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Multiple Laplace Transform of the Product of Generalized H- Functions

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Abstract: ***In this paper a relation between multiple Mellin transform and multiple Laplace transform is established. Using this relation a multiple Laplace transform involving the product of two H-functions of 'r' variables with different arguments is evaluated.***

Special cases include the result is given by Gupta and Goyal [1], Nair and Samar [4].

Notations and Results Used.

i) $(a)_n = a(a+1)..........(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}; n \geq 1.$

ii) $(a)_p = a_1,.........,a_p$

iii) $(a_j, A_j)_{1,p} = (a_1, A_1),.........,(a_p, A_p)$

iv) $(a_j;\alpha_j, A_j)_{1,p} = (a_1;\alpha_1, A_1),.........,(a_p;\alpha_p, A_p)$

Mathai and Saxena [p, 10-11,159].

$$H_{1,2}^{2,0}\left[x\left|\begin{matrix}(a-\lambda+1,1)\\(a\pm\mu+\frac{1}{2},1)\end{matrix}\right.\right] = x^a e^{-\frac{x}{2}} W_{\lambda,\mu}(x) \quad (1.2.1)$$

Where $W_{\lambda,\mu}(x)$ denote the Whittaker function.

Erdelyi:[9,P.307].

$$\int_0^\infty x^{s-1}\left(\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} g(s)x^{-s}ds\right)dx = g(s) \qquad (1.2.2)$$

PRELIMINARY RESULT:

$$\int_0^\infty ......\int_0^\infty x_1^{\eta_1-1}.....x_r^{\eta_r-1} H[u_1 x_1^{\sigma_1},.....,u_r x_r^{\sigma_r}] H^/[v_1 x_1^{\mu_1},.....,v_r x_r^{\mu_r}] dx_1....dx_r$$

$$= \frac{v_1^{\frac{-\eta_1}{\mu_1}}.......v_r^{-\frac{\eta_r}{\mu_r}}}{\mu_1.......\mu_r} \times A_{P+Q^/,P^/+Q:(p_1+q_1^/,p_1^/+q_1);.......;(p_r+q_r^/,p_r^/+q_r)}^{N^/\qquad N:(m_1+n_1^/,m_1^/+n_1);......;(m_r+n_r^/,m_r^/+n_r)}$$

$$\left[\begin{matrix}u_1 v_1^{\frac{-\sigma_1}{\mu_1}}\\ .\\ .\\ u_r v_r^{\frac{-\sigma_r}{\mu_r}}\end{matrix}\left|\begin{matrix}(a_j,\alpha_j^{(1)},......,\alpha_j^{(r)})_{1,N},(1-b_j^/-\frac{\eta_1}{\mu_1}\beta_j^{/(1)}-......-\frac{\eta_r}{\mu_r}\beta_j^{/(r)};\frac{\sigma_1}{\mu_1}\beta_j^{/(1)},.....,\frac{\sigma_r}{\mu_r}\beta_j^{/(r)})_{1,Q^/},\\ \\ \\ (b_j,\beta_j^{(1)},......,\beta_j^{(r)})_{1,Q},(1-a_j^/-\frac{\eta_1}{\mu_1}\alpha_j^{/(1)}-.....-\frac{\eta_r}{\mu_r}\alpha_j^{/(r)};\frac{\sigma_1}{\mu_1}\alpha_j^{/(1)},......,\frac{\sigma_r}{\mu_r}\alpha_j^{/(r)})_{1,P^/}:\end{matrix}\right.\right.$$

$$(a_j;\alpha_j^{(1)},......,\alpha_j^{(r)})_{N+1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,n_1}(1-d_j^{/(1)}-\frac{\eta_1}{\mu_1}\delta_j^{/(1)},\frac{\sigma_1}{\mu_1}\delta_j^{/(1)})_{1,q_1^/},(c_j^{(1)},\gamma_j^{(1)})_{n_1+1,p_1};$$

$$:(d_j^{(1)},\delta_j^{(1)})_{1,m_1}(1-c_j^{/(1)}-\frac{\eta_1}{\mu_1}\gamma_j^{/(1)},\frac{\sigma_1}{\mu_1}\gamma_j^{/(1)})_{1,p_1^/},(d_j^{(1)},\delta_j^{(1)})_{m_1+1,q_1};$$

$$:\ .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .$$

$$:\ .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad ..$$

$$:(c_j^{(r)},\gamma_j^{(r)})_{1,n_r}(1-d_j^{/(r)}-\frac{\eta_r}{\mu_r}\delta_j^{/(r)},\frac{\sigma_r}{\mu_r}\delta_j^{/(r)})_{1,q_r^/},(c_j^{(r)},\gamma_j^{(r)})_{n_r+1,q_r}$$

$$\left.:(d_j^{(r)},\delta_j^{(r)})_{1,m_r}(1-c_j^{/(r)}-\frac{\eta_r}{\mu_r}\gamma_j^{/(r)},\frac{\sigma_r}{\mu_r}\gamma_j^{/(r)})_{1,p_r^/},(d_j^{(r)},\delta_j^{(r)})_{m_r+1,p_r}\right] \quad (1.2.3).$$

Where $H(z_1,........,z_r)$ and $H^/(z_1,........,z_r)$ are the H-function of 'r' variables given by Srivastava and Panda [5], $A(z_1,........,z_r)$ is the A- function of 'r' variables is given by

Gautam and Goyal [6].

PROOF:

Express the H- function of 'r' variables in volved in the left hand side of (1.2.2) as a Contour integral given by Srivastava and Panda [5] and put $\rho_i t_i = -v_i, (i = 1,2,3,.....,r)$, then change the order of integration and evaluate the inner integral using(1.2.2) to obtain the right hand side of (1.2.3).

If $F(p_1,........,p_r)$ is the Laplace transform of $f(t_1,......,t_r)$, then ,

$$F(p_1,........,p_r)=\int_0^\infty....\int_0^\infty e^{-p_1t_1}.......e^{-p_rt_r}f(t_1,......,t_r)dt_1....dt_r \quad (1.2.4)$$

If $G(s_1,........,s_r)$ be the Mellin transform of $f(t_1,......,t_r)$, then,

$$G(s_1,........,s_r)=\int_0^\infty....\int_0^\infty t_1^{s_1-1}.......t_r^{s_r-1}f(t_1,......,t_r)dt_1....dt_r \quad (1.2.5).$$

1.3. Relation Between Multiple Mellin And Multiple Laplace Transform.

If $F(p_1,......,p_r)$ be the Laplace transform and $G(s_1,......,s_r)$ be the Mellin transform of $f(t_1,......,t_r)$, then,

$$F(p_1,........,p_r)=\sum_{s_1=0}^\infty......\sum_{s_r=0}^\infty\frac{(-p_1)^{s_1}}{s_1!}.......\frac{(-p_r)^{s_r}}{s_r!}G(s_1+1,......,s_r+1) \quad (1.3.1)$$

Provided,

i) $f(t_1,......,t_r)$ is continuous for all values of $t_1,......,t_r$,

ii) the Laplace transform of $|f(t_1,.....t_r)|$ exist,

iii)the series on the right hand side of $F(p_1,........,p_r)$ converges.

PROOF:

$$F(p_1,\dots\dots, p_r) = \int_0^\infty \dots\int_0^\infty e^{-p_1 t_1}\dots\dots e^{-p_r t_r} f(t_1,\dots\dots,t_r)dt_1\dots dt_r$$

$$= \sum_{s_1=0}^\infty\dots\dots\sum_{s_r=0}^\infty \frac{(-p_1)^{s_1}}{s_1!}\dots\dots\frac{(-p_r)^{s_r}}{s_r!}\int_0^\infty\dots\dots\int_0^\infty t_1^{s_1}\dots\dots t_r^{s_r}dt_1\dots\dots dt_r$$

Which on using the Mellin transform of $f(t_1,\dots\dots,t_r)$ gives the required result.

When r=1, (1.3.1) reduces the result given by Nair and Samar [4].

1.4. Multiple Laplace Transform of the Product of Two H- Function of 'R' Variables.

Theorem

$$\int_0^\infty\dots\dots\int_0^\infty e^{-p_1 t_1}\dots\dots e^{-p_r t_r} t_1^{\rho_1-1}\dots\dots t_r^{\rho_r-1} \times H[h_1 t_1^{\sigma_1},\dots\dots,h_r t_r^{\sigma_r}]$$

$$\times H^{/}[k_1 t_1^{\mu_1},\dots\dots,k_r t_r^{\mu_r}]dt_1\dots\dots dt_r$$

$$= \frac{k_1^{\frac{-\rho_1}{\mu_1}}\dots\dots k_r^{-\frac{\rho_r}{\mu_r}}}{\mu_1\dots\dots\mu_r}\sum_{s_1=0}^\infty\dots\dots\sum_{s_r=0}^\infty \frac{\left(\frac{-p_1}{k_1^{1/\mu_1}}\right)^{s_1}}{s_1!}\dots\dots\frac{\left(\frac{-p_r}{k_r^{1/\mu_r}}\right)^{s_r}}{s_r!}$$

$$\times A^{N^{/}\quad N:(m_1+n_1^{/},m_1^{/}+n_1);\dots\dots;(m_r+n_r^{/},m_r^{/}+n_r)}_{P+Q^{/},P^{/}+Q:(p_1+q_1^{/},p_1^{/}+q_1);\dots\dots;(p_r+q_r^{/},p_r^{/}+q_r)}$$

$$\left[\begin{matrix} h_1 k_1^{\frac{-\sigma_1}{\mu_1}} \\ \cdot \\ \cdot \\ h_r k_r^{\frac{-\sigma_r}{\mu_r}}\end{matrix}\left|\begin{matrix}(a_j,\alpha_j^{(1)},\dots\dots,\alpha_j^{(r)})_{1,N},(1-b_j^{/}-\frac{(s_1+\rho_1)}{\mu_1}\beta_j^{/(1)}-\dots\dots-\frac{(s_r+\rho_r)}{\mu_r}\beta_j^{/(r)};\frac{\sigma_1}{\mu_1}\beta_j^{/(1)},\dots\dots,\frac{\sigma_r}{\mu_r}\beta_j^{/(r)})_{1,Q^{/}}, \\ \\ (b_j,\beta_j^{(1)},\dots\dots,\beta_j^{(r)})_{1,Q},(1-a_j^{/}-\frac{(s_1+\rho_1)}{\mu_1}\alpha_j^{/(1)}-\dots\dots-\frac{(s_r+\rho_r)}{\mu_r}\alpha_j^{/(r)};\frac{\sigma_1}{\mu_1}\alpha_j^{/(1)},\dots\dots,\frac{\sigma_r}{\mu_r}\alpha_j^{/(r)})_{1,P^{/}}:\end{matrix}\right.\right.$$

$$(a_j;\alpha_j^{(1)},\dots\dots,\alpha_j^{(r)})_{N+1,P}:(c_j^{(1)},\gamma_j^{(1)})_{1,n_1}(1-d_j^{/(1)}-\frac{(s_1+\rho_1)}{\mu_1}\delta_j^{/(1)},\frac{\sigma_1}{\mu_1}\delta_j^{/(1)})_{1,q_1^{/}},(c_j^{(1)},\gamma_j^{(1)})_{n_1+1,p_1};$$

$$:(d_j^{(1)},\delta_j^{(1)})_{1,m_1}(1-c_j^{/(1)}-\frac{(s_1+\rho_1)}{\mu_1}\gamma_j^{/(1)},\frac{\sigma_1}{\mu_1}\gamma_j^{/(1)})_{1,p_1^{/}},(d_j^{(1)},\delta_j^{(1)})_{m_1+1,q_1};$$

$$:\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

$$:\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\cdot$$

$$:(c_j^{(r)},\gamma_j^{(r)})_{1,n_r}(1-d_j^{/(r)}-\frac{(s_r+\rho_r)}{\mu_r}\delta_j^{/(r)},\frac{\sigma_r}{\mu_r}\delta_j^{/(r)})_{1,q_r^{/}},(c_j^{(r)},\gamma_j^{(r)})_{n_r+1,q_r}$$

$$:(d_j^{(r)},\delta_j^{(r)})_{1,m_r}\,(1-c_j^{/(r)}-\frac{(s_r+\rho_r)}{\mu_r}\gamma_j^{/(r)},\frac{\sigma_r}{\mu_r}\gamma_j^{/(r)})_{1,p_r^{/}},(d_j^{(r)},\delta_j^{(r)})_{m_r+1,p_r}\Bigg]$$

(1.4.1).

Provided,

i) $\sigma_i,\mu_i,\mathrm{Re}(p_i)>0; \quad (i=1,2,3,4,.....,r)$

ii) $-\sigma_i\min\limits_{1\le j\le m_i}\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)-\mu_i\min\limits_{1\le j\le m_i^{/}}\mathrm{Re}\left(\frac{d_j^{/(i)}}{\delta_j^{/(i)}}\right)<\mathrm{Re}(\rho_i)<\sigma_i\min\limits_{1\le j\le n_i}\left(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right)+\mu_i\min\limits_{1\le j\le n_i^{i}}\left(\frac{1-c_j^{/(i)}}{\gamma_j^{/(i)}}\right),$

iii) $A_i\le 0,\Delta_i>0,|\arg h_i|<\frac{1}{2}\Delta_i\pi,\quad A_i^{/}>0,\Delta_i^{/}>0,\quad |\arg k_i|<\frac{1}{2}\Delta_i^{/}\pi,$

where,

$$A_i=\sum_{j=1}^{P}\alpha_j^{(i)}-\sum_{j=1}^{Q}\beta_j^{(i)}+\sum_{j=1}^{p_i}c_j^{(i)}-\sum_{j=1}^{q_i}d_j^{(i)},$$

$$A_i^{/}=\sum_{j=1}^{P^{/}}\alpha_j^{/(i)}-\sum_{j=1}^{Q^{/}}\beta_j^{/(i)}+\sum_{j=1}^{p_i^{/}}\gamma_j^{/(i)}-\sum_{j=1}^{q_i^{/}}\delta_j^{/(i)},$$

$$\Delta_i=-\sum_{j=N+1}^{P}\alpha_j^{(i)}+\sum_{J=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}-\sum_{j=1}^{Q}\beta_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=m_i+1}^{q_i}\delta_j^{(i)},$$

$$\Delta_i^{/}=-\sum_{j=N^{/}+1}^{P^{/}}\alpha_j^{/(i)}+\sum_{J=1}^{n_i^{/}}\gamma_j^{/(i)}-\sum_{j=n_i^{/}+1}^{p_i^{/}}\gamma_j^{/(i)}-\sum_{j=1}^{Q^{/}}\beta_j^{/(i)}+\sum_{j=1}^{m_i^{/}}\delta_j^{/(i)}-\sum_{j=m^{/}{}_i+1}^{q_i^{/}}\delta_j^{/(i)},$$

iv) the integrals involved in (1.4.1) are absolutely convergent.

PROOF: In (1.3.1) put

$$f(t_1,....,t_r)=t_1^{\rho_1-1}.....t_r^{\rho_r-1}\times H[a_1t_1^{\sigma_1},....,a_rt_r^{\sigma_r}]\times H^{/}[b_1t_1^{\mu_1},....,b_rt_r^{\sigma_r}]$$

and use (1.2.2), hence $F(p_1,........,p_r)=$ the right hand side of (1.4.1).

SPECIAL CASES:

Put

$$N^{/}=P^{/}=Q^{/}=0,\quad m_i^{/}=q_i^{/}=2,\quad p_i^{/}=1,n_i^{/}=0,\gamma_i^{/}=1,\quad \delta_1^{/(i)}=\delta_2^{/(i)}=1,$$

$$c_1^{/(i)}=\mu_i-\lambda_i+1,\quad d_1^{/(i)}=\mu_i+\upsilon_i+\frac{1}{2},\quad d_2^{/(i)}=\mu_i-\upsilon_i+\frac{1}{2},\quad (i=1,2,3,4,......,r)$$

and using (1.2.1), (1.4.1) becomes,

$$\int_0^\infty .....\int_0^\infty e^{-(k_1+2p_1)\frac{t_1}{2}} ....e^{-(k_r+2p_r)\frac{t_r}{2}} t_1^{\mu_1}......t_r^{\mu_r} \times W_{\lambda_1,\upsilon_1}(k_1t_1).........W_{\lambda_r,\upsilon_r}(k_rt_r)$$

$$\times H^{0,N:(m_1,n_1);.......;(m_r,n_r)}_{P,Q:(p_1,q_1);.......;(p_r,q_r)} \left[\begin{array}{l|l} h_1 t_1^{\sigma} & (a_j,\alpha_j^{(1)},.......,\alpha_j^{(r)})_{1,P},(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};.........;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ . & \\ . & \\ h_r t_r^{\sigma_r} & (b_j,\beta_j^{(1)},.......,\beta_j^{(r)})_{1,Q},(d_j^{/(1)},\delta_j^{/(r)})_{1,q_1};........;(d_j^{/(1)},\delta_j^{/(r)})_{1,q_r} \end{array}\right] dt_1.......dt_r$$

$$= k_1^{-(\mu_1+1)}.....k_r^{-(\mu_r+1)} \sum_{s_1=0}^{\infty}.....\sum_{s_r=0}^{\infty} \frac{\left(\frac{-p_1}{k_1}\right)^{s_1}}{s_1!}......\frac{\left(\frac{-p_r}{k_r}\right)^{s_r}}{s_r!}$$

$$\times H^{0,N:(m_1,n_1+2);.......;(m_r,n_r+2)}_{P,Q:(p_1+2,q_1+1);.......;(p_r+2,q_r+1)}$$

$$\left[\begin{array}{l|l} h_1 k_1^{-\sigma_1} & (a_j,\alpha_j^{(1)},.......,\alpha_j^{(r)})_{1,P},(-\mu_1-\vartheta-\frac{1}{2}-s_1,\sigma_1),(-\mu_1+\vartheta_1-\frac{1}{2}-s_1,\sigma_1),(c_j^{(1)},\gamma_j^{(1)})_{1,p_1} \\ . & \\ . & \\ h_r k_r^{-\sigma_r} & (b_j,\beta_j^{(1)},.......,\beta_j^{(r)})_{1,Q},(d_j^{(1)},\delta_j^{(r)})_{1,q_1};(-\mu_1+\lambda_1-\frac{1}{2}-s_1,\sigma_1); \end{array}\right.$$

$$\begin{array}{cccccc} . & . & . & . & . & . \\ . & . & . & . & . & . \end{array}$$

$$:(-\mu_r-\vartheta_r-\frac{1}{2}-s_r,\sigma_r),(-\mu_r+\vartheta_r-\frac{1}{2}-s_r,\sigma_r),,(c_j^{(r)},\gamma_j^{(r)})_{1,p_r};$$

$$\left.(d_j^{(r)},\delta_j^{(r)})_{1,q_r};(-\mu_r+\lambda_r-1-s_r,\sigma_r)\right] \qquad (1.4.2).$$

Provided,

$$\sigma_i,\quad \mu_i>0,\quad (i=1,2,3,......,r)$$

$$\mathrm{Re}\left(\mu_i \pm \vartheta_i + \min \sigma_i \frac{d_j^{(i)}}{\delta_{jj}^{(i)}}+\frac{3}{2}\right)>0,\quad (j=1,2,3,4,........,m_i),\quad (i=1,2,......r)$$

$$\mathrm{Re}(k_i 2p_i)>0,\quad A_i\le 0,\quad \Delta_i>0,\quad |\arg h_i|<\frac{1}{2}\Delta_i\pi,\quad (i=1,2,......,r)$$

Where

$$\Delta_i = -\sum_{j=N+1}^{P}\alpha_j^{(i)}+\sum_{J=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}-\sum_{j=1}^{Q}\beta_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=m_i+1}^{q_i}\delta_j^{(i)},$$

$$A_i = \sum_{j=1}^{P} \alpha_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)}.$$

In (1.4.2) make $p \to 0$, it reduces the multiple Wittaker transform of H- function of 'r' variable as:

$$\int_0^\infty \dots \int_0^\infty e^{-(k_1 t_1 + \dots\dots k_r t_r)\frac{1}{2}} t_1^{\mu_1} \dots\dots t_r^{\mu_r} W_{\lambda_1, \upsilon_1}(k_1 t_1) \dots\dots\dots W_{\lambda_r, \upsilon_r}(k_r t_r)$$

$$\times H^{0,N:(m_1,n_1);\dots\dots;(m_r,n_r)}_{P,Q:(p_1,q_1);\dots\dots;(p_r,q_r)}$$

$$\left[\begin{matrix} h_1 t_1^{\sigma_1} \\ \cdot \\ \cdot \\ h_r t_r^{\sigma_r} \end{matrix}\left| \begin{matrix} (a_j, \alpha_j^{(1)}, \dots\dots, \alpha_j^{(r)})_{1,P}, (c_j^{(1)}, \gamma_j^{(1)})_{1,p_1}; \dots\dots\dots; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r} \\ \\ \\ (b_j, \beta_j^{(1)}, \dots\dots, \beta_j^{(r)})_{1,Q}, (d_j^{(1)}, \delta_j^{(r)})_{1,q_1}; \dots\dots\dots; (d_j^{(1)}, \delta_j^{(r)})_{1,q_r} \end{matrix}\right.\right] dt_1 \dots\dots dt_r$$

$$= k_1^{-(\mu_1+1)} \dots\dots k_r^{-(\mu_r+1)}$$

$$k_1^{-(\mu_1+1)} \dots\dots k_r^{-(\mu_r+1)} \times H^{0,N:(m_1,n_1+2);\dots\dots;(m_r,n_r+2)}_{P,Q:(p_1+2,q_1+1);\dots\dots;(p_r+2,q_r+1)}$$

$$\left[\begin{matrix} h_1 k_1^{-\sigma_1} \\ \cdot \\ \cdot \\ h_r k_r^{-\sigma_r} \end{matrix}\left| \begin{matrix} (a_j, \alpha_j^{(1)}, \dots\dots, \alpha_j^{(r)})_{1,P}, (-\mu_1 - \vartheta - \frac{1}{2}, \sigma_1), (-\mu_1 + \vartheta_1 - \frac{1}{2}, \sigma_1), (c_j^{(1)}, \gamma_j^{(1)})_{1,p_1} \\ \\ \\ (b_j, \beta_j^{(1)}, \dots\dots, \beta_j^{(r)})_{1,Q}, (d_j^{(1)}, \delta_j^{(r)})_{1,q_1}; (-\mu_1 + \lambda_1 - \frac{1}{2}, \sigma_1); \end{matrix}\right.\right.$$

. . . . .

. . . . . .

$$: (-\mu_r - \vartheta_r - \frac{1}{2}, \sigma_r), (-\mu_r + \vartheta_r - \frac{1}{2}, \sigma_r),, (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r};$$

$$\left.(d_j^{(r)}, \delta_j^{(r)})_{1,q_r}; (-\mu_r + \lambda_r - 1, \sigma_r)\right] \quad (1.4.3).$$

Provided,

$i) \sigma_i, \mu_i > 0, \quad (i = 1,2,3,4,\dots\dots r),$

$$ii) \operatorname{Re}\left(\mu_i \pm \vartheta_i + \min \sigma_i \frac{d_j^{(i)}}{\delta_j^{(i)}} + \frac{3}{2}\right) > 0, \quad (j = 1,2,3,4,\dots\dots\dots, m_i), \quad (i = 1,2,\dots\dots r)$$

$$iii) \operatorname{Re}(k_i) > 0, \quad A_i \le 0, \quad \Delta_i > 0, \quad |\arg h_i| < \frac{1}{2}\Delta_i \pi, \quad (i = 1,2,\dots\dots, r)$$

Where

$$\Delta_i = -\sum_{j=N+1}^{P} \alpha_j^{(i)} + \sum_{J=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(i)},$$

$$A_i = \sum_{j=1}^{P} \alpha_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)}.$$

When

$$\lambda_i = 0, \vartheta_i = \frac{1}{2}, \mu_i = \mu_i - 1, k_i = p_i, (i = 1,2,3,4,........, r)\, in \quad (1.4.3),$$

to get the Laplace transform of H- function of 'r' variables as:

$$\int_0^\infty .....\int_0^\infty e^{-(p_1 t_1 + ... + p_r t_r)} t_1^{\mu_1 - 1} ......t_r^{\mu_r - 1}$$

$$\times H^{0,N:(m_1,n_1);............;(m_r,n_r)}_{P,Q:(p_1,q_1),............;(p_r,q_r)} \left[\begin{matrix} h_1 t_1^{\sigma} \\ . \\ . \\ h_r t_r^{\sigma_r} \end{matrix} \middle| \begin{matrix} (a_j, \alpha_j^{(1)}, ..., \alpha_j^{(r)})_{1,P}, (c_j^{(1)}, \gamma_j^{(1)})_{1,p_1}; ....; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r} \\ \\ \\ (b_j, \beta_j^{(1)}, ..., \beta_j^{(r)})_{1,Q}, (d_j^{/(1)}, \delta_j^{/(r)})_{1,q_1}; ....; (d_j^{/(1)}, \delta_j^{/(r)})_{1,q_r} \end{matrix} \right] dt_1 ....dt_r$$

$$= p_1^{-\mu_1} .....p_r^{-\mu_r} \times H^{0,N:(m_1,n_1+1);............;(m_r,n_r+1)}_{P,Q:(p_1+1,q_1),............;(p_r+1,q_r)} \left[\begin{matrix} h_1 p_1^{-\sigma_1} \\ . \\ . \\ h_r p_r^{-\sigma_r} \end{matrix} \middle| \begin{matrix} (a_j, \alpha_j^{(1)}, ..., \alpha_j^{(r)})_{1,P}, (1-\mu_1, \sigma_1), (c_j^{(1)}, \gamma_j^{(1)})_{1,p_1}; ......; (1-\mu_r, \sigma_r), (c_j^{(r)}, \gamma_j^{(r)})_{1,p_1} \\ \\ \\ (b_j, \beta_j^{(1)}, ..., \beta_j^{(r)})_{1,Q}, (d_j^{(1)}, \delta_j^{(r)})_{1,q_1}; ....(d_j^{(r)}, \delta_j^{(r)})_{1,q_r} \end{matrix} \right] \quad (1.4.4)$$

Provided,

$$i) \operatorname{Re}\left(\mu_i + \sigma_i \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) > 0, \quad (j = 1,2,3,4,........, m_i), \quad (i = 1,2,......r)$$

$$ii) \sigma_i > 0, \quad \operatorname{Re}(p_i) > 0, \quad |\arg h_i| < \frac{1}{2} \Delta_i \pi, A_i \leq 0, \Delta_i > 0 (i = 1,2,3,4,......r),$$

Where

$$\Delta_i = -\sum_{j=N+1}^{P} \alpha_j^{(i)} + \sum_{J=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(i)},$$

$$A_i = \sum_{j=1}^{P} \alpha_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)}.$$

When r = 2, (1.4.4) reduces to the double Laplace transform of H- function of two variables given by Gupta and Goyal [1, p.149].

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