

# A Cylindrical Model for Muco-Ciliary Transport in The Human Trachea: Effects of Mucus-Visco Elasticity and Porosity

M. Chitra<sup>1</sup>, S. Radhakrishnan<sup>2</sup>

<sup>1</sup>Associate Professor, Department of Mathematics, Thiruvalluvar University, Serkadu, Vellore, Tamilnadu

<sup>2</sup>Research Scholar, Department of Mathematics, Thiruvalluvar University, Serkadu, Vellore, Tamilnadu

**Abstract:** A Cylindrical two layer fluid model for the transport of serous and mucus layer in the human trachea and the steady state condition due to cilia beating, some immotile cilia forming porous bed in serous sub -layer in contact with the epithelium and air motion by considering mucus as a visco-elastic fluid are studied. The effect of air-motion due to forced expiration and other processes is considered by prescribing shear stress at mucus air interface. It is shown that the transport of mucus increases as the pressure drop, the velocity generated by cilia tips and porosity parameter increase. It is noted that the effect of gravity is similar to that of the pressure drop. It is observed that the transport of mucus decreases as the viscosity of serous layer fluid or mucus increases, but any increase in mucus viscosity at its higher values does not seem to affect the mucus transport. It is also found that for given total depth of serous layer and mucus layer, there exists a serous fluid layer thickness for which mucus transport is maximum.

**Keywords:** cilia, porosity, serous sub layer, mucus layer, visco-elasticity.

## 1. Introduction

The physiology of the respiratory tract has interested in man for centuries. The movement of mucus by cilia in the upper part of the respiratory tract, and in particular the mechanism of this movement. Many micro-organisms are covered with cilia, the movement of which results in their vital locomotion. The main functions of this mucus sub-layer is in cleaning the inspired air of unwanted particles (e.g. aerosols, bacteria, viruses, and carcinogens in tobacco smoke) and bringing the air to near body temperature while the relative humidity is brought close to saturation. Failure of the cilia, which leads to in-efficient removal of the collected particles, may lead to disease.

The respiratory system has several means protecting itself from dust. As the inhaled air is drawn in through the mouth or nose, it starts to travel a path that has numerous branches. While small particles move with the air around then bends and branches, larger particles strike the sides of the airways at these bends. Here they get stuck in the mucus lining and are removed from the system. Several other features enhance the effectiveness of the structure, keeping dust out of the system, or working to remove it after entry.

Hairs in the nose filter out large particles of dust and deposit them in sticky mucus, also called phlegm, which slowly moves towards the exterior of the body and is periodically sneezed, spit out, coughed up, or swallowed. Mucus is produced by the mucus membranes of the windpipe and the bronchi. It traps large particles of dust as they flow by in inhaled air. Mucus is continually being moved forward towards the mouth by cilia, which are tiny hair like projections. Waving in time with each other, the cilia move a layer of mucus forward to carry most of the trapped particles up the windpipe. When the mucus reaches the mouth, it is spit out or swallowed.

The mucus layer that is secreted from the underlying cells has a highly viscous quality that enables it to capture the air born particles and to remain on the outside of the respiratory tubes around the cilia. Mucus covers the ciliated epithelium of the respiratory tract, which includes the nose, trachea, sinuses, and the proximal bronchioles. The mucus continually moves upwards the upper end of the trachea. The regular airflow reversals are obviously very important in contributing to the particle deposition on the surface of the mucus layer which is one of the main functions of the mucus.

In some models of the ciliated epithelia, the upper layer has been assumed to be visco-elastic while the serous sub layer (which contains the cilia) consists of a Newtonian viscous fluid. It consists of three layers namely: a mucus layer, a serous layer and the cilia forming porous bed in serous sub layer which are small hair-like projections lining with the epithelium of the bronchial respiratory tracts. The serous layer fluid is considered as a Newtonian fluid while mucus as a visco-elastic fluid. It has been pointed out that in general mucus transport depends upon the structure of cilia, the force imparted by cilia tips in the serous sub layer fluid, the thicknesses and the viscosities of the serous fluid and mucus and the interaction of mucus with the serous layer fluid. Mucus transport is also dependent on the pressure drop in the airways generated by the processes such as inspiration, expiration, coughing etc. and gravitational force(Blake[9] and Sleight et al.[6]).

In recent decades, the mucus flow in the trachea has been studied by several researchers. In particular, an analytical model has been presented by Barnton and Raynor[11] by considering the cilium as an oscillating cylinder with a greater height during the effective stroke and a smaller height during the recovery stroke. Blake and Winet[8] suggested that if the cilia just penetrate the upper much more viscous

layer, then the mucus flow rate is substantially enhanced. King et al.[4] presented a two-layer steady state mathematical model for mucus transport by introducing cilia tip velocity, and mucus interface is studied by Nirmala P Ratchagar., M.Chitra.[13].

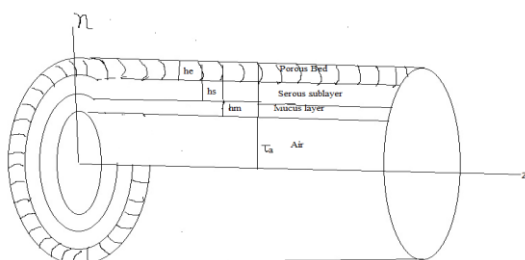
In their model, Agarwal and Verma[3] and Verma[1,2] have studied the mucus transport by analyzing the effect of porosity due to the formation of porous matrix bed by immotile cilia. Very little attention has been paid to explain these observations using mathematical models by considering mucus as a visco-elastic fluid. Ross and Corrsin[10] modeled muco-ciliary pumping by representing the beating of cilia by a travelling surface wave(envelop) and predicted that mucus behaves like an elastic slab and its transport decreases as the fractional depth of serous layer fluid decreases. King et al.[4] have given a planar two layer model by considering mucus as a visco-elastic fluid. They have taken the effect of cilia beating air-motion due to forced expiration and other processes by prescribing the mucus-air interface.

In view of the above, we are interested to study the muco-ciliary transport in the human trachea by taking the following aspects into account:

- (1) The serous layer fluid is considered as incompressible Newtonian fluid while mucus layer is considered as a visco-elastic fluid.
- (2) The serous layer fluid is divided into two sub-layers, one in contact with the epithelium and other in contact with the mucus. It is assumed that cilia during beating impart a velocity at the mean level of their tips, causing the serous sub – layer in contact with mucus to undergo motion. It is also assumed that certain cilia are immotile and form a porous matrix bed in serous sub layer in contact with the epithelium where flow may occur due to pressure gradient as considered by Beavers and Joseph[3]. No net flow is assumed in the serous sub layer in contact with epithelium.
- (3) The effects of net flow of velocity of serous sub-layer and mucus layer relating with the shear stress of mucus air interface as a boundary condition are also taken into consideration in the model.
- (4) The effects of pressure gradients and gravitational force are also taken into consideration in the model.

## 2. Mathematical Formulation

The physical situation of the transport of serous fluid and mucus in the trachea may be represented by a circular two-layer fluid model.



**Figure 1:** Cylindrical model for muco-ciliary transport in human trachea

In the serous sub layer  $0 \leq y \leq h_e$ , no net flow of the fluid is assumed. However, in the serous sub layer  $h_e \leq y \leq h_s$  and in the mucus layer  $h_s \leq y \leq h_m$ , the flow of respective fluid is governed by interactions of cilia, air motion in contact with the mucus, pressure gradient present in the fluid and gravitational force. The equations governing the motion of the serous layer fluid and the mucus under steady state and low Reynold's number flow approximations, by taking the effect of gravitational force in the direction of flow, can be written as follows:

Region – I, Serous layer ( $h_e \leq y \leq h_s$ )

$$\mu_s \left[ \frac{1}{y} \frac{\partial}{\partial y} \left( y \frac{\partial u_s}{\partial y} \right) \right] = \frac{\partial p}{\partial x} - \rho_s g \cos \alpha \quad \text{-----(1)}$$

Region – II, Mucus Layer ( $h_s \leq y \leq h_m$ )

$$\mu_m \left[ \frac{1}{y} \frac{\partial}{\partial y} \left( y \frac{\partial u_m}{\partial y} \right) \right] = \frac{\partial p}{\partial x} - \rho_m g \cos \alpha \quad \text{-----(2)}$$

Where p is the pressure that is constant across the layers;  $u_s$  and  $u_m$  are the velocity components of serous sub-layer fluid and mucus in x-direction respectively;  $\rho_s, \mu_s, \rho_m$  and  $\mu_m$  are their respective densities and viscosities; g is the acceleration due to gravity and  $\alpha$  is the angle by which the airway in trachea is inclined with the vertical. Here,  $h_e$  is the mean thickness the measured from the surface of the epithelium to the tips of cilia during beating i.e, the interface between measured from the surface of the epithelium to the interface between serous sub layer and mucus and  $h_m$  is the thickness measured form the surface of the epithelium to the mucus air interface.

The following boundary and matching conditions are taken for the system of equations,

## 3. Boundary Conditions

$$u_s = U_0 + \beta \frac{\partial u_s}{\partial y}, \quad \text{at } y = h_e \quad \text{-----(i)}$$

$$-\mu_m \frac{\partial u_m}{\partial y} = -\mu_a \frac{\partial u_a}{\partial y} \quad \text{at } y = h_m \quad \text{-----(ii)}$$

Where  $U_0$  is the mean velocity imparted by cilia tips during beating in the serous sub layer at  $y=h_e$  and  $\beta$  is the porosity parameter due to immotile cilia forming porous matrix bed in the serous sub-layer in contact with epithelium where flow may occur due to pressure gradient.

Matching conditions:

$$u_s = u_m = U_1 \quad \text{at } y = h_s \quad \text{-----(iii)}$$

$$\mu_s \frac{\partial u_s}{\partial y} = \mu_m \frac{\partial u_m}{\partial y} \quad \text{at } y = h_s \quad \text{-----(iv)}$$

Where  $U_1$  is the mucus-serous sub layer interface velocity to be determined by using equation (iv). The conditions (iii) and (iv) imply that the velocities shear stress are continuous at mucus-serous layer interface.

Solving the equation (1), We get,

$$\therefore \mu s u_s = \frac{\varphi_s y^2}{4} + A \log y + B \text{-----(3)}$$

And applying the B.C (i) we get,

$$\therefore \mu s U_0 = -\frac{\varphi_s}{4} (h_s^2 - 2\beta h_s^2) + A \left( \log h_s - \frac{\beta}{h_s} \right) + B \text{-----(4)}$$

Solving equation(2) and applying B.C (iii), we get,

$$\mu m(u_m) = \varphi_m \frac{y^2}{4} + C \log y + D \text{-----(5)}$$

Applying BC (iii), From equ (3) & (5), we get

$$\mu m(U_1) = \varphi_m \frac{h_s^2}{4} + C \log h_s + D \text{-----(6)}$$

$$\mu s(U_1) = \frac{\varphi_s h_s^2}{4} + A \log h_s + B \text{-----(7)}$$

Applying BC (iv), we get values A,B,C,D and substitute the equation (3) gives

$$u_s = \frac{\varphi_s}{4\mu s} (y^2 + h_s^2 + 2\beta h_s) + U_0 + \frac{1}{\mu s} \left( \log y - \log h_s + \frac{\beta}{h_s} \right) \left[ (\varphi_m - \varphi_s) \frac{h_s^2}{2} + \mu a \frac{\partial u_a}{\partial y} h_m - \frac{\varphi_m h_m^2}{2} \right] \text{-----(8)}$$

From equ (5)

$$u_m = \frac{\varphi_s}{4\mu m} (y^2 - h_s^2 - 2\log y h_m^2 + 2(\log h_s) h_s^2) + \frac{(\mu m - \mu s) U_1}{\mu m} + \frac{\varphi_s}{4\mu m} [h_s^2 - 2h_s^2(\log h_s) - h_s^2 - 2\beta h_s] + \frac{\mu a}{\mu m} \log y \frac{\partial u_a}{\partial y} h_m - \frac{\log h_s - \beta}{\mu m} \left[ \mu a \frac{\partial u_a}{\partial y} h_m - \frac{\varphi_m h_m^2}{2} \right] + \frac{\mu s}{\mu m} U_0 \text{-----(9)}$$

Applying M.C (iii)

$$U_1 = U_0 + \frac{\varphi_s}{4\mu s} \left( h_s^2 - h_s^2 + 2\beta h_s - 2h_s^2 \log h_s + 2h_s^2 \log h_s - \frac{2\beta h_s^2}{h_s} \right) + \frac{\mu a \partial u_a}{\mu s \partial y} h_m \left( \log h_s - \log h_s + \frac{\beta}{h_s} \right) + \frac{\varphi_m}{2\mu s} \left[ (h_s^2 - h_m^2) \left( \log h_s - \log h_s + \frac{\beta}{h_s} \right) \right] \text{-----(10)}$$

Using dimensionless quantities,

$$\eta = \frac{y}{h_m}, \bar{\beta} = \frac{\beta}{h_m}, \bar{h}_s = \frac{h_s}{h_m}, \bar{h}_m = \frac{h_m}{h_m}, \bar{\mu}_s = \frac{\mu_s}{\mu_0}, \bar{\varphi}_s = \frac{\varphi_s h_m^2}{\mu_0 U_0}, \bar{\varphi}_m = \frac{\varphi_m h_m^2}{\mu_0 U_0}, \bar{h}_m = \frac{\mu_m}{\mu_0}, \bar{\varphi}_{s0} = \frac{\varphi_{s0} h_m^2}{\mu_0 U_0}, \bar{\varphi}_{m0} = \frac{\varphi_{m0} h_m^2}{\mu_0 U_0}, \bar{\tau}_a = \frac{\tau_a h_m}{\mu_0 U_0}, \bar{Q}_m = \frac{Q_m}{h_m U_0}$$

$$u_s = \frac{\bar{\varphi}_s U_0}{4\bar{\mu}_s \bar{h}_m^2} [h_m^2 \eta^2 - \bar{h}_s^2 h_m^2 + 2\bar{\beta} h_m^2 \bar{h}_s] + U_0$$

$$- \frac{1}{\mu m} \left( \log h_s - \frac{\beta}{h_s} \right) (h_m - h_s) \left[ (\varphi_m - \varphi_s) \frac{h_s^2}{2} + \mu a \frac{\partial u_a}{\partial y} h_m - \frac{\varphi_m h_m^2}{2} \right] \text{-----(14)}$$

$$\bar{Q}_m = \frac{\bar{\varphi}_{m0}}{4\bar{\mu}_m} \left[ \frac{\bar{h}_m}{3} - \frac{\bar{h}_s}{3} + \bar{h}_s^3 - h_s^2 + 2\bar{h}_s^2 h_m \log \bar{h}_s h_m - 2\bar{h}_s^3 h_m \log \bar{h}_s h_m \right] + \frac{\bar{\mu}_s}{\bar{\mu}_m} (1 - \bar{h}_s) + \left( \frac{(\bar{\mu}_m - \mu s)}{\bar{\mu}_m U_0} \right) (1 - \bar{h}_s)$$

$$+ \frac{1}{\bar{\mu}_s \bar{\mu}_0} \left[ \log(h_m \eta) - \log(\bar{h}_s h_m) + \frac{\bar{\beta}}{\bar{h}_s} \right] \left[ (\bar{\varphi}_m - \bar{\varphi}_s) \frac{\mu_0 U_0}{h_m^2} \frac{\bar{h}_s^2 h_m^2}{2} + \frac{\mu a}{h_m} \frac{\partial u_a}{\partial \eta} h_m - \frac{\bar{Q}_m \mu_0 U_0}{h_m^2} \right] \text{-----(11)}$$

$$\varphi_{s0} = \left( \frac{\Delta p}{\Delta L} + \rho_s g \cos \alpha \right)$$

$$\varphi_{m0} = \left( \frac{\Delta p}{\Delta L} + \rho_m g \cos \alpha \right)$$

Where  $\Delta p = p_0 - p_L$ ,  $p = p_0$  at  $Z=Z_0$ ,  $p = p_L$  at  $Z=L$ . It is noted that the effect of acceleration due to gravity is similar to that of the pressure drop.

$$u_m = \frac{\bar{\varphi}_{m0} h_m^2}{4\bar{\mu}_0 \bar{\mu}_m \bar{\mu}_0} [h_m^2 \eta^2 - \bar{h}_s^2 h_m^2 + 2h_m^2 \log(h_m \eta) + 2 \log(\bar{h}_s h_m) \bar{h}_s^2 h_m^2] + \frac{\bar{\mu}_m - \bar{\mu}_s}{\bar{\mu}_m} U_1 + \bar{\tau}_a \mu_0 U_0 \log(h_m \eta) + \frac{\bar{\mu}_s U_0}{\bar{\mu}_m} + \frac{\bar{\varphi}_{s0} U_0}{4\bar{h}_m^2 \bar{\mu}_m} [h_s^2 h_m^2 - 2\bar{h}_s^2 h_m^2 \log(\bar{h}_s h_m) - \bar{h}_s^2 h_m^2 + 2\bar{\beta} h_m^2 \bar{h}_s] - \left[ \frac{\log(\bar{h}_s h_m) - \frac{\bar{\beta}}{\bar{h}_s}}{\bar{\mu}_m \bar{\mu}_0} \right] \left[ (\bar{\varphi}_{m0} - \bar{\varphi}_{s0}) \frac{h_s^2 \mu_0 U_0}{2\bar{h}_m^2} + \bar{\tau}_a \mu_0 U_0 - \frac{\varphi_{m0} \mu_0 U_0}{2} \right] \text{-----(12)}$$

$$Q_s = \int_{h_s}^{h_m} u_s dy$$

$$Q_s = \frac{\varphi_s}{4\mu s} (h_s - h_m) \left[ \frac{1}{3} (h_s^2 + h_s h_m + h_m^2) - h_s^2 + 2\beta h_s \right] + U_0 (h_s - h_m) + \frac{1}{\mu s} [h_s \log h_s - h_s \log h_s - h_s + h_s + \left( \frac{\beta}{h_s} - \log h_s \right) (h_s - h_m)] \left[ \tau_a h_m + \varphi_m \left( \frac{h_s^2}{2} - \frac{h_m^2}{2} \right) - \frac{\varphi_s}{2} h_s^2 \right] \text{-----(13)}$$

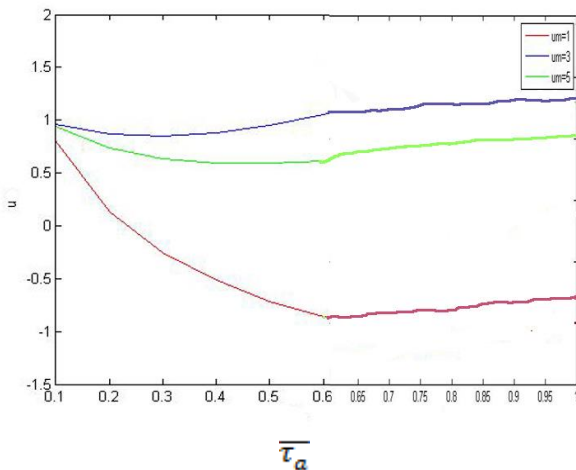
$$Q_m = \int_{h_s}^{h_m} u_m dy$$

$$Q_m = \frac{\varphi_m}{4\mu m} \left[ \frac{h_m^3}{3} - \frac{h_s^3}{3} + h_s^3 - h_s^2 h_m + 2h_s^2 h_m \log h_s - 2h_s^3 \log h_s \right] - 2h_m^2 \frac{\varphi_m}{4\mu m} [h_m (\log h_m - 1) - h_s (\log h_s - 1)] + \frac{(\mu m - \mu s)}{\mu m} U_1 (h_m - h_s) + \frac{\mu s}{\mu m} U_0 (h_m - h_s) + \frac{\varphi_s}{4\mu m} [h_s^2 - h_s^2 + 2\beta h_s - 2h_s^2 \log h_s] (h_m - h_s) + \frac{\mu a \partial u_a}{\mu s \partial y} h_m [h_m (\log h_m - 1) - h_s (\log h_s - 1)]$$

$$\begin{aligned}
 & + \frac{1 - \bar{h}_s}{4\bar{\mu}_m} [\bar{h}_s^2 - \bar{h}_s^2 - 2\bar{h}_s^2 \log \bar{h}_s \bar{h}_m + 2\beta \bar{h}_s] \\
 & + \mu_0 \bar{\tau}_a \left[ \log(\bar{h}_m - 1) - \bar{h}_s (\log \bar{h}_s \bar{h}_m - 1) \right] \\
 & - \frac{1}{\bar{\mu}_m \bar{h}_m} \left[ \log \bar{h}_s \bar{h}_m - \frac{\beta}{\bar{h}_s} \right] \left[ 1 - \bar{h}_s \right] \left[ \frac{(\bar{\varphi}_{m_0} - \bar{\varphi}_{s_0}) \bar{h}_s \bar{h}_m}{2} + \bar{\tau}_a - \frac{\bar{\varphi}_{m_0}}{\bar{h}_m} \right]
 \end{aligned}
 \tag{15}$$

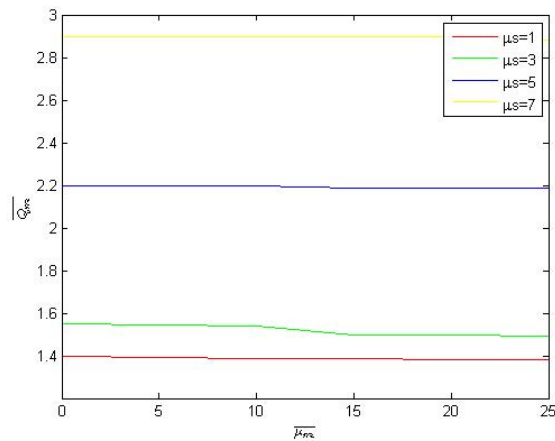
#### 4. Results

To study the effect of velocity of the serous sub layer ( $u_s$ ), mucus layer ( $u_m$ ) and mucus flow rate ( $Q_m$ ) are related with viscosity of mucus ( $\mu_m$ ), viscosity of serous sub layer ( $\mu_s$ ), and shear stress ( $\tau_a$ ) of airways are discussed. Where  $\mu_0$  is the viscosity of the serous sub layer fluid in contact with epithelium. Expression for  $Q_m$  given by (15) is plotted in Fig. 2 to 7 using the following set of parameters which have been calculated by using typical values of various characterizes related to airways (King et al.[4], Agarwal and Verma[3]).



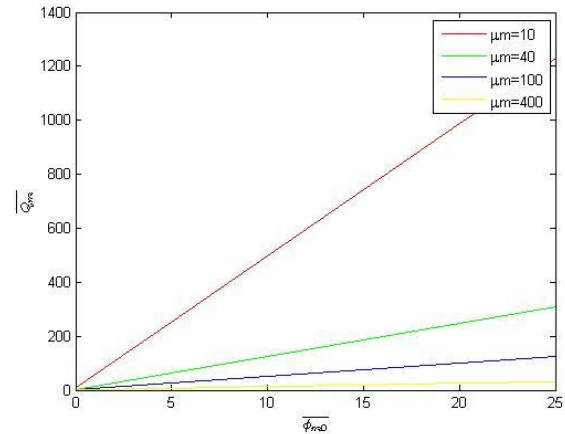
**Figure 2:** Variation of velocity of muco-ciliary ( $u = u_s + u_m$ ) with shear stress  $\bar{\tau}_a$  for different values of  $\bar{\mu}_s$  and  $\bar{\mu}_m$

Figure.2 shows that the velocity of muco-ciliary transport  $u$  ( $u = u_s + u_m$ ) increases as the shear stress  $\bar{\tau}_a$  generated by air-motion at mucus- air interface increases. Also it decreases as the viscosity of serous  $\bar{\mu}_s$  and viscosity of mucus layer increases  $\bar{\mu}_m$ .



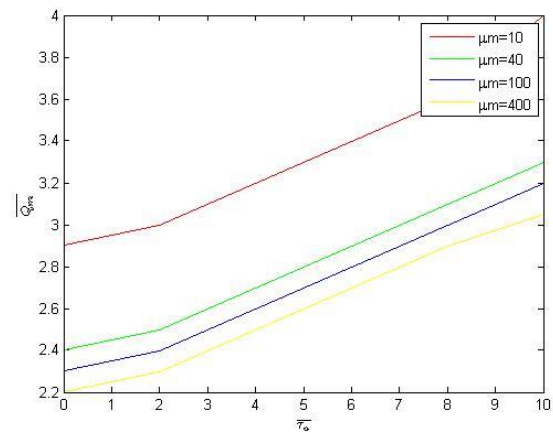
**Figure 3:** Variation of  $Q_m$  with  $\bar{\mu}_m$  for different values of  $\bar{\mu}_s$

Fig.3 shows that the variation of flow rate of mucus  $Q_m$  with  $\bar{\mu}_m$  for different values of  $\bar{\mu}_s$  and for fixed values of  $\beta = 0.02$ ,  $\bar{h}_s = 0.1$ ,  $\bar{h}_s = 0.2$ ,  $\bar{\tau}_a = 5$ ,  $\bar{\varphi}_{s_0} = 1$ ,  $\bar{\varphi}_{m_0} = 20$ ,  $\bar{\lambda}_0 = 0.05$ . The figure illustrates that the flow rate of mucus transport decreases as the viscosity of the serous layer fluid or mucus increases; However, increase in mucus viscosity at higher values do not have any significant effect on its transport.



**Figure 4:** Variation of  $Q_m$  with  $\bar{\varphi}_{m_0}$  for different values of  $\bar{\mu}_m$

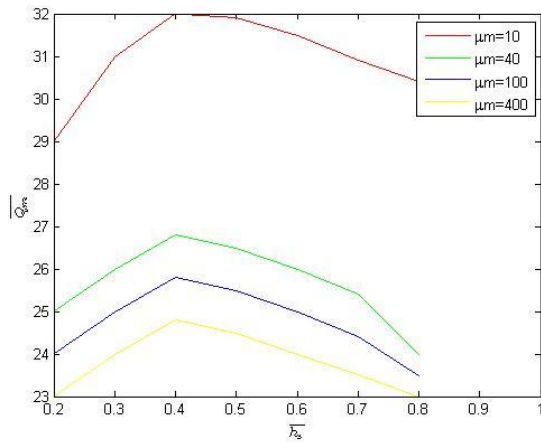
Fig.4 shows that the variation of flow rate of mucus  $Q_m$  with  $\bar{\varphi}_{m_0}$  for different values of  $\bar{\mu}_m$  and fixed values of  $\beta = 0.02$ ,  $\bar{h}_s = 0.1$ ,  $\bar{h}_s = 0.2$ ,  $\bar{\tau}_a = 5$ ,  $\bar{\varphi}_{s_0} = 1$ ,  $\bar{\lambda}_0 = 0.05$ . The figure illustrates that the mucus transport rate increases as the pressure drop or force due to gravity increases but it decreases with increase in mucus viscosity, the relative decrease being larger at larger values of pressure drop.



**Figure 5:** Variation of  $Q_m$  with  $\bar{\tau}_a$  for different values of  $\bar{\mu}_m$

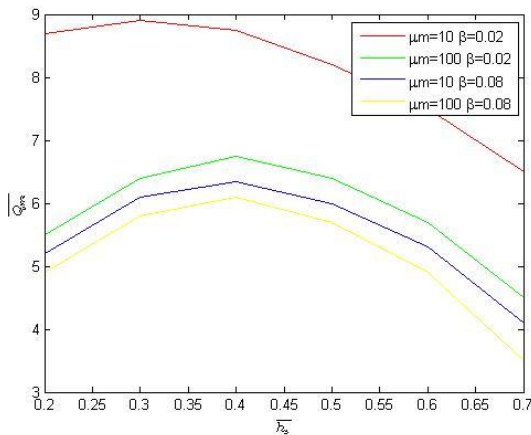
Fig.5 shows the variation of flow rate of mucus  $Q_m$  with  $\bar{\tau}_a$  for different values of  $\bar{\mu}_m$  and fixed values of  $\beta = 0.02$ ,  $\bar{h}_s = 0.1$ ,  $\bar{h}_s = 0.2$ ,  $\bar{\tau}_a = 1$ ,  $\bar{\varphi}_{s_0} = 1$ ,  $\bar{\varphi}_{m_0} = 20$ ,  $\bar{\lambda}_0 = 0.05$ . The figure illustrates that the mucus transport

rate increases as generated by air-motion at mucus-air interface increases, but it decrease as its viscosity increases, the relative decrease being larger at higher values of the shear stress.



**Figure 6:** Variation of  $\overline{Q_m}$  With  $\overline{h_s}$  for different values of  $\overline{\mu_m}$

Fig.6 shows the variation of flow rate of mucus  $\overline{Q_m}$  with  $\overline{h_s}$  for different values of  $\overline{\mu_m}$  and fixed values of  $\overline{\beta} = 0.02$ ,  $\overline{h_s} = 0.1$ ,  $\overline{\mu_s} = 1$ ,  $\overline{\varphi_{m_0}} = 20$ ,  $\overline{\tau_a} = 5$ ,  $\overline{\lambda_0} = 0.05$ . The figure illustrates that mucus transport rate increases as  $\overline{h_s}$  increases up to critical values of  $\overline{h_s} (\approx 0.4)$ , after which it starts decreasing with increasing  $\overline{h_s}$ . Since  $\overline{Q_m}$  approaches zero as  $\overline{h_s}$  tends to unity, this implies that for a fixed total thickness of mucus and serous layer, there exist a maximum of  $\overline{Q_m}$  for some values of serous layer thickness. The results being dependent on magnitudes of various parameter involved in equation(15). We further note that the  $\overline{Q_m}$  decreases as  $\overline{\mu_m}$  increases.



**Figure 7:** Variation of  $\overline{Q_m}$  with  $\overline{h_s}$  for different values of  $\overline{\mu_m}$  and  $\overline{\beta}$

Fig.7 shows the variation of flow rate of mucus  $\overline{Q_m}$  with  $\overline{h_s}$  for different values of  $\overline{\mu_m}$  and  $\overline{\beta}$  fixed values of,  $\overline{h_s} = 0.1$ ,  $\overline{\mu_s} = 1$ ,  $\overline{\varphi_{s_0}} = 1$ ,  $\overline{\varphi_{m_0}} = 20$ ,  $\overline{\tau_a} = 5$ ,  $\overline{\lambda_0} = 0.05$ . The figure illustrates that mucus transport rate  $\overline{Q_m}$  increases as

$\overline{h_s}$  increases up to optimum value of  $\overline{h_s}$  after which it starts decreasing with increasing value of  $\overline{h_s}$ , but it decreases with increase in its viscosity and increases as porosity parameter increases, since  $\overline{Q_m}$  approaches zero as  $\overline{h_s}$  tends to unity, this implies that for a fixed total thickness of mucus and serous layer there is maximum of  $\overline{Q_m}$  for some values of serous layer thickness.

## 5. Conclusion

The objective of this paper is to study the basic equation mechanisms between the serous sub-layer and the mucus region and to predict the shear stress at mucus interface. A simplified case of cylindrical two layer modal of the mucus transport in the human trachea under steady State condition due to cilia beating, some immotile cilia forming porous matrix bed in serous sub-layer in contact with the epithelium and air motion by considering mucus as a visco-elastic fluid are obtained. There is general agreement that only those particles experience slow bronchial clearance, which are transported to the sol layer or to the epithelial surface. slow bronchial clearance was attributed to three mechanisms.(i)uptake by macrophages,(ii)transfer through the epithelium to blood or intracellular accumulation, and (iii)re-transfer from the sol to the gel phase and subsequent removal by muco ciliary action. Airborne dust most often enters the body by being inhaled. The most hazardous dust particles are those of repairable size. When inhaled, they can reach the air sacs in the lungs. The health hazard presented by a particular dust depends on the size and shape of the dust particles, their chemical composition and the intensity and duration of exposure.

The health effects caused by some dusts are quite specific. The silica in quartz dust can cause silicosis. Asbestos fibers can cause asbestosis, lung cancer or mesothelioma, an otherwise rare cancer of the abdominal cavity. Cement dust can cause dermatitis, a disease of the skin.

Health effects of over exposure to toxic dusts may be respiratory or non-respiratory. That is, they may affect the lungs and other parts of the respiratory system, or may attack other body organs. Except for skin diseases, most occupational diseases are caused by the inhalation of minute particles of materials used in the work area. Lung tissue has a surface area of some 55 to 75 square meters (m<sup>2</sup>), and thus easily captures and absorbs airborne contaminants. Moreover, the air sacs deep in lungs are separated from the bloodstream by only one thin layer of cells. These factors can result in the rapid and widespread distribution of toxic materials throughout the body.

When a toxic dust is breathed in, the toxin may be taken into the lungs and subsequently dissolved in the lung fluids. It may have a localized effect on the lung tissue or pass through the cellular barrier in the alveoli to enter the bloodstream. Then it can be transported throughout the body to points where it can exert its harmful effects. Certain sites may become “target organs”. For example, when lead dust reaches the circulatory system, it acts on the liver, kidneys, or central nervous system.

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