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LQR Controller Design for Stabilization of Cart Model Inverted Pendulum

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Abstract: ***Optimal response of the controlled dynamical systems is desired hence for that is the optimal control. Linear Quadratic Regulator (LQR), an optimal control method, and PID control which are generally used for control of the linear dynamical systems have been used in this paper to control the nonlinear dynamical system. The inverted pendulum, a highly nonlinear unstable system is used as a benchmark for implementing the control methods. In this paper the modeling and control design of nonlinear inverted pendulum-cart dynamic system with disturbance input using PID control & LQR have been presented. The nonlinear system states are fed to LQR which is designed using linear state-space model. Here PID & LQR control methods have been implemented to control the cart position and stabilize the inverted pendulum in vertically uprightposition. The MATLAB-SIMULINK models have been developed for simulation of the control schemes. The simulation results justify the comparative advantages of LQR control methods.***

Keywords: Inverted Pendulum, Nonlinear System, PID Control,Optimal Control, LQR, Disturbance Input

1. Introduction

Inverted pendulum system is a typical model of multivariable, nonlinear, essentially unsteady system, which is perfect experiment equipment not only for pedagogy but for research because many abstract concepts of control theory can be demonstrated by the system-based experiments [1]. The research on such a complex system involves many important theory problems about system control, such as nonlinear problems, robustness, ability and tracking problems. Therefore, as an ideal example of the study, the inverted pendulum system in the control system has been universal attention. And it has been recognized as control theory, especially the typical modern control theory research and test equipment. So it is not only the best experimental tool but also an ideal experimental platform. The research of inverted pendulum has profound meaning in theory and methodology, and has valued by various countries' scientists [2].

The problem is referred in classical literature as pole balancer control problem, cart-pole problem, broom balancer control problem, stick balancer control problem, inverted pendulum control problem [3]. Control of inverted pendulum resembles the control systems that exist in some of the real time applications such as rockets and missiles, heavy crane lifting containers and self-balancing robots. According to control purposes of inverted pendulum, the control of inverted pendulum can be divided into three aspects. The first aspect widely researched is the swing-up control of Inverted Pendulum (IP) [4,5]. The second aspect is the stabilization of the inverted pendulum [6-7]. The third aspect is tracking control of the inverted pendulum [8]. In practice, stabilization and tracking control is more useful for plenty of real time applications. There are several problems to be solved in the control of inverted pendulum, such as swinging up and catching the pendulum from its stable pending position to the upright unstable position, and then balancing the pendulum at the upright position during disturbances,and further move the cart to a specified position on the rail [9]. Several methods for achieving swing-up and stabilization of pendulum system have been proposed in literature. However, in practical setups, there is an inherent limitation on the cart length and the magnitude of control force that can be applied. This gives the motivation to find out energy based methods for controlling and stabilizing the cart position with restricted cart length and restricted control force.In addition, the inverted pendulum has always been adopted as a classical control example to test the advantages and disadvantages of various control algorithms such as PID control, state feedback control, fuzzy control, neural network control, adaptive control and genetic algorithms.

2. Mathematical Modelling

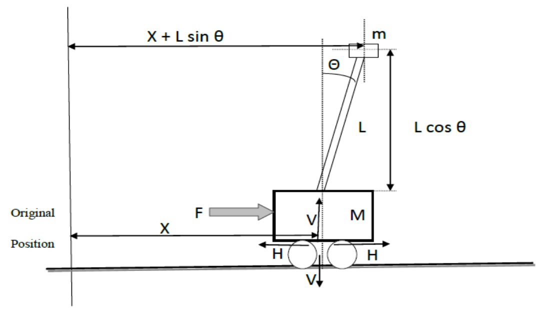

Figure 1: Analysis of Forces on Cart and Pendulum

Let H the horizontal component of reaction force and V be vertical component of reaction force. Let x1 be the horizontal component of co-ordinates of Centre of Gravity (COG) and y1 be the vertical component of co-ordinates of COG.

$$x_1 = x + l \sin \theta$$

$$x_1 = x + l\,\theta \cos \theta$$

$$y_1 = -l\,\theta \sin \theta$$

$$y_1 = l \cos \theta$$

Define the angle of the rod from the vertical (reference) line as θ and displacement of the cart as x. Also assume the force applied to the system is F, g be the acceleration due to gravity and l be the half length of the pendulum rod, v , and w be the translational and angular velocity of the cart and pendulum. Let H be the horizontal component of reaction force and V be vertical component of reaction force. So the horizontal reaction force H becomes:-

$$H = m\,x_1$$

$$H = m(\ddot{x} + \ddot{\theta} l.\cos\theta + \dot{\theta}^2 l(-\sin\theta))$$

The forced F applied on the cart equals the sum of the force due to acceleration, friction component of force that opposes the linear motion of the cart and the horizontal reaction.

$$F = M\ddot{x} + b\dot{x} + H$$

Thus we get

$$F = (m + M)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta$$

The Vertical reaction V can be expressed as:-

$$V = m\ddot{y}_1 = -m\ddot{\theta} l\sin\theta - m\dot{\theta}\; l\cos\theta$$

Torque equation is,

$$-H\cos\theta l + (V + mg)\sin\theta = I\ddot{\theta} + b\dot{\theta}$$

Thus,

$$0 = (I + ml^2)\ddot{\theta} - mgl\sin\theta + ml\ddot{x}\cos\theta + d\dot{\theta}$$

The following is the parameter table that gives the value of the various parameters that has been adopted from the Feedback Digital Pendulum Manual.

Table 1: Parameters from Feedback Digital Pendulum Manual

Symbol	*Parameter*	*Value*	*Unit*
M	Cart Mass	2.4	Kg
m	Pole Mass	0.23	Kg
l	Pole Length	0.4	m
b	Cart Friction coefficient	0.05	Ns/m
d	Pendulum damping coefficient	0.005	Nms/m
I	Moment of Inertia of the pole	0.099	Kg.m^2
g	Gravity	9.81	m/s^2

Using above equations and substituting the parameters, we get the following state space equation

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0195 & 0.2381 & 0.0013 \\ 0 & 0 & 0 & 1 \\ 0 & 0.0132 & 6.807 & -0.0377 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.3895 \\ 0 \\ -0.2638 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

3. LQR Design

A system can be expressed in state variable form as x = Ax + Bu with x(t)∈R N, u(t)∈ R N . The initial condition is x(0) . We assume here that all the states are measurable and seek to find a State-Variable Feedback (SVFB) control.

$$u = -Kx + v$$

This equation gives desired closed-loop properties. The closed-loop system using this control becomes

$$x = (A - BK)\,x + B$$
$$v = Ac\,x + Bv$$

With Ac the closed-loop plant matrix and v the new command input. Ackermann's formula gives a SVFB K that places the poles of the closed-loop system at desired location. Todesign a SVFB that is optimal, we may define the performance index J as

$$J = \frac{1}{2}\int_0^\infty x^T(Q + K^T RK)x\,dt$$

Optimal solution for closed loop system is obtained by

$$x = Ax + Bu,$$
$$x(t_0) = x_0$$
$$\dot{\lambda} = -Qx - A^T\lambda$$
$$Ru + B^T\lambda = 0$$

As the above equations are linear we can connect them as λ=Px

Here, P is the solution to Algebraic Riccati Equation given as

$$A^T P + PA + Q - PBR^{-1}B^T P = 0$$

Optimal control law is given as,

u= - Kx

But from equation (a),

$$u = R^{-1}B^T\lambda$$

Hence,

$$K = R^{-1}B^T P$$

Where, K is Optimal Feedback Gain Matrix.

4. Simulations and Results

The MATLAB-SIMULINK models for the simulation of modeling, analysis, and control of nonlinear inverted pendulum-cart dynamical system with disturbance input have been developed. The typical parameters of inverted pendulum cart system setup are selected as [16,20]: mass of the cart (M):2.4 kg, mass of the pendulum (m): 0.23 kg, length of the pendulum (l): 0.36 m, length of the cart track (L): ± 0.5 m, friction coefficient of the cart & pole rotation is assumed negligible. The disturbance input parameters which has been taken in simulation are [21]: Band Limited White Noise Power= 0.001, Sample Time = 0.01, Seed = 23341.

Here three control schemes have been implemented for optimal control of nonlinear inverted pendulum-cart dynamical system with disturbance input: 1. PID control method having two PIDs i.e. angle PID & cart PID, 2. Two PIDs (i.e. angle PID & cart PID) with LQR control method, 3. One PID (i.e.cart PID) with LQR control method.

In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, pendulum angle θ , angular velocity θ, cart position x , and cart velocity x have been considered available for measurement which are directly fed to the LQR. The LQR is designed using the linear state-space model of the system. The optimal control value of LQR is added negatively with PID control value to have a resultant optimal control. The tuning of the PID controllers which are used here either as PID control method or PID+LQR control methods is done by trial & error method and observing the responses achieved to be optimal. The SIMULINK model for control of inverted pendulum system with disturbance input using PID control method fornon linear plant model is shown in Fig.4.1.

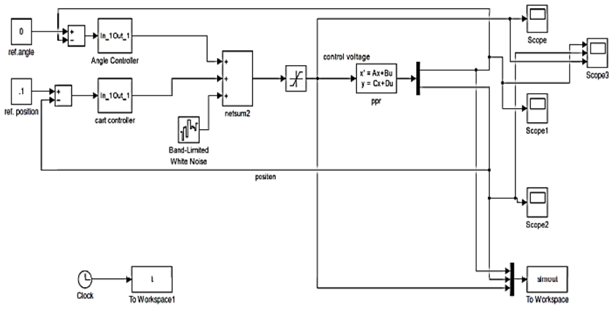

Figure 4.1: PID Control of Nonlinear IP System with Disturbance Input.

Here only pendulum angle θ and cart position *x* have been considered for the measurement. The band limited white noise has been added as the disturbance input to the system. The reference angle has been set to 0 (rad), and reference cart position is set to 0.1 (m).The simulation results are shown in Fig.4.2.

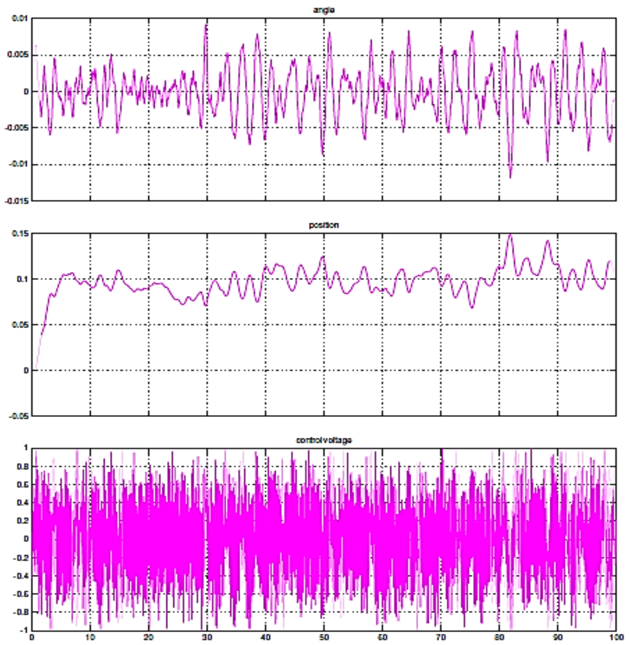

Figure 4.2: Responses of PID Control of Nonlinear IP System with Disturbance Input.

It is observed that the pendulum stabilizes in vertically upright position with minor oscillations, and also the cart position *x* reaches the desired position of 0.1 (m) quickly with minor oscillations. The control input u is bounded in range [-1 1]. The simulation results justify the effectiveness of the PID control. The SIMULINK model for optimal control of nonlinear inverted pendulum-cart system with disturbance input using two PID controllers (angle PID & cart PID) with LQR control method is shown in Fig. 4.3.

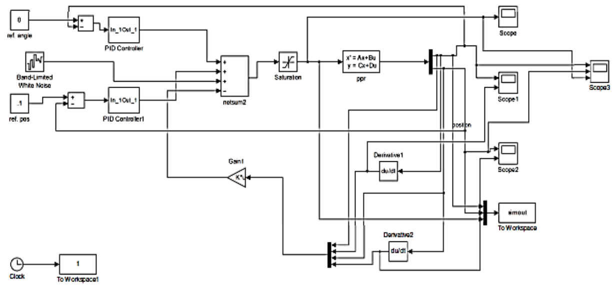

Figure 4.3: Angle PID, Cart PID & LQR Control of Nonlinear IP System with Disturbance Input.

In this approach all the states of the system θ, $\dot{\theta}$, *x* and $\dot{x}$ are fed to LQR, which is designed using the linear state-space model of the system. Here also the angle θ & cart position *x* have been taken as variables of interest for control. The band limited white noise has been added as the disturbance input to the system. The reference angle is set to 0(rad), and the reference cart position has been set to 0.1 (m).The simulation results are shown in Fig. 4.4.

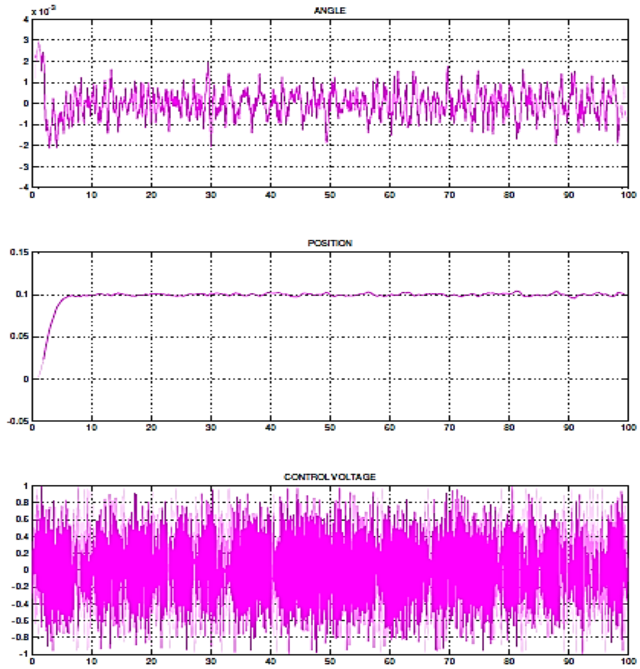

Figure 4.4 Angle PID, Cart PID & LQR Control ofNonlinear IP System with Disturbance Input.

Here responses of angle θ , angular velocity $\dot{\theta}$, cart position *x* , cart velocity $\dot{x}$,and control *u* have been plotted. It is observed that the pendulum stabilizes in vertically upright position with minute oscillations, and the angular velocity oscillates by approx. +/-0.01 (rad/s) remaining at most in range approx +/-0.02 (rad/s).The cart position *x* reaches smoothly the desired position of0.1 (m) quickly in approx. 6 seconds, and the cart velocity oscillates near to zero. The control input *u* is bounded in range[-1 1]. The simulation results justify the effectiveness of the 2PID+LQR control. The SIMULINK model for optimal control of nonlinear inverted pendulum-cart system with disturbance input using one PID controller (cart PID) with LQR control method is shown in Fig.4.5.

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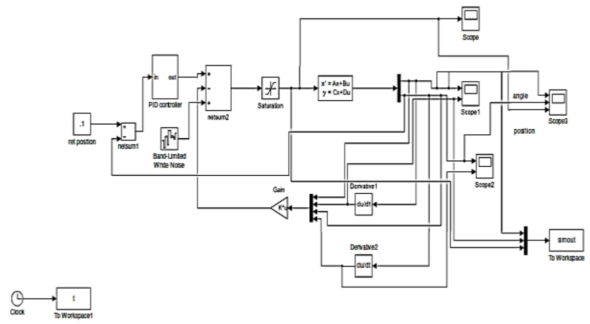

Figure 4.5: Cart PID & LQR Control of Nonlinear Inverted Pendulum System with Disturbance Input.

This control method is similar to 2 PID+LQR control method in all respect of control techniques but differs only in number of PID controllers used. Here only cart PID controller has been used, and angle PID controller has not been used. Here only cart position x has been taken as variable of interest for control. The reference cart position has been set to0.1 (m). The desired angle to be zero is directly taken care of by state feedback control of LQR which is designed using the linear state-space model of the system with vertically upright position as reference. The band limited white noise has been added as the disturbance input to the system as same. The simulation results are shown in Fig. 4.6. Here also responses of angle θ, angular velocity$\dot{\theta}$, cart position x, cart velocity $\dot{x}$, and control u have been plotted. It is observed that the pendulum stabilizes in vertically upright position with minute oscillations, and the angular velocity oscillates by approx. +/-0.01 (rad/s) remaining at most in range approx. +/-0.02(rad/s).The cart position x reaches the desired position of 0.1(m)quickly in approx. 6 seconds, and the cart velocity oscillates very near to zero. The control input u is bounded in range [-11]. The simulation results justify the effectiveness of the cart PID+LQR control.

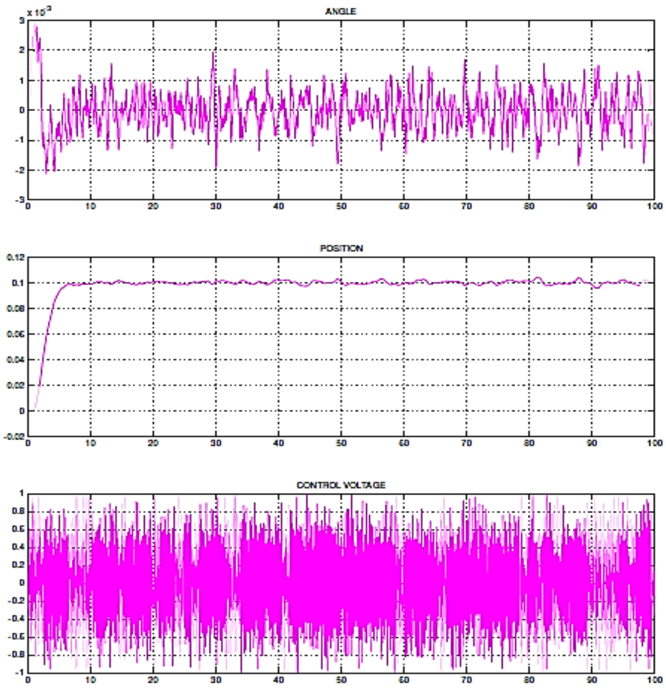

Figure 4.6: Responses of Cart PID & LQR Control of Nonlinear IP System with Disturbance Input.

Comparing the results as shown in Fig.4.7 it is observed that the responses of both alternatives of PID+LQR control method are better than PID control, which are smooth & fast also. It is also observed that the responses of 2 PID+LQR control and cart PID+LQR control are similar. Just the cart position response of 2PID+LQR control is smoother than cart PID+LQR control and so it is slightly better, which is due to the additional degree of freedom of control added by the angle PID controller. But the cart PID+LQR control has structural simplicity in its credit. The analysis of the performances of the control schemes of PID control, 2 PID+LQR control, and cart PID+LQR control for the nonlinear inverted pendulum-cart dynamical system with disturbance input gives that these control schemes are effective & robust.

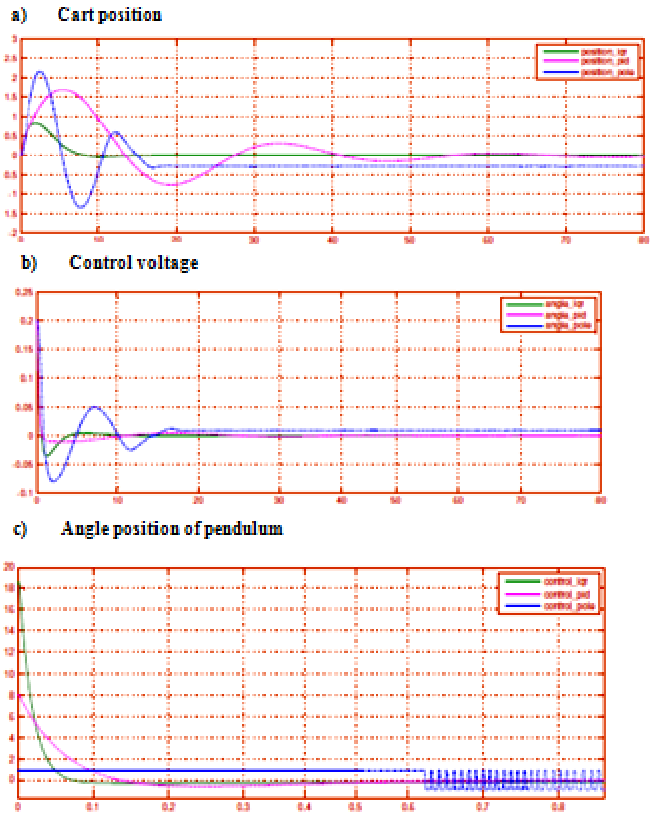

Figure 4.7: Comparison of responses of three control methods

5. Conclusion

Modeling of Inverted Pendulum shows that system is unstable with non-minimum phase zero. Results of applying state feedback controllers show that the system can be stabilized. LQR controller method is cumbersome because of selection of constants of controller. Constant of the controllers can be tuned by some soft computing techniques for better result. PID control, and LQR, an optimal control technique to make the optimal control decisions, have been implemented to control the nonlinear inverted pendulum-cart system with disturbance input. To compare the results PID control has been implemented. In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, are considered available for measurement, which are directly fed to the LQR. The LQR is designed using the linear statespace model of the system.

The optimal control value of LQR is added negatively with PID control value to have a resultant optimal control. The MATLAB-SIMULINK models have been developed for simulation of the control schemes. The tuning of the PID controllers which are used here either as PID control method or PID+LQR control methods is done by trial & error method and observing the responses achieved to be optimal. The simulation results justify the comparative advantages of optimal control using LQR method. The pendulum stabilizes in upright position with acceptable minor oscillations and cart approaches the desired position even under the continuous disturbance input such as wind force justify that the control schemes are effective & robust. The response of PID controller using LQR is better than PID controller.

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