Performance Analysis of MUSIC Algorithms for Evaluation of Coherent and Non-Coherent Signals

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Abstract: ***The demand for increased capacity in wireless communication system is improved using Direction-of-arrival (DOA) estimation algorithm which corresponds to finding the direction in which desired user and the interference lie for directional antenna array. Many DOA algorithms exists amongst which Multiple Signal Classification (MUSIC), Improved MUSIC are investigated. The proposed work investigates and compares MUSIC and Improved MUSIC DOA estimation algorithm on the uniform linear array (ULA) which are used in design of smart antenna system. MUSC algorithm is high resolution subspace based method which is used to estimate DOA of incoherent signals. Spatial smoothing of MUSIC is introduced to estimate DOA of coherent and completely correlated signal. For closely spaced users this optimization is very important and will be helpful while increasing the mobile services and number of users in a limited area. As a result the performance of cellular systems will be enhanced.***

Keywords: Direction-of-arrival, MUSIC, Improved MUSIC, Smart Antenna System

1. Introduction

Adaptive Antenna Array processing techniques have been used widely for the estimation of Direction-of-Arrival (DOA) of incident source signals [5]. Adaptive Antenna, also known as Smart Antenna, is capable of adjusting its radiation pattern according to the surroundings. In actual, antennas are not Smart Antenna, systems that are used is smart. Generally co-located with a base station, a smart antenna system unites an antenna array with a digital signal-processing capability to transmit and receive in an adaptive manner. In other words, such a system can automatically adapt the directionality of its radiation patterns in response to its signal environment. This can increase the performance characteristics such as capacity of a wireless system.

Smart Antennas used to identify the Direction of arrival (DOA) of the signal, which is needed to track and locate the intended mobile set. Direction of arrival (DOA) estimation is the process of determining the direction of an incoming signal from mobile devices to the Base Transceiver Station. In this way, Signal-to-Interference-and-Noise Ratio (SINR) improves by producing nulls along the direction of interference. Hence DOA estimation can also use in many field such as radar, sonar, radio, astronomy, under water surveillance to estimate source location.

Many resolution algorithms exist amongst which Conventional spectral-based method and sub-spaced based method were used for DOA estimation. Conventional method for DOA estimation is based on the concepts of beamforming and null-steering. This technique electronically steer beams in all directions and look for the peaks in the output power.Thesum and delay or Bartlett and Minimum Variance Distortionless Response (MVDR) or Capon etc. are conventional spectral-based method. These methods are highly dependent on physical size of an array antenna, require low computational complexity. Nevertheless, these methods have low resolution and accuracy that leads to the introduction of sub-spaced based method. DOA estimation of these algorithm is based on Eigen values and steering vectors. High resolutions Multiple Signal Classification (MUSIC), Improved MUSIC, Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT) are sub-spaced based method having superior accuracy and resolution performance over the conventional methods.

Performance MUSIC algorithm depend on the source number estimation, number of snapshots, number of array elements and high computational complexity. Multiple Signal Classification (MUSIC) algorithm is high resolution subspace based method which is used to estimate DOA of incoherent signals. Under the impulse noise environment, the performance of the MUSIC algorithm will degenerate [5]. Spatial smoothing of MUSIC is introduced to estimate DOA of coherent and completely correlated signal. For coherent signals improved MUSIC remove the correlation between incident signals. MUSIC and Improved MUSIC algorithms were developed and simulated in MATLAB software. The Paper is organized as follows- Firstly, a system model is developed for DOA estimation in Section 2. The MUSIC algorithm is described in details in Section 3. The proposed Improved MUSIC for coherent signal is described in details in Section 4.In Section 5, results of MUSIC and Improved MUSIC algorithm are presented and compared. The conclusion is given in the further section.

2. Signal Model

Fig. 1 shows a uniform linear array (ULA) M with equispaced sensors.

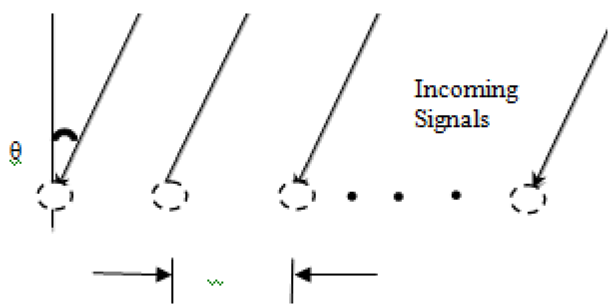

Figure 1: Uniform linear array with M-Element

- Number of elements = M
- Inter-element spacing = d
- Number of incident signals = D
- Number of Data samples = k

There are D signals incident on the array, the received input data vector at an M-element arrayseparated by a distance d can be expressed as a linear combination of the D incident waveforms and noise.

$$\mathbf{u(k)} = \sum_{l=1}^{D} \mathbf{a(\theta_l)\, s_l(k) + n(k)} \quad (1)$$

$$\mathbf{u(k)} = [\mathbf{a(\theta_1)..a(\theta_D)}]\begin{bmatrix} \mathbf{s_l(k)} \\ . \\ . \\ . \\ \mathbf{s_D(k)} \end{bmatrix} + \mathbf{n(k)} \quad (2)$$

$$\mathbf{u(k) = A\, s(k) + n(k)} \quad (3)$$

Where $\mathbf{A} = [\mathbf{a(\theta_1)\, a(\theta_2) \ldots \ldots . a(\theta_D)}]$is the matrix of steering vectors, $\mathbf{s^T(t)} = [\mathbf{s_1(k) s_2(k) \ldots . . s_D(k)}]$ is the signal vector and $\boldsymbol{n(t)} = [\boldsymbol{n_1(k) n_2(k) \ldots . . n_D(k)}]$ is a noise vector with components of variance σ_n^2.

Theinput covariance matrix is

$$\mathbf{R_{uu}} = \mathbf{A\, E}[ss^{\mathbf{H}}]\mathbf{A^H} + \mathbf{E[nn^H]} \quad (4)$$

$$\mathbf{R_{uu}} = \mathbf{AR_{ss}A^H} + \boldsymbol{\sigma_n^2} \quad (5)$$

Where R_{ss} is the signal correlation matrix.

3. MUSIC Algorithm

The MUSIC algorithm was proposed by Schmidt in 1979 [13]. MUSIC algorithm is a high resolution Multiples Signal Classification subspace DOA estimation algorithm which based on exploiting the Eigen structure of the input covariance matrix.[8]

Figure 2: Shows the flowchart of MUSIC algorithm. Direction of arrival estimation is the process of determining the direction of an incoming signal from mobile devices to the Base Transceiver Station.

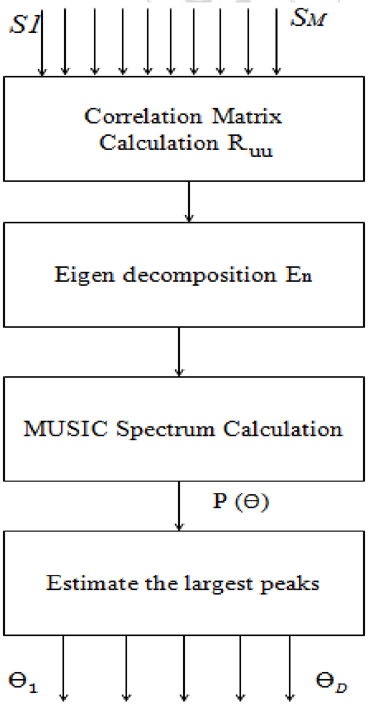

Figure 2: MUSIC implementation flow chart.

MUSIC deals with the decomposition of correlation matrix into two orthogonal matrices, signal-subspace and noise subspace. Estimation of direction is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated. This makes the correlation matrix diagonal [4]. The correlation matrix from equation (5) is

$$\mathbf{R_{uu}} = \mathbf{AR_{ss}A^H} + \boldsymbol{\sigma_n^2}$$

Where H – "Hermitian" means conjugate transpose. The eigenvectors of the covariance matrix R_{uu} belong to either of the two orthogonal subspaces, signal subspace and the noise subspace. The array correlation matrix has M Eigen values $(\lambda_1, \lambda_2, \lambda_3, \ldots., \lambda_M)$along with M Eigenvectors $(E_1, E_2, E_3, \ldots., E_M)$[4].

If the Eigen values are arranged in largest to smallest order, the next step is dividing the matrix E into two subspaces $[E_N E_S]$. $E_N$is the $\mathrm{M} \times (\mathrm{M}-\mathrm{D})$noise subspace composed of Eigenvectors associated with the noise , whileE_S is the M ×D signal subspace composed of Eigenvectors associated with the arriving signal. Due to the orthogonality of noise subspace and the array steering vector corresponding to signal components at the angles of arrival $(\theta_l, \theta_2, \theta_3, \ldots \ldots, \theta_M)$, the matrix product $a^H(\theta)E_N {E_N}^H a(\theta)$ is zero for θ corresponding to the DOA of a multipath component. Then the DOAs of the multiple incident signals can be find out by locating the peak of a MUSIC spatial spectrum given by

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)E_n {E_n}^H a(\theta)} \quad (6)$$

Orthogonality between array steering vectors a (θ) and noise subspace $E_N$will minimize the denominator and hence will create sharp peaks at the angle of arrival.

4. Improved MUSIC Algorithm

MUSIC algorithm assumes that incoming signalsare uncorrelated. The covariance matrix R_{uu} is a non-singular matrix i.e. full-rank matrix as long as the incoming signals on the array are not coherent. The covariance matrix R_{uu} will lose its non-singularity property in case of the correlated signals. Thus the performance of MUSIC algorithm degrades severely in coherent or highly correlated signal environment as encountered in multipath propagation where multiple version of same signal arrive within the symbol duration. Improved MUSIC involves modification of covariance matrix through a preprocessing scheme. Spatial smoothing is required to remove correlation between the signals. The idea behind this spatial smoothing technique was proposed by Evans in 1982. [13]

Uniform linear array with M identical elements are divided into overlapping sub arrays of size p and introducing phase shift between these as shown in Figure 3. Such that the array elements (0,., P-1) form the first subarray an (1, , p) form the second subarray.

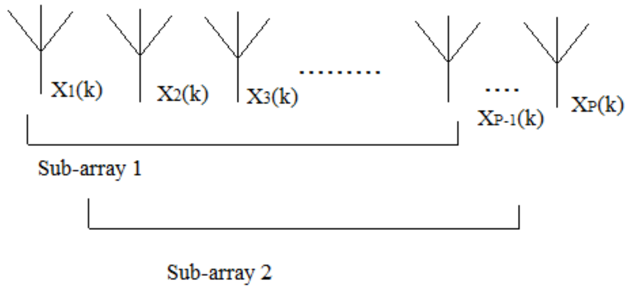

Figure 3: Uniform linear array is divided into several sub-arrays

The vector of received signals at the k^{th} forward subarrays is expressed as

$$\boldsymbol{x_k^F} = \boldsymbol{AD^{(k-1)}s(t)} + \boldsymbol{n_k(t)} \quad (7)$$

Where (k-1) denotes k^{th} power of the diagonal matrix D given by:

$$\boldsymbol{D} = \boldsymbol{diag}\{\boldsymbol{e}^{-j\frac{2\pi}{\lambda}sin\theta_1}, \ldots, \boldsymbol{e}^{-j\frac{2\pi}{\lambda}sin\theta_m}\} \quad (8)$$

The spatial correlation matrix R of the sensor array is then defined as the sample mean of the covariance matrices of the forward sub-arrays:

$$\boldsymbol{R} = \frac{\boldsymbol{1}}{\boldsymbol{L}}\sum_{\boldsymbol{k=0}}^{\boldsymbol{L-1}} \boldsymbol{R_k^F}$$

Here L is the number of overlapping sub-arrays. By applying spatial smooth MUSIC algorithm, M element sensor array can detect up to M/2 correlated signals.

5. Simulation Results

A. Simulation Results of Music Algorithm

The MUSIC and Improved MUSIC method of DOA estimation are simulated using MATLAB. In these simulations, it is considered a linear array antenna formed by elements that are equally spaced with the distance of$\lambda/2$. The SNR=20db.The simulation has been run for four independent narrow band signals,angle of arrival is -40^0 ,10^0, $50^{0,}$ 80^0 The performance has been analyzed for different value of snapshots and array elements

Case 1: MUSIC spectrum for varying number of array elements

As shown in Figure 4 and 5, it is clear that as number of array elements increases (M=150), Peaks of spectrum become sharper and MUSIC can accurately estimate the DOA of the incoming signals. And if number of array element (M=10) decreases, then resolution of Music algorithm decreases.

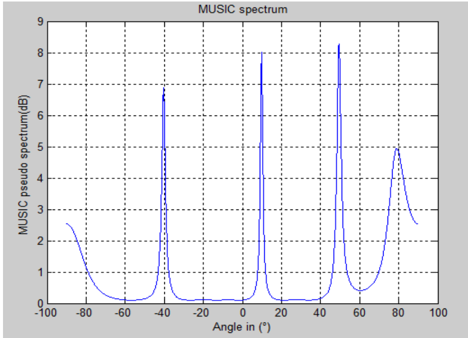

Figure 4: M=10, Snapshots = 200

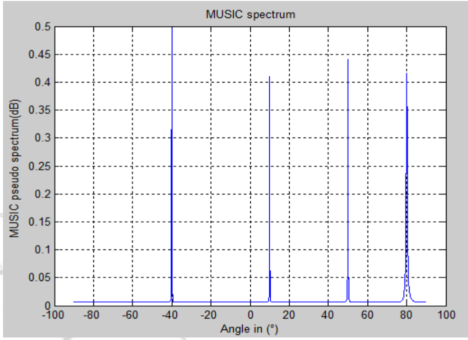

Figure 5: M=150, Snapshots = 200

Figure 4 and 5: MUSIC Spectrum for varying number of array element.

Case.2MUSIC spectrum for varying number of snapshots. As shown in Figure 6 and 7, as number of snapshots (k=500) increases,Precise detection of incoming signals and Resolution capacity increases This is because increase the number of snapshots, will make narrower beam width around incoming signals directions. If number of snapshot (k=50) then resolution of Music algorithm decreases.

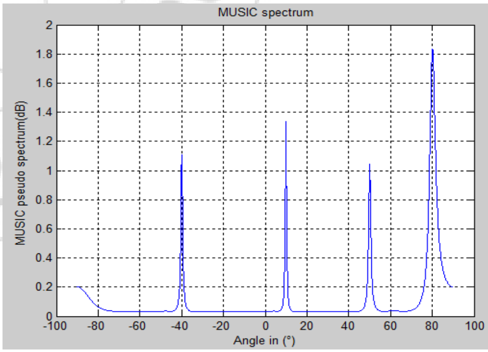

Figure 6: Snapshots= 50, M= 30

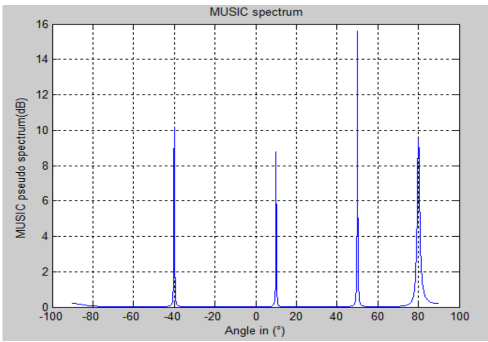

Figure 7: Snapshots= 500, M= 30

Figure 6 and 7: MUSIC Spectrum for varying Snapshots.

B. Simulation Results of Improved Music Algorithm:
When the signals are coherent, let the incident angle is same (-40°, 10°, 50° and 80°), the SNR is 20dB, the element spacing is half of the input signal wavelength, array element number is 10, and the number of snapshots is 200. The simulation results are shown in Figure 8.

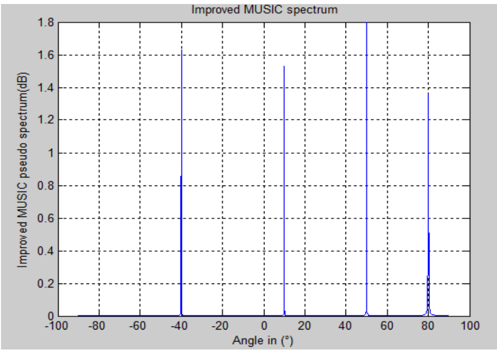

Figure 8: M=10, Snapshots = 200

C. Simulation Results for Comparison of MUSIC and Improved MUSIC Algorithm
In these simulations, it is considered a linear array antenna formed by 14 elements that are equally spaced with the distance of $\lambda / 2$. The SNR is 20dB and number of snapshots is 100. The simulation has been run for two independent narrow band signals,angle of arrival is 20^0,-30^0 and 35^0.

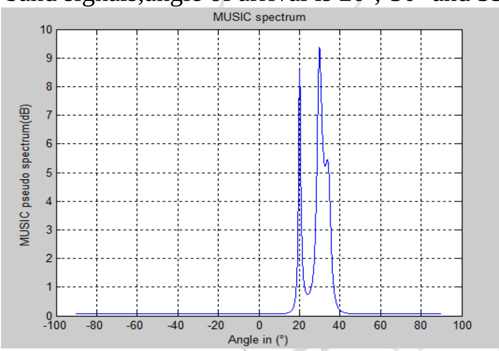

Figure 9: Snapshots= 100, M= 14

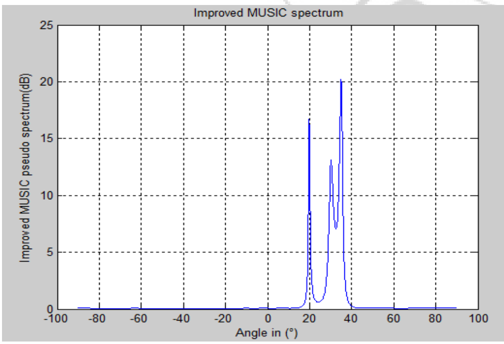

Figure 10: Snapshots= 100, M= 14
Figure 9 and 10: MUSIC and Improved MUSIC Spectrum for same specification.

6. Conclusion

In this paper we presented a subspace based estimation using MUSIC and improved MUSIC algorithm. The simulation result of Music algorithm shows that angular resolution of Music algorithm improves with more no. of elements in the array, with large snapshot of signals.MUSIC can estimate uncorrelated signal very well but it fails to detect correlated signals. Improved MUSIC estimates accurate DOA of signal under coherent condition.

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