Comparative Study of PCA and LDA

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Abstract: Face recognition gains a lot of courtesy in recent years due to its various applications in our societies. In the appearancebased face recognition classical principal component analysis (PCA) and linear component analysis (LDA) algorithms are widely used. Small database marks the robust result of these algorithms principally. These algorithms are mainly used for feature point extraction and dimensionality reduction in 2D face recognition. We present a comparison of both the algorithms and combination of these two algorithms.

Keywords: Principle component analysis; linear discriminant analysis; face recognition, dimensionality reduction, Eigenfaces

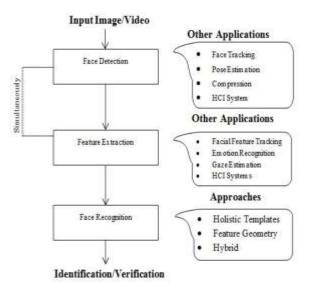
1. Introduction

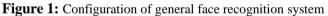
Distinct definition of face recognition is that the capability of a computer to scan, store, and recognize human faces for use in identifying people. Typical categorization of face recognition is [1]:

- 1. Holistic methods: these approaches recognize a face using the entire face images as an input. How to address the extremely small size problem is the main challenge faced by these methods is how to address the extremely small size problem.
- 2. Feature-based methods: these approaches used the local facial features for recognition. At the time of incorporation of global configuration information into local face methods care should be taken.
- 3. Hybrid Methods: these approaches used both featurebased and holistic features for face recognition. These methods results in robust result than individual one.

Face recognition performance reflected by two main modalities shape and texture respectively. Most of the face recognition techniques focus on the shape modality but texture modality also plays an important role in face recognition. Majorly appearance-based matching works on the texture modality. There are numerous algorithms works on the appearance-based matching LDA and PCA are the most likely used algorithms out of them. Configuration of face recognition is looks like a Fig. 1 [17].

This paper arranged as section 2 describes Principle Component Analysis (PCA) followed by section 3 describes Linear Discriminant Analysis. Section 4 states the comparative study of both the algorithms and analyses the results of both the algorithms. Finally we conclude in section 5.





2. Principle Component Analysis

Principle component analysis (PCA) is basically the dimensionality reduction procedure created by extracting the preferred number of principle components of the multidimensional data [3]. The first principle component is nothing but a linear grouping of the true dimensions that has the extreme variance; the n th principal component is the linear grouping with the uppermost variance, focus to being orthogonal to the n - 1 first principle components [4].

Eigenface approach adopted for face recognition is the key factor of PCA implementation [5], [6]. Early days different methods have seemed which gives robust result with Eigenfaces limited to certain constraints. There are number of applications of PCA such as, recognition of handprint, recognition of human made objects, mobile and industry robotics [7, 8, 9 and 10].

In the Eigenface approach-based PCA method face appearances are denoted as vectors of line-by-line concatenation of the pixels of the image. After that signifies mean face by calculating average vector. Likewise, a difference vector is calculated for each user to meet the requirements of mean face differences. Then calculate the covariance matrix from the difference vectors. Lastly, covariance matrix's eigen decomposition can be acquired by principle axes. Keep the highest eigen values offered by beginning N eigenvectors and signifies the most weighted features. Finally, weighted sum of coefficients equivalent to each eigenface is calculated to characterize each user model [11]. Training the system is the main objective of PCA. This method gives robust results at the time of new face images testing. In short simple understanding of PCA is shown in Fig.1 which is attractively and presently described by Patil and Kolhe [12].

PCA algorithm steps are:

1. Average Mean calculation by subtracting test image from individual images in the training samples

 $\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i (1)$

2. Make a column wise order of each pixel sample to create a co-variance matrix and compute the co-variance matrix with the use of followed formula

$$C = \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X}) (X_i - \bar{X})^T (2)$$

3. Discover the eigenvectors and its equivalent eigenvalues from the co-variance matrix by

 $CV = \lambda V$ (3)

where λ and V are respectively, eigenvector and eigenvalue respectively.

- 4. High to low order taxonomy of the eigenvector according to their corresponding eigen values can be done.
- 5. Every test image should be mean centered in the testing, then assign the test image into the similar eigen space as assigned in the training phase.
- 6. Compare estimated training copy of image in eigen space with estimated image.
- 7. Similarity is the measurement parameter used for image comparison. The training image which is nearest to the test image will be complemented and mark to use for identification [13].

Another approach of principle component analysis (PCA) is the kernel principle component analysis (K-PCA) which performs nonlinear mapping contradictory as PCA [14]. K-PCA uses the kernel methods. In which mapping is applied from input space to the feature space.

3. Linear Discriminant Analysis

When significant variations in illumination and expression are present, ample of the dissimilarity in the data is due to these variations. The PCA techniques primarily select a subspace that preserves best of that variation, and consequently the match in the face space is not really determined by the identity [4].

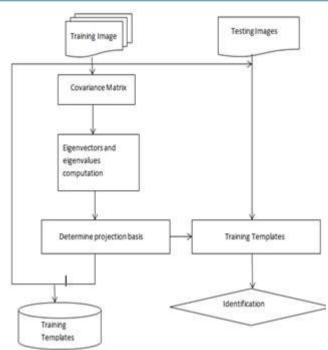


Figure 2: Simple flowchart of PCA algorithm



Figure 3: Simple flowchart of PCA algorithm

Belhumeur [15] proposed a method to solve this problem with "fisher faces", an application of Fisher's linear discriminant (FLD) which picks the linear subspace Ø, so it also called as linear discriminant analysis (LDA). In short simple understanding of LDA is shown in Fig.2 which is attractively and presently described by Patil and Kolhe [12]. Application of LDA is the mobile robotics [10].

LDA algorithms steps are:

1. Assume that C is the known design classes W_1, W_2, \dots, W_c and N is the training samples. $X = [X_j^i], i = 1, 2, \dots, I_c$ and $j = 1, 2, \dots, c$ is a set

of samples with $(m \times n)$ dimension.

$$\sum_{i=1}^{c} I_j = N \tag{4} 5.$$

where I_j is the total of training trials of class j.

2. Compute the average matrix of training appearance

$$\bar{X} = \frac{1}{N} \sum_{j=1}^{c} \sum_{i=1}^{l_j} X_j^i$$
(5)

3. LDA maximizes the ratio $\frac{\phi^T S_b \phi}{\phi^T S_b \phi}$

where

$$S_{b} = \sum_{i=1}^{m} N_{i} (\overline{x_{i}} - \overline{x}) (\overline{x_{i}} - \overline{x})^{T}$$
(7)

$$S_w = \sum_{i=1}^m \sum_{x \in X_i} (x - \overline{x_i}) (x - \overline{x_i})^T$$
(8)

where S_b and S_w are the between-class and within-class scatter matrix respectively; m is the number of classes in the gallery.

- 4. Inevitably, LDA finds the estimate of the data where classes are linearly separable. It can be demonstrated that the dimension of ϕ is at most m 1.
- 5. For the reason that S_w is generally singular, the LDA algorithm first eases the dimensionality of the data by using PCA algorithm. So, eq. (6) can be calculated and then applies LDA to further reduce the dimensionality to m-1.
- 6. The recognition is then trained by a Neural Network kind of classifier in this complete subspace.

4. Comparative and Result Analysis

In the appearance-based face recognition consider that LDA is superior as compared to PCA. But Martinez and Kak [16] proposed an example to show that this is not the always true consideration. PCA can outperform LDA when the training dataset is small as well as PCA is less sensitive to different kinds of training images.

In training, it is also not surprising to use both LDA and PCA in combination. For example, PCA is used for dimensionality reduction followed by an LDA. When the training set is large LDA outperforms PCA and when the training set is small PCA outperforms LDA. The percentage of LDA recognition is higher than PCA when the numbers of samples in the database are same.

Table I states the comparison of principle component analysis (PCA) algorithm and algorithm for the linear discriminant analysis (LDA).

Table 1:	Comparison	of PCA	and LDA
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PCA	LDA		
Unsupervised Algorithm	Supervised Algorithm		
Ignores class labels and its aim is to find out the principle components which maximize the variance in the dataset.	Calculates the linear discriminants that will represent the axes which maximize the separation between multiple classes.		
Feature classification	Data classification		

5. Conclusion

Principle component analysis and linear discriminant analysis are the most preferably used algorithms in the circumstance of appearance-based matching in face recognition. When the number of samples in the database are small these algorithms gives robust results. Both algorithms are outperformed by each other depends on the training dataset as discussed in the result analysis section.

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