}\right)_{1\leq i,j\leq n}$ where $a_{ii} = 2$ and $a_{ij} = 1$ for all $i \neq j$;
$b = (b_i)_{1\leq i\leq n}$ where $b_i = r\left(\frac{c_i}{pKq_i} - 1\right)$
Here the upper subscript T denotes the transpose vector.

Now we set $z = (\bar{E}, 0)^T$ and $w = (\bar{u}, \bar{v})^T$ then (17) can be rewritten in the following form

$$(\text{LCP}) \begin{cases} w = Mz + q \geq 0 \\ z \geq 0 \\ z^T w = 0 \end{cases}$$

where $M = \begin{bmatrix} A & 0 \\ -1 & 1 \end{bmatrix}$ and $q = \begin{bmatrix} b \\ r \end{bmatrix}$

Here -1 denotes the vector $(-1, \cdots, -1)$ and 0 denotes the vector $(0, \cdots, 0)^T$.

The last row in (LCP) problem indicates that we require z and w to be complementarity non-negative variables.

We note that the (LCP) problem is called a Linear Complementarity Problem. LCP(M, q) is to find a vectors z in $\mathbb{R}^m$ and w in $\mathbb{R}^m$ satisfying $w = Mz + q,\ z^T w = 0,\ z \geq 0\ and\ w \geq 0$ where $M = \left(m_{ij}\right) \epsilon \mathbb{R}^{m\times m}$ and $q \epsilon \mathbb{R}^m$ are given, for more details see C. E. Lemke [13], R. W. Cottle et al. [5], [6], Y. El Foutayeni et al. [7], [8]).

For solving the linear complementarity problem LCP (M, q) we can demonstrate that the matrix M is P-matrix (Recall that a matrix M is called P-matrix if all of its principal minors are positive) and we will use the following result:

A linear complementarity problem LCP(M, q) has a unique solution for every q if and only if M is a P-matrix (For demonstration we can see K. G. Murty [17]).

To prove this result (the matrix M is P-matrix), we note by $(M_i)_{i=1,\cdots,n+1}$ the submatrix of M, then we obtain $det(M_i) = i + 1 > 0$ for all $i = 1, \cdots, n+1$.

So the matrix M is P-matrix and therefore the LCP(M, q) admits one and only one solution (see Y. El Foutayeni et al. [9]) . This solution is given by:
For all $i = 1, \cdots, n$

$$\begin{cases} z_i = \frac{r}{(n+1)pK \prod_{k=1}^{n} q_k} \left[\prod_{j=1, j\neq i}^{n} q_j (pKq_i - nc_i) + \sum_{j=1, j\neq i}^{n} c_j \prod_{k=1, k\neq j}^{n} q_k \right] \\ w_i = 0 \end{cases}$$

and

$$\begin{cases} z_{n+1} = 0 \\ w_{n+1} = r - \frac{r}{(n+1)pK\prod_{k=1}^{n} q_k} \sum_{i=1}^{n} \prod_{j=1, j\neq i}^{n} q_j \left(pKq_i - c_i\right) \end{cases}$$

Furthermore, the Generalized Nash Equilibrium point is given by
for all $i = 1, \cdots, n$

$$E_i^* = \frac{r}{(n+1)pK\prod_{k=1}^{n} q_k} \frac{\left(\prod_{j=1, j\neq i}^{n} q_j (pKq_i - nc_i) + \sum_{j\neq i}^{n} c_j \prod_{k=1, k\neq j}^{n} q_k\right)}{q_i} \quad (18)$$

and the profit of fisherman i is then given by
for all $i = 1, \cdots, n$

$$\pi_i(E^*) = \frac{r}{pK(n+1)^2 \prod_{k=1}^{n} q_k^2} \left(\prod_{j=1, j\neq i}^{n} q_j \left(pKq_i - nc_i\right) + \sum_{j\neq i}^{n} c_j \prod_{k=1, k\neq j}^{n} q_k\right)^2 \quad (19)$$

It is important to remark that if $c_i = pKq_i$ then

$$\prod_{j=1, j\neq i}^{n} q_j \left(pKq_i - nc_i\right) + \sum_{j\neq i}^{n} c_j \prod_{k=1, k\neq j}^{n} q_k = 0$$

and therefore $E^* = \pi^* = 0$.

4. Discussion

We begin our discussion with the study of steady states of the system. When the fish population is at biological equilibrium, i.e. the variation of the biomass of fish population is zero: $\dot{B}(t) = 0$, thus losses by natural and fishing mortalities are compensated by the fish population increase due to individual growth and recruitment. The equation can be defined as

$$\mathrm{rB}\left(1 - \frac{\mathrm{B}}{\mathrm{K}}\right) - \mathrm{qEB} = 0 \quad (20)$$

The solutions of this equation are $B_1^* = 0$ and $B_2^* = K\left(1 - \frac{\mathrm{q}}{\mathrm{r}}\mathrm{E}\right)$.

Note that B_1^* and B_2^* are not only solutions of the algebraic equation (20), but also represent constant solutions of the differential equation (8), since all we have done in equation (20) is to algebraically manipulate the right-hand side of equation (8) to equal forms. That is, it has been shown that the differential equation (8) can be rewritten in the form

$$\dot{\mathrm{B}}(\mathrm{t}) = \mathrm{rB}\left(1 - \frac{\mathrm{q}}{\mathrm{r}}\mathrm{E}\right)\left[1 - \frac{B}{K\left(1 - \frac{q}{r}E\right)}\right] \quad (21)$$

Furthermore, it is easy to show that the constant solution $B_1^* = 0$ is unstable and that the constant solution $B_2^* = K\left(1 - \frac{\mathrm{q}}{\mathrm{r}}\mathrm{E}\right)$ is asymptotically stable. After (6) and (20) was

$$\begin{cases} B_s = k_s\left(1 - \frac{\mathrm{q}}{\mathrm{r}}\mathrm{E}\right) \\ B_i = k_i\left(1 - \frac{\mathrm{q}}{\mathrm{r}}\mathrm{E}\right) \end{cases}$$

more was

$$\begin{cases} B_s = k_s\left(1 - \frac{\mathrm{q}}{\mathrm{r_s}}\mathrm{E_s}\right) \\ B_i = k_i\left(1 - \frac{\mathrm{q}}{\mathrm{r_i}}\mathrm{E_i}\right) \end{cases} \quad (22)$$

where

$$\begin{cases} E_s = \frac{r_s}{r}E \\ E_i = \frac{r_i}{r}E \end{cases} \quad (23)$$

it is plain that $\mathrm{E} = \frac{\mathrm{k_s}}{\mathrm{K}}\mathrm{E_s} + \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{\mathrm{k_i}}{\mathrm{K}}\mathrm{E_i}$.
Every zone i, i ϵ [1, N], has two equilibrium points: $B_i^* = K\left(1 - \frac{\mathrm{q}}{\mathrm{r}}\mathrm{E}\right)$ is asymptotically stable and $B_i^* = 0$ is unstable, also for the free zone where both equilibrium points are $B_s^* = K\left(1 - \frac{\mathrm{q}}{\mathrm{r}}\mathrm{E}\right)$ is asymptotically stable and $B_s^* = 0$ is unstable.

In this part, we are drawing attention to growth rate; most of the works considered a fixed growth rate noted r_1. In this work, we considered that fish population could have different growth rate for every site i, which we called r_i, because in the reality, zones cannot have the same growth rate, see A. Kamili [12]. For example, we take data of exploitation of sardines on Moroccan Atlantic coast given by INRH (Morocco) ("l'Institut National des Recherches Halieutiques" i.e. "the National Institute of Fisheries Research") (Table 1 and Fig. 2).

The sardine distributed off the coast of north-western Africa, mainly between the Cape Spartel and Cape Blanc, is exploited in four fishing zones:

North Zone:	35°45'-32°N	(Cape Spartel-Eljadida)
Zone A:	32°N-29°N	(Safi-Sidi Ifni)
Zone B:	29°N-26°N	(Sidi Ifni-Cape Bojador)
Zone C:	26°N- to south	(Cape Bojador- to the southerrn extent of the species)

Table 1 : Exploitation's zone of sardines from Morocco

	Central zone (Zone A & B)	Zone C
Growth rate r	1.53	1.15
Carrying capacity k	1703	5723
Catchability coefficient q	0.025	0.0035

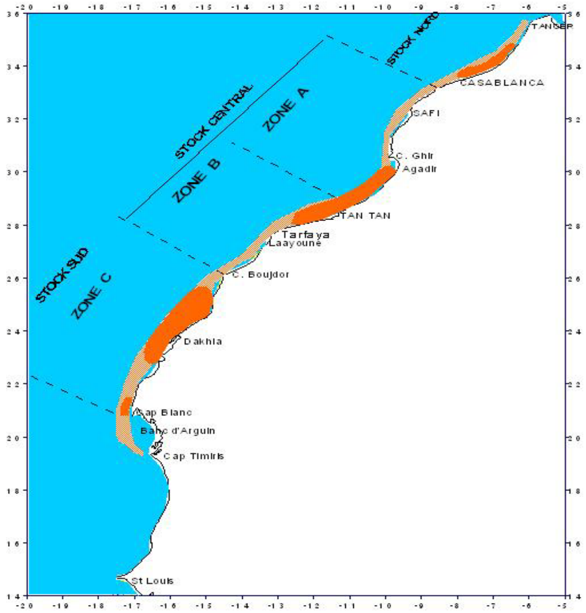

Figure 2: Representation of the distribution of exploitation's zones of sardines in coast of north-western Africa

We would stress the fact that the price of fish population which we have considered, in this work, is constant. We know that the price of fish population from a particular stock is hardly affected by the quantity fished, if the fish is sold in a competitive market with many sellers and buyers in competition with similar types of fish from other stocks. So, we can define function where price depends on the level of effort and biomass stock of each fish population giving, which we called p_i. We will take that the price of fish population depends on the quantity harvested; especially we assumed that the price of the fish population increases with decreasing harvest and the price of the fish population decreases with the increase of the harvest (Fig. 3), but the minimum price is equal to a fixed positive constant. The price p_i of fish population i ϵ [1,N] is given by the equation:

$$p_i(H_i) = \frac{a_i}{H_i} + p_{0i} \qquad (24)$$

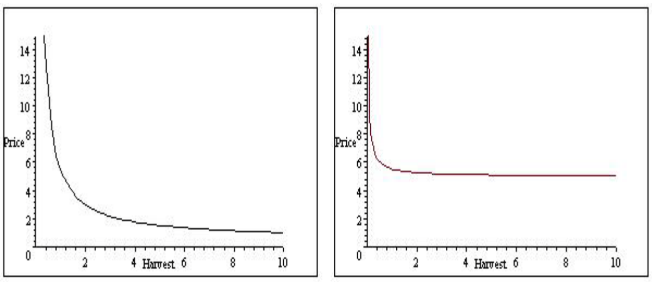

Figure 3: The first figure illustrates the price with parameters $a_i = 5$ and $p_{0i} = 0.5$. The second figure illustrates the price with parameters $a_i = 0.5$ and $p_{0i} = 5$.

After the introduction of the price's function (24), the problem (14) can be written easily in the following form. For all $i = 1, \dots, n$. Each fisherman i must solve problem (P_i)

$$\begin{cases} \max \pi_i(E) = -\frac{p_{0i}K}{r} q_i^2 E_i^2 + a_i + p_{0i}Kq_i\left(1 - \frac{c_i}{p_{0i}Kq_i} - \frac{1}{r}\sum_{j=1, j\neq i}^{n} q_j E_j\right)E_i \\ \textit{Subject to} \\ \frac{1}{r} q_i E_i < -\frac{1}{r}\sum_{j=1, j\neq i}^{n} q_j E_j + 1 - \frac{B_{min}}{K} \\ \quad E_i \geq 0 \\ (E_j)_{j=1,\dots,n;\, j\neq i} \ \textit{are given} \end{cases} \qquad (25)$$

In this case, the Generalized Nash Equilibrium point, for all $i = 1, \dots, n$, is given by

$$E_i^* = \frac{r}{(n+1)p_{0i}K\prod_{k=1}^{n} q_k} \frac{\left(\prod_{j=1, j\neq i}^{n} q_j (p_{0i}Kq_i - nc_i) + \sum_{j\neq i}^{n} c_j \prod_{k=1, k\neq j}^{n} q_k\right)}{q_i};$$

and for all $i = 1, \dots, n$, the profit of fisherman i is then given by

$$\pi_i(E^*) = \frac{r}{(n+1)^2 p_{0i}K\prod_{k=1}^{n} q_k^2} \left(\prod_{j=1, j\neq i}^{n} q_j (p_{0i}Kq_i - nc_i) + \right.$$
j≠incjk=1, k≠jnqk2

Contrary to works which consist in maximizing captures, in this work we maximize the profits of the fishermen, according to the work of Y. El Foutayeni et al. [10] when they prove making double capture does not mean winning double. We can easily verify this result by using the data of the Table 1 and Table 2.

Table 2: Estimation of the parameters (source INRH)

	Class 1	Class 2
Average price Dh/kg	1.08	
Cost per unit of effort	7237	11014

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