International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

Kantowski -Sachs Inflationary Cosmological Model with Bulk Viscosity and Varying Cosmological Constant in General Relativity

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Abstract: ***Kantowski-Sachs inflationary cosmological model with bulk viscosity is investigated. To get the deterministic model of the universe, we assume the relation between metric potentials $R = S^n$ where R and S are metric potentials and n is a constant. It is found that the cosmological constant Λ is positive and is a decreasing function of time, which is supported by results from recent supernova observations. To get an inflationary universe, we have considered flat region in which the potential V is constant. Some physical and geometrical properties of the models are also discussed.***

Keywords: Kantowski-Sachs, inflationary universe and variable cosmological constant.

1. Introduction

Any physical theory can be studied easily though the exact solutions of its mathematical structure. Therefore the exact solutions of relativistic model carry important role than those obtained through approximation scheme and numerical computation. Moreover the relativistic use various symmetries to get physical viable information from the complicated structure of the field equations in Einstein's theory. The origin of structure in the universe is one of the greatest cosmological mysteries even today. The present day observations indicate that the universe at large scale is homogeneous and isotropic and it is accelerating phase of the universe (recently detected experimentally) [1]. It is well known that exact solutions of general theory of relativity for homogeneous space times belongs to either Bianchi types or Kantowski-Sachs [2].

In particular models play an important role in solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. Guth [3] introduced the idea of early inflationary phase in context of grand unified field theories. Linde and La and Steinhardt [4-5] are some of the authors who have investigated several aspects of inflationary universe in general relativity. Wald, Gron, Barrow[6-8] discussed inflationary scenario in Fridmann-Robertson-Walker (FRW) model which is already homogeneous and isotropic. In general theory of relativity, the effect of bulk viscosity on the cosmology evolution has been studied by many researchers like Zimdahl, Sahni and Starobinski, Saha, Singh et al.[9-13]. Also Zeldovich, Bertolami Pradhan et al.[14-19] have investigated more significant cosmological models with cosmological constant Λ. Ratra and Peebles[20] discussed the cosmology with a time varying cosmological constant. Stein-Schabes [21] has shown that inflationary scenario is possible when potential $V(\phi)$ has flat region and Higgs field (ϕ) evolves slowly but the universe expands in an exponential way due to vacuum field energy. The role of self interacting scalar fields in inflationary cosmology has been investigated by Bhattacharjee and Baruha, Bali and Jain and Rahaman et al.[22-24] Reddy et al.[25-26] have discussed inflationary universe in general relativity in four and five dimensions and also presented a plane symmetric Bianchi type-I inflationary universe in general relativity. Adhav et.al [27] have studied Kantowski-Sachs cosmological model in presence of perfect fluid coupled with massless scalar field. Kantowski-Sachs inflationary universe in general relativity have been investigated by Katore et.al.[28]. Recently Mete et.al. [29-30] have presented Kantowski-Sachs bulk viscous fluid with time dependent Λ-term and also studied two fluid model in Kantowski-Sachs universe.

2. Metric and Field Equations

We consider the Kantowski- Sachs cosmological model in the form

$$ds^2 = dt^2 - R^2 dx^2 - S^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (1)$$

where R and S are functions of cosmic time t only. In case of gravity minimally coupled to a scalar field $V(\phi)$, we have

$$S = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right] d^4x \qquad (2)$$

which on variation of S with respect to dynamical fields leads to Einstein field equation

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij} + \Lambda g_{ij} \qquad (3)$$

with energy momentum tensor (T_{ij}) for scalar field is given by Guth [3] in presence of viscosity is given by

$$T_{ij} = \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_l\phi\partial^l\phi + V(\phi)\right]g_{ij} - \xi\theta(g_{ij} + v_i v_j) \qquad (4)$$

and $$\frac{1}{\sqrt{-g}}\partial_i\left[\sqrt{-g}\partial^i\phi\right]=-\frac{dV}{d\phi}\ , \tag{5}$$

where V is the effective potential, ϕ the Higgs field, ξ the coefficient of bulk viscosity and θ the expansion in the model.

We assume the coordinate to be commoving so that $v^1=0=v^2=v^3, v^4=1$.

The Einstein field equation (3) for the metric (1) leads to

$$2\frac{S_{44}}{S}+\frac{S_4^2}{S^2}+\frac{1}{S^2}+\Lambda=-8\pi\left[\frac{\phi_4^2}{2}-K-\beta\right] \tag{6}$$

$$\frac{R_{44}}{R}+\frac{S_{44}}{S}+\frac{R_4S_4}{RS}+\Lambda=-8\pi\left[\frac{\phi_4^2}{2}-K-\beta\right] \tag{7}$$

$$2\frac{R_4S_4}{RS}+\frac{S_4^2}{S^2}+\frac{1}{S^2}+\Lambda=8\pi\left[\frac{\phi_4^2}{2}+K\right] \tag{8}$$

For deterministic model of inflationary universe, we have assumed that $V(\phi)=K$ (constant) and $\xi(\theta)=\beta$ (constant)

We use the ansatz $\xi(\theta)=$constant because it has significant role to connect with occurrence of Little Rip (LR) cosmology using FRW metric as given by Brevik et al.[31].

Equation (5) leads to

$$\phi_{44}+\left(\frac{R_4}{R}+2\frac{S_4}{S}\right)\phi_4=0 \tag{9}$$

which gives

$$\phi_4=\frac{\alpha}{RS^2}\ , \tag{10}$$

where α is integrating constant and the subscript '4' denotes differentiation with respect to *t* equations (6) and (8) leads to

$$2\frac{S_{44}}{S}-2\frac{R_4S_4}{RS}=-8\pi\frac{\alpha^2}{R^2S^4}+8\pi\beta\ . \tag{11}$$

The above equation contains two unknowns R and S. To get the deterministic solution, we assume the shear (σ) is proportional to expansion (θ) which leads to

$$R=S^n\ , \tag{12}$$

where n is constant, Equation (11) and (12) leads to

$$2S_{44}-2n\frac{S_4^2}{S}=-8\pi\frac{\alpha^2}{S^{2n+3}}+8\pi\beta S \tag{13}$$

solving this equation we get

$$S_4^2=aS^{-2(n+1)}+bS^2\ , \tag{14}$$

where

$$a=\frac{4\pi\alpha^2}{2n+1}, b=\frac{4\pi\beta}{1-n} \tag{15}$$

equation (14) leads to

$$\frac{S^{n+1}ds}{\sqrt{a+bS^{2(n+2)}}}=dt \tag{16}$$

which leads to

$$S=l^{\frac{1}{n+2}}\sinh^{\frac{1}{n+2}}mT \tag{17}$$

and

$$R=l^{\frac{n}{n+2}}\sinh^{\frac{n}{n+2}}mT\ , \tag{18}$$

$$\text{where } l=\sqrt{\frac{a}{b}}, m=(n+1)\sqrt{b}, T=t+t_0 \tag{19}$$

Therefore the metric (1) reduces the form

$$ds^2=dT^2-l^{\frac{2n}{n+2}}\sinh^{\frac{2n}{n+2}}mT dr^2-l^{\frac{2}{n+2}}\sinh^{\frac{2}{n+2}}mT\left(d\theta^2+\sin^2\theta d\varphi^2\right) \tag{20}$$

3. The Physical and Kinematical Parameters

Equation (10) leads to

$$\phi=\frac{\alpha}{l}\log\tan\left(\frac{mT}{2}\right)+T_0 \tag{21}$$

The spatial Volume $V=a^3=l\sin\theta\sinh mT$ (22)

Expansion scalar

$$\theta=u^i_{;i}=m\tanh mT \tag{23}$$

$$\text{Shear scalar } \sigma=\sqrt{\frac{2}{3}}\frac{m(n-1)}{(n+2)}\tanh mT \tag{24}$$

Hubble Parameter

$$H=\frac{m\tanh mT}{3} \tag{25}$$

Cosmological constant

$$\Lambda(t)=8\pi\left(\frac{\alpha^2}{2l^2\sinh^2 mT}+K\right)-\frac{m^2(2n+1)}{(n+2)^2}\tanh^2 mT-\frac{1}{l^{\frac{2}{n+2}}\sinh^{\frac{2}{n+2}}mT} \tag{26}$$

The spatial volume for the model (20) increases with time exponentially, hence inflationary scenario exists in the model. This means that the model starts expanding with big-bang at *T = 0*.The cosmological constant in the model is decreasing function of time as time increases.

4. Conclusion

In this paper we have constructed the Kantowski–Sachs inflationary cosmological model with bulk viscosity and flat potential in presence of cosmological term Λ in general relativity. It is observed that the model is free from singularity and does not approach isotropy for large value of T. Since $\lim_{T\to\infty}\frac{\sigma}{\theta}=$constant, the model is not isotropic for large value of T .The Hubble parameter is initially large but tends to finite limit for large value of T.

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