Two-Dimensional Transient Thermoelastic Problem for A Hallow Cylinder with Internal Heat Generation

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Abstract: ***The present paper deals with the determination of temperature distribution, displacement and thermal stresses of hallow cylinder with internal heat generation. I apply integral transform techniques to find the thermoelastic solution. The results are obtained as series of Bessel's functions. Numerical calculations are carried out for hallow cylinder made up of aluminum metal and illustrated graphically.***

Keywords: Thermoelastic problem, hallow cylinder, temperature distribution, thermal stress, integral transform, internal heat generation.

1. Introduction

As a result of the increased usage of industrial and construction materials the interest in isotropic thermal stress problems has grown considerably. However there are only a few studies concerned with the two-dimensional steady state thermal stress. **Nowacki** [**4**] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. **Wankhede** [**7**] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there are not many investigations on transient state. **Roy Choudhuri** [**6**] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In a recent work **Noda et al. [5]** have been solved some problems. **Dange et al. [3]** have determined two-dimensional transient problem for a thick annular disc in thermoelasticity. **N.W. Khobragade and Warsha K. Dange [2]** have determined two dimensional transient problems for a thick solid circular plate due to thermal shock**. Warsha K. Dange [1]** has determined Thermal Stresses of Two-Dimensional Transient Thermoelastic Problem for A Hallow Cylinder. This paper concerned with transient thermoelastic problem of hallow cylinder with internal heat generation. The heat conduction equation is solved by using Marchi – Zgrablich transform and Laplace transform techniques. The temperature distribution, displacement and thermal stresses of hallow cylinder occupying the space D: $a \le r \le b,\ 0 \le z \le h,$ with stated boundary conditions are obtained in terms of infinite series of Bessel's functions. Numerical calculations are carried out for hallow cylinder made of aluminum metal.

2. Statement of the Problem

Consider the hallow cylinder occupying the space D: $a \le r \le b,\ 0 \le z \le h,$ where *a* and *b* are the internal and external radii respectively and (r, z, t) are cylindrical coordinates. The third kind boundary conditions are maintained zero at inner and outer curved surface of the hallow cylinder. The arbitrary temperature $u(r,t)$ on the lower surface $z = 0$ and $f(r,t)$ on the upper surface at $z = h$. Under these more realistic prescribed condition the temperature distribution, displacement and thermal stresses are required to be determined.

The differential equation governing the displacement function $U(r, z, t)$ is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+v)\, a_t T \tag{1}$$

$$\text{With } U = 0 \text{ at } r = a \text{ and } r = b \tag{2}$$

Where v and a_t are Poisson's ratio and the linear coefficient of thermal expansion of the material of the hallow cylinder respectively.

The equation for $T(r, z, t)$, the temperature is:

$$\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

Subject to the initial and boundary conditions

$$M_t(T, 1,\ 0, 0) = 0 \ \text{ for all } a \le r \le b,\ 0 \le z \le h \tag{4}$$

$$M_r(T, 1,\ k_1, a) = 0,\ \ M_r(T, 1,\ k_2, b) = 0 \quad \text{for all } 0 \le z \le h,\ t > 0 \tag{5}$$

$$M_z(T, 1, 0, 0) = u(r,t), \ \text{ for all } a \le r \le b,\ t > 0 \tag{6}$$

$$M_z(T,1,0,h) = f(r,t), \text{ for all } a \le r \le b,\ t > 0 \tag{7}$$

and

$$g(r,z,t) = \delta(r - r_0)\cosh(\mu_n(z+h)e^{-\omega t} \tag{8}$$

The most general expression for these conditions can be given by

$$M_v(f,k,\bar{\bar{k}},\$) = (\bar{k}f + \bar{\bar{k}}\hat{f})_{v=\$}$$

Where the prime (^) denotes differentiation with respect to v, k_1 and k_2 are radiation constants on the curved surfaces, α is the thermal diffusivity, k is thermal conductivity of the material of the hallow cylinder.

The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu\frac{1}{r}\frac{\partial U}{\partial r} \tag{9}$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2 U}{\partial r^2} \tag{10}$$

where μ is the Lame's constant, while each of the stress functions σ_{rz}, σ_{zz} and $\sigma_{\theta z}$ are zero within the hallow cylinder in the plane state of stress.

The equations (1) to (10) constitute the mathematical formulation of the problem under consideration.

3. Solution of the Problem

Results Required:

First introduce the integral transform of order *n* over the variable *r*. Let *n* be the parameter of the transform, then the integral transform and its inversion theorem are written as

$$\bar{g}(n) = \int_a^b r\, g(r)\, S_p(k_1,k_2,\mu_n r)\,dr, \tag{11}$$

$$g(r) = \sum_{n=1}^{\infty}(\bar{g}_p(n)/C_n)S_p(k_1,k_2,\mu_n r) \tag{12}$$

where $\bar{g}_p(n)$ is the transform of $g(r)$ with respect to nucleus $S_p(k_1,k_2,\mu_n r)$ which is defined as.

$$S_p(k_1,k_2,\mu_n r) = J_p(\mu_n r)[Y_p(k_1,\mu_n a) + Y_p(k_2,\mu_n b)]$$
$$-Y_p(\mu_n r)[J_p(k_1,\mu_n a) + J_p(k_2,\mu_n b)]$$

$$\left.\begin{aligned} J_p(k_i,\mu_n r) &= J_p(\mu_n r) + k_i\,\mu_n\,J'_p(\mu_n r)\\ Y_p(k_i,\mu_n r) &= Y_p(\mu_n r) + k_i\,\mu_n\,Y'_p(\mu_n r)\end{aligned}\right\} \text{ for}$$

$i = 1,2$

And

$$C_n = \int_a^b r[S_p(k_1,k_2,\mu_n b)]^2\,dr$$

in which $J_p(\mu_n r)$ and $Y_p(\mu_n r)$ are Bessel functions of first and second kind of order p respectively.

Determination Temperature Function $T(r,z,t)$:

By applying finite Marchi – Zgrablich transform as defined in (11) to the equations (3) and (4) (6) and (7) and (8) and using (5) to reduce differential equation in Marchi – Zgrablich transform domain and then applying laplace transform and making use of respective inversion as in (12) and Sneddon over the heat conduction equation one obtains the expression for temperature distribution function as

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{2\alpha\pi}{h^2}\sum_{m=1}^{\infty}\frac{m}{\cos m\pi}\right.$$
$$\int_0^t\left[\bar{u}(n,t')\sin\frac{m\pi}{h}(z-h) - \bar{f}(n,t')\sin\frac{m\pi}{h}z\right]e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right](t-t')}dt'$$
$$-\frac{2\alpha\pi r_0 S_0(k_1,k_2,\mu_n r_0)}{k^2h^2}\times$$
$$\sum_{m=1}^{\infty}\frac{m\left[\cosh(\mu_n h)\sin\frac{m\pi}{h}(z-h) - \cosh(2\mu_n h)\sin\frac{m\pi}{h}z\right]}{\cos m\pi\left[\omega - k\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right]\right]\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right]}e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right]t}$$
$$-\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega}\times$$
$$\left[\left(\frac{\cosh(\mu_n h)\sinh p(z-h)}{\sinh ph} - \frac{\alpha\cosh(\mu_n h)\sinh\mu_n(z-h)}{\sinh\mu_n h}\right)\right.$$
$$\left.+\left(\frac{\cosh(2\mu_n h)\sinh pz}{\sinh ph} - \frac{\alpha\cosh(2\mu_n h)\sinh\mu_n z}{\sinh\mu_n h}\right)\right]$$
$$\left.+\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega}(1-e^{-\omega t})\cosh(\mu_n(z+h)\right\}S_0(k_1,k_2,\mu_n r)$$

(13)

Determination of Thermoelastic Displacement Function $\cup(r,z,t)$:

Substituting the value of $T(r,z,t)$ from equation (13) to equation (1) one obtains the thermoelastic displacement $\cup(r,z,t)$ as.

$$\cup(r,z,t) = \alpha_t(1+\nu)\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{2\alpha\pi}{h^2}\sum_{m=1}^{\infty}\frac{m}{\mu_n^2\cos m\pi}\right.$$
$$\int_0^t\left[\bar{u}(n,t')\sin\frac{m\pi}{h}(z-h) - \bar{f}(n,t')\sin\frac{m\pi}{h}z\right]e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right](t-t')}dt'$$
$$-\frac{2\alpha\pi r_0 S_0(k_1,k_2,\mu_n r_0)}{k^2h^2}$$
$$\sum_{m=1}^{\infty}\frac{m\left[\cosh(\mu_n h)\sin\frac{m\pi}{h}(z-h) - \cosh(2\mu_n h)\sin\frac{m\pi}{h}z\right]}{\mu_n^2\cos m\pi\left[\omega - k\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right]\right]\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right]}e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^2\right]t}$$
$$-\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega\mu_n^2}$$
$$\left[\left(\frac{\cosh(\mu_n h)\sinh p(z-h)}{\sinh ph} - \frac{\alpha\cosh(\mu_n h)\sinh\mu_n(z-h)}{\sinh\mu_n h}\right)\right.$$

$$+\left(\frac{\cosh(2\mu_n h)\sinh pz}{\sinh ph}-\frac{\alpha\cosh(2\mu_n h)\sinh \mu_n z}{\sinh \mu_n h}\right)\Bigg]$$

$$\left.+\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega\mu_n^{\ 2}}(1-e^{-\omega t})\cosh(\mu_n(z+h)\right\}S_0(k_1,k_2,\mu_n r) \quad (14)$$

Determination Of Stress Functions σ_{rr} and $\sigma_{\theta\theta}$:

Using equation (14)) to equation (9) and (10) one obtains the stress function σ_{rr} and $\sigma_{\theta\theta}$ as,

$$\sigma_{rr}=-2\mu\alpha_t(1+\nu)\sum_{n=1}^{\infty}\frac{1}{rC_n}\left\{\frac{2\alpha\pi}{h^2}\sum_{m=1}^{\infty}\frac{m}{\mu_n^{\ 2}\cos m\pi}\right.$$

$$\int_0^t\left[\bar{u}(n,t')\sin\frac{m\pi}{h}(z-h)-\bar{f}(n,t')\sin\frac{m\pi}{h}z\right]e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right](t-t')}dt'$$

$$-\frac{2\alpha\pi r_0 S_0(k_1,k_2,\mu_n r_0)}{k^2h^2}\times$$

$$\sum_{m=1}^{\infty}\frac{m\left[\cosh(\mu_n h)\sin\frac{m\pi}{h}(z-h)-\cosh(2\mu_n h)\sin\frac{m\pi}{h}z\right]}{\mu_n^{\ 2}\cos m\pi\left[\omega-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]\right]\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]}e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]t}$$

$$-\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega\mu_n^{\ 2}}\times$$

$$\Bigg[\left(\frac{\cosh(\mu_n h)\sinh p(z-h)}{\sinh ph}-\frac{\alpha\cosh(\mu_n h)\sinh \mu_n(z-h)}{\sinh \mu_n h}\right)$$

$$+\left(\frac{\cosh(2\mu_n h)\sinh pz}{\sinh ph}-\frac{\alpha\cosh(2\mu_n h)\sinh \mu_n z}{\sinh \mu_n h}\right)\Bigg]$$

$$\left.+\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega\mu_n^{\ 2}}(1-e^{-\omega t})\cosh(\mu_n(z+h)\right\}S_0'(k_1,k_2,\mu_n r) \quad (15)$$

$$\sigma_{\theta\theta}=-2\mu\alpha_t(1+\nu)\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{2\alpha\pi}{h^2}\sum_{m=1}^{\infty}\frac{m}{\mu_n^{\ 2}\cos m\pi}\right.$$

$$\int_0^t\left[\bar{u}(n,t')\sin\frac{m\pi}{h}(z-h)-\bar{f}(n,t')\sin\frac{m\pi}{h}z\right]e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right](t-t')}dt'$$

$$-\frac{2\alpha\pi r_0 S_0(k_1,k_2,\mu_n r_0)}{k^2h^2}\times$$

$$\sum_{m=1}^{\infty}\frac{m\left[\cosh(\mu_n h)\sin\frac{m\pi}{h}(z-h)-\cosh(2\mu_n h)\sin\frac{m\pi}{h}z\right]}{\mu_n^{\ 2}\cos m\pi\left[\omega-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]\right]\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]}e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]t}$$

$$-\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega\mu_n^{\ 2}}\times$$

$$\Bigg[\left(\frac{\cosh(\mu_n h)\sinh p(z-h)}{\sinh ph}-\frac{\alpha\cosh(\mu_n h)\sinh \mu_n(z-h)}{\sinh \mu_n h}\right)$$

$$+\left(\frac{\cosh(2\mu_n h)\sinh pz}{\sinh ph}-\frac{\alpha\cosh(2\mu_n h)\sinh \mu_n z}{\sinh \mu_n h}\right)\Bigg]$$

$$\left.+\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega\mu_n^{\ 2}}(1-e^{-\omega t})\cosh(\mu_n(z+h)\right\}S''_0(k_1,k_2,\mu_n r) \quad (16)$$

4. Numerical Results, Discussion and Remarks

To interpret the numerical computations, I consider material properties of Aluminum metal, which can be commonly used in both wrought and cast forms. The low density of aluminum results in its extensive use in the aerospace industry.

Table 1: Material properties and parameters used in this study

Poisson ratio, υ	0.281
Thermal expansion coefficient, α_t (cm/cm-^{0}C)	25.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec)	0.86
Inner radius, a (cm)	1
Outer radius, b (cm)	2
Height h (cm)	3

In the foregoing analysis are performed by setting the radiation constants, $k_1=k_2=1$ Set:

$$u(r,t)=\delta(r-r_1)(1-e^t)$$

$$f(r,t)=\delta(r-r_1)(1-e^t)e^h$$

In the equation (13), one obtains the expression for temperature distribution function as

$$T(r,z,t)=\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{2\alpha\pi}{h^2}\sum_{m=1}^{\infty}\frac{m}{\cos m\pi}\right.$$

$$r_1S_0(k_1,k_2,\mu_n r_1)\left[\sin\frac{m\pi}{h}(z-h)-\sin\frac{m\pi}{h}z\right]$$

$$\left\{\frac{\left[1-e^{-k\left[\mu_n^{\ 2}+\left(\frac{m\pi}{h}\right)^2\right]t}\right]\left[k\left[\mu_n^{\ 2}+\left(\frac{m\pi}{h}\right)^2\right]+1\right]-e^tk\left[\mu_n^{\ 2}+\left(\frac{m\pi}{h}\right)^2\right]}{k\left[\mu_n^{\ 2}+\left(\frac{m\pi}{h}\right)^2\right]\left[k\left[\mu_n^{\ 2}+\left(\frac{m\pi}{h}\right)^2\right]+1\right]}\right\}$$

$$-\frac{2\alpha\pi r_0 S_0(k_1,k_2,\mu_n r_0)}{k^2h^2}\times$$

$$\sum_{m=1}^{\infty}\frac{m\left[\cosh(\mu_n h)\sin\frac{m\pi}{h}(z-h)-\cosh(2\mu_n h)\sin\frac{m\pi}{h}z\right]}{\cos m\pi\left[\omega-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]\right]\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]}e^{-k\left[\frac{m^2\pi^2}{h^2}+\mu_n^{\ 2}\right]t}$$

$$-\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega}\times$$
$$\left[\left(\frac{\cosh(\mu_n h)\sinh p(z-h)}{\sinh ph}-\frac{\alpha\cosh(\mu_n h)\sinh\mu_n(z-h)}{\sinh\mu_n h}\right)\right.$$
$$\left.+\left(\frac{\cosh(2\mu_n h)\sinh pz}{\sinh ph}-\frac{\alpha\cosh(2\mu_n h)\sinh\mu_n z}{\sinh\mu_n h}\right)\right]$$
$$\left.+\frac{\alpha r_0 S_0(k_1,k_2,\mu_n r_0)}{k\omega}(1-e^{-\omega t})\cosh(\mu_n(z+h))\right\}S_0(k_1,k_2,\mu_n r)$$
(17)

The derived numerical results from equation (13) (14) and (15) and (16) has been illustrated graphically in figures 1 to 8.

Figure 1 represents graph of temperature **T(r,z,t)** versus **r** for different values of **t**. It is observed that T(r,z,t) goes on decreasing from r=1**. T(r,z,t)** develops tensile stress from r=1.3to r=1.5 and r=1.7to r=1.9 and compressive stress in circular region r=1.1 to r=1.3 and r=1.5 to r=1.7 and then T(r,z,t) goes on decreasing from r=1.7 in hallow cylinder for different values of t.

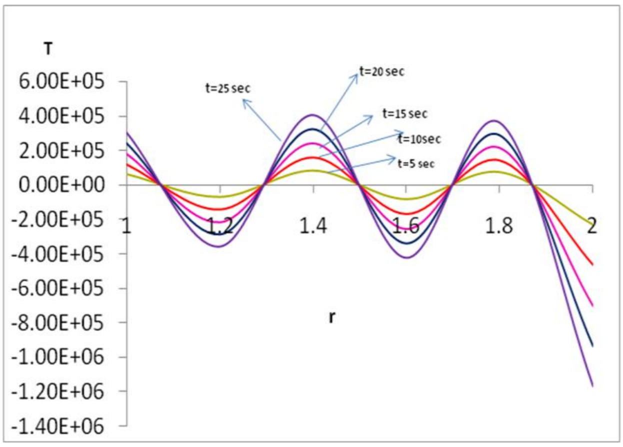

Figure 1: Graph of T(r,z,t) versus r for different values of t

Figure 2 represents graph of U(r,z,t) versus **r** for different values of t. It is observed that U(r,z,t) goes on decreasing from r=1 to r=1.1. U(r,z,t) develops tensile stress from r=1.3to r=1.5 and r=1.7to r=1.9 and compressive stress in circular region r=1.1 to r=1.3 and r=1.5 to r=1.7 and then U(r,z,t) goes on decreasing from r=1.7 to r=2 in hallow cylinder for different values of t.

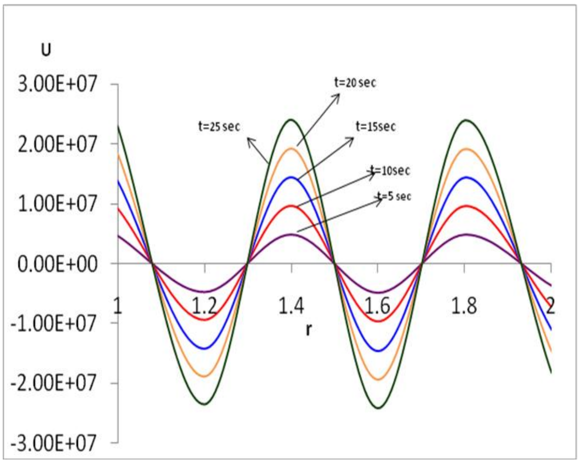

Figure 2: Graph of U(r,z,t) versus r for different values of t

Figure 3 represents graph of σ_{rr} versus **r** for different values of **t**. It is observed that σ_{rr} goes on increasing r=1 to r=1.1**.** σ_{rr} develops tensile stress from r=1.1to r=1.4 and compressive stress in circular region r=1.4 to r=1.8 and then σ_{rr} goes on increasing from r=1.8 to r=2 in hallow cylinder for different values of t.

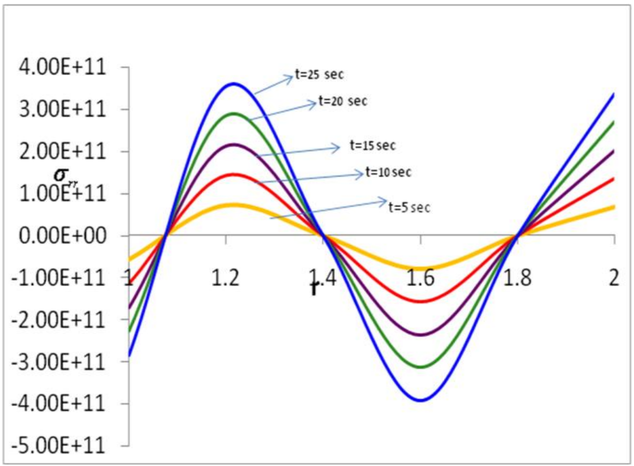

Figure 3: Graph of σ_{rr} versus r for different values of t

Figure 4 represents graph of $\sigma_{\theta\theta}$ versus r for different values of t. It is observed that $\sigma_{\theta\theta}$ goes on increasing from r=1 to r=1.2.It is also observed that $\sigma_{\theta\theta}$ develops tensile stress from r=1.2to r=1.6 and compressive stress in circular region r=1.6 to r=1.9 and then $\sigma_{\theta\theta}$ goes on increasing in the region of hallow cylinder for different values of t.

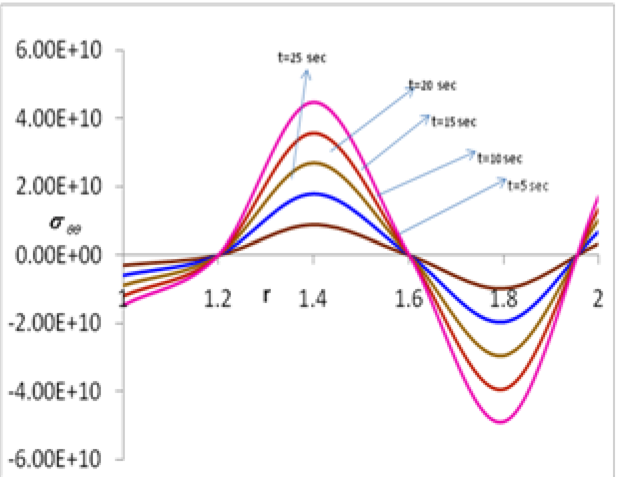

Figure 4: Graph of $\sigma_{\theta\theta}$ versus r for different values of t

Figure 5 represents graph of temperature T(r,z,t) versus z for different values of r. It is observed that T(r,z,t) increases uniformly in the region of hallow cylinder for different values of r.

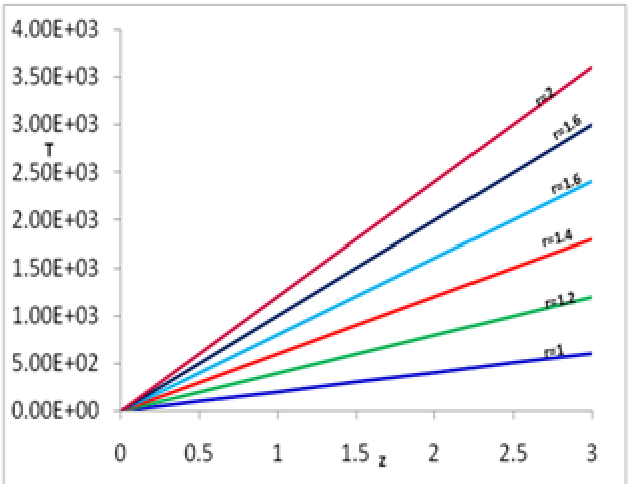

Figure 5: Graph of T(r,z,t) versus z for different values of r

Figure 6: represents graph of U(r,z,t) versus z for different values of r. It is observed that U(r,z,t) goes on increasing in the region of hallow cylinder for different values of r.

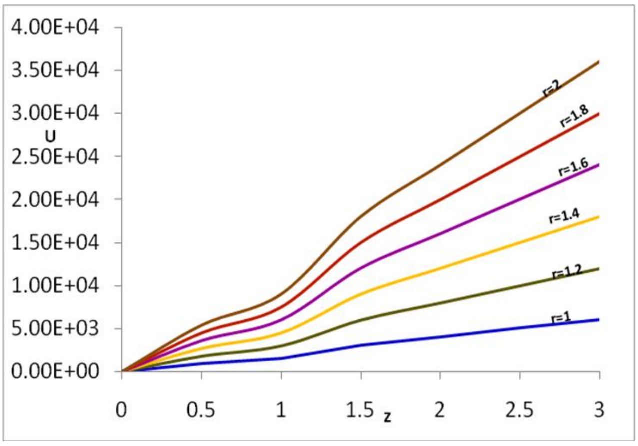

Figure 6: Graph of U(r,z,t) versus z for different values of r

Figure 7: represents graph of σ_{rr} versus z for different values of r. It is observed that σ_{rr} goes on decreasing **in** the region of hallow cylinder for different values of r.

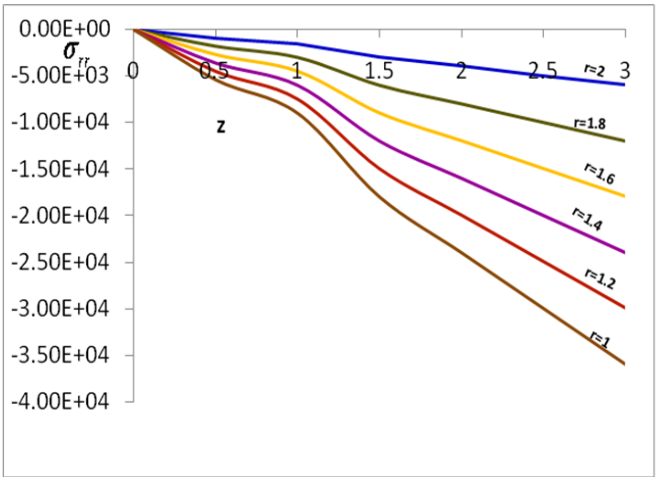

Figure 7: Graph of σ_{rr} versus z for different values of r

Figure 8: represents graph of $\sigma_{\theta\theta}$ versus z for different values of r. It is observed that $\sigma_{\theta\theta}$ is zero upto z=1 then from z=1 to z=3, $\sigma_{\theta\theta}$ goes on decreasing in the region of hallow cylinder for different values of r.

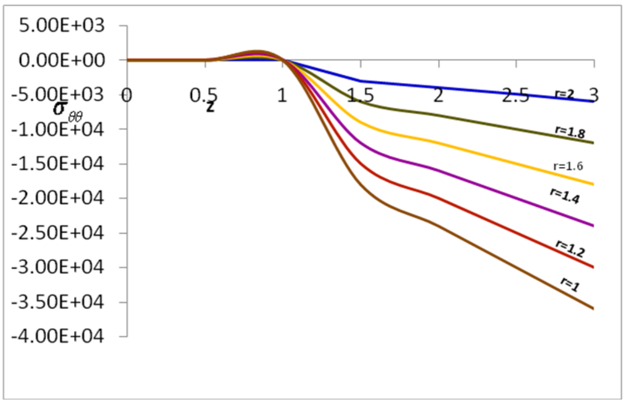

Figure 8: Graph of $\sigma_{\theta\theta}$ versus z for different values of r

5. Conclusion

In this study I treated the two-dimensional thermoelastic problem of the hallow cylinder with internal heat generation and temperature distribution $T(r,z,t)$,thermoelastic displacement $U(r,z,t)$ and thermal stresses have been determined with the help of finite Marchi – Zgrablich transform techniques. The results are obtained in terms of Bessel's function in the form of infinite series. Moreover, assigning suitable values to the parameters and functions in the equations of temperature, displacements and stresses respectively, expressions of special interest can be derived for any particular case. We may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behavior and illustrated graphically.

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