Geometric Decomposition of Spider Tree

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Abstract: *Let $G = (V, E)$ be a simple connected graph with p vertices and q edges. If $G_1, G_2, G_3, \ldots, G_n$ are connected edge disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, G_3, \ldots, G_n)$ is said to be a decomposition of G. A decomposition $(G_1, G_2, G_3, \ldots, G_n)$ of G is said to be an Arithmetic Decomposition if each G_i is connected and $|E(G_i)| = a + (i-1)d$, for every $i = 1, 2, 3, \ldots, n$ and $a, d \in N$. In this paper, we introduced a new concept Geometric Decomposition. A decomposition $(G_a, G_{ar}, G_{ar^2}, G_{ar^3}, \ldots, G_{ar^{n-1}})$ of G is said to be a Geometric Decomposition(GD) if each $G_{ar^{i-1}}$ is connected and $|E(G_{ar^{i-1}})| = ar^{i-1}$, for every $i = 1, 2, 3, \ldots, n$ and $a, r \in N$. Clearly $q = \frac{a(r^n-1)}{r-1}$. If $a = 1$ and $r = 2$, then $q = 2^n-1$. In this paper we study the Geometric Decomposition of spider tree.*

Keywords: Decomposition, Arithmetic Decomposition(AD), Geometric Decomposition(GD), Geometric Path Decomposition(GPD), Geometric Star Decomposition(GSD).

1. Introduction

In this paper, we consider simple undirected graph without loops or multiple edges. For all other standard terminology and notations we follow Harary [1].

N.Gnanadhas and J.Paulraj Joseph introduced the concept of Continuous Monotonic Decomposition (CMD) of graphs [2]. E. Ebin Raja Merly and N. Gnanadhas introduced the concept of Arithmetic Odd Decomposition (AOD) of spider tree [3].

Definition: 1.1

Let $G = (V, E)$ be a simple connected graph with p vertices and q edges. If $G_1, G_2, G_3, \ldots, G_n$ are connected edge disjoint subgraphs of G with

$E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \ldots \cup E(G_n)$, then

$(G_1, G_2, G_3, \ldots, G_n)$ is said to be a decomposition of G.

Definition: 1.2

A decomposition $(G_1, G_2, G_3, \ldots, G_n)$ of G is said to be an Arithmetic Decomposition(AD) if each G_i is connected and $|E(G_i)| = a+(i-1)d$, for every $i = 1, 2, 3, \ldots, n$ and $a, d \in N$.

Definition: 1.3

Let G be a connected graph. The diameter of G is defined as $\max\{d(u, v) : u, v \in V(G)\}$ and is denoted by diam(G).

2. Geometric Decomposition of Graphs

Definition: 2.1

A decomposition $(G_a, G_{ar}, G_{ar^2}, G_{ar^3}, \ldots, G_{ar^{n-1}})$ of G is said to be a Geometric Decomposition(GD) if each $G_{ar^{i-1}}$ is connected and $|E(G_{ar^{i-1}})| = ar^{i-1}$, for every $i = 1, 2, 3, \ldots, n$ and $a, r \in N$. Clearly $q = \frac{a(r^n-1)}{r-1}$. If $a = 1$ and $r = 2$, then $q = 2^n-1$.

We know that 2^n-1 is the sum of $2^0, 2^1, 2^2, 2^3, \ldots, 2^{n-1}$. That is, $2^n - 1$ is the sum of 1, 2, 4, 8, …, 2^{n-1}. Thus we denote the GD as $(G_1, G_2, G_4, \ldots, G_{2^{n-1}})$.

Example: 2.2

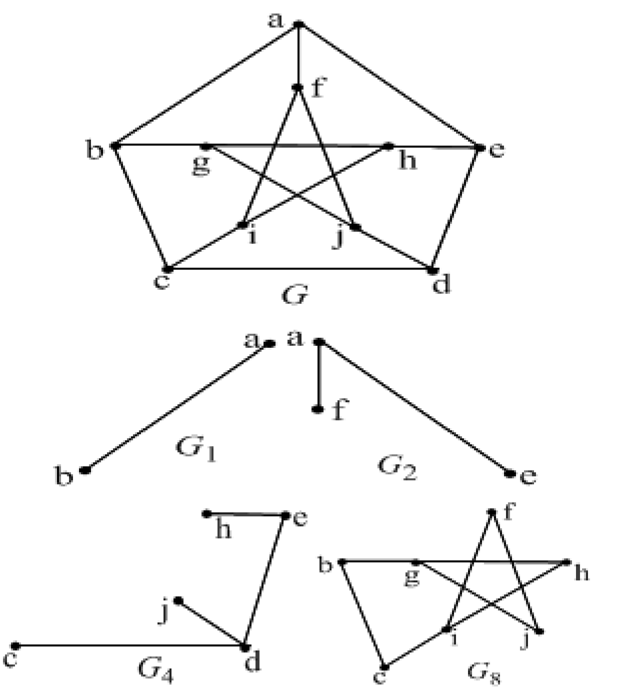

Figure 1: A Petersen graph admits GD (G_1, G_2, G_4, G_8) of G.

Theorem 2.3: A graph G admits GD $(G_1, G_2, G_4, \ldots, G_{2^{n-1}})$ if and only if $q = 2^n - 1$ for each $n \in N$.

Proof:

Let G be a connected graph with $q = 2^n - 1$. Let u, v be two vertices of G such that $d(u,v)$ is maximum. Let $N_r(u) = \{v \in V / d(u,v) = r\}$. If $d(u) = 2^{n-1}$, choose 2^{n-1} edges incident with u. Let $G_{2^{n-1}}$ be a subgraph induced by these 2^{n-1} edges. If $d(u) < 2^{n-1}$, then choose 2^{n-1} edges incident with

u vertices of $N_1(u)$, $N_2(u)$, $\cdots$ successively such that the subgraph $G_{2^{n-1}}$ induced by these edges is connected. In both cases G - $G_{2^{n-1}}$ has a connected component H_1 with $2^n - 2^{n-1} - 1$ edges.

Now, consider H_1 and proceed as above to get $G_{2^{n-2}}$ such that H_1 - $G_{2^{n-2}}$ has a connected component H_2 of size $2^n - 2^{n-1} - 2^{n-2} - 1$ edges. Proceeding like this we get a connected subgraph G_2 such that $H_{2^{n-2}}$ is a graph with one edge taken as G_1. Thus (G_1, G_2, G_4, …, $G_{2^{n-1}}$) is a GD of G.

Conversely, Suppose G admits GD (G_1, G_2, G_4, …, $G_{2^{n-1}}$). Then obviously, $q(G) = 1 + 2 + 4 + … + 2^{n-1} = 2^n - 1$ for each $n \in$ N.

Definition 2.4:
A GD in which each $G_{2^{i-1}}$ is a path of size 2^{i-1} is said to be a Geometric Path Decomposition (GPD).

Example 2.5:

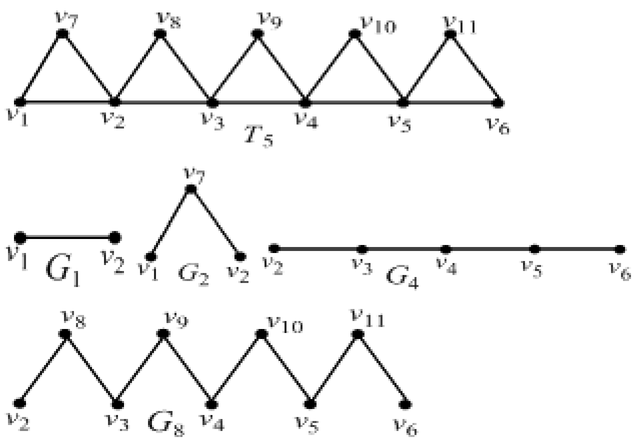

Figure 2: A triangular snake graph T_5 admits GPD.

Definition: 2.6
A GD in which each $G_{2^{i-1}}$ is a star of size 2^{i-1} is said to be a Geometric star Decomposition (GSD).

Example: 2.7

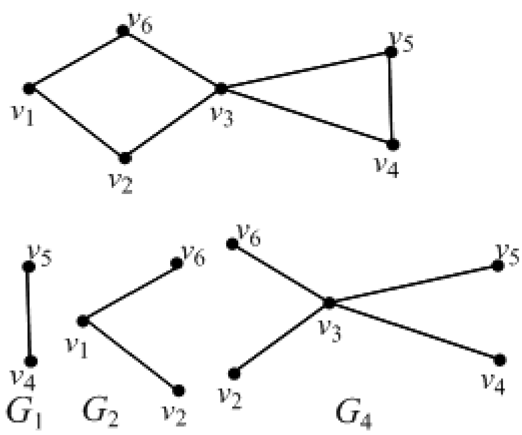

Figure 3: Fish graph admits GSD.

3. Geometric Decomposition of Spider Graphs

Definition 3.1: A tree T with exactly one vertex of degree ≥ 3 is called a Spider tree.

Notation 3.2: Let W denote the set of pendent vertices of T and u be the vertex of degree ≥ 3 in T.

Theorem 3.3 : If T is a spider tree with diam(T) = t, 2 ≤ t ≤ 5 with d(u) = (2^n - 1) - (t - 2), then T admits GSD.

Proof:
Case (i): t =2. Since diam (T) = 2, T is a star. Also, since d(u) = 2^n - 1, T is $K_{1, 2^n-1}$. Therefore, q(T) = 2^n - 1. Hence T admits GSD.

Case (ii) t =3. Since diam(T) = 3 and $d(u)$ = (2^n - 1) - 1, there are (2^n - 1) - 2 pendent edges incident with u. Let S_1 = e. Then T- e is a star $K_{1,(2^n-1)-1}$ and q(T- e) = (2^n - 1) -1. Then we can easily decompose T - e into S_2, S_4, S_8, …, $S_{2^{n-1}}$. Hence T admits GSD.

Case (iii) t=4.
Subcase (i): u is the origin of P_3.
Let u_1 be a non pendent vertex adjacent to u and u_2 be a terminus of u - u_2 path of length 3. Let $S_1 = u_1u$ and $S_2 = u_2 - u_1$. Then the remaining edges of tree is a star which can be decomposed into S_4, S_8, S_{16}, …, $S_{2^{n-1}}$.

Subcase(ii): u is not the origin of P_3.
Let u_1 and u_2 be the two non pendent vertices adjacent to u and let v_1 and v_2 be the pendent vertices adjacent to u_1 and u_2 respectively. Then $S_1 = u_1v_1$ and $S_2 = u - v_2$ path in T and the remaining edges form a star $K_{1,(2^n-1)-3}$. Then we can easily decomposed into S_4, S_8, S_{16}, …, $S_{2^{n-1}}$.

Case (iv) t=5.
Subcase (i): u is the origin of P_4.
Let u_1 be a non pendent vertex adjacent to u and u_2 be a terminus of u - u_2 path of length 4. Then u_2 - u_1 path can be decomposed in to S_1, S_2 and the remaining edges is a star. Clearly q(T - { S_1 S_2}) =(2^n - 1) - 3. Then T can easily decompose S_4, S_8, S_{16}, …, $S_{2^{n-1}}$.

Subcase(ii): u is not the origin of P_4.
Let u_1 and u_2 be the two non pendent vertices adjacent to u and v_1 be a pendent vertex adjacent to u_1. Let v_2 be the pendent vertex of T such that there is a u_2 - v_2 path of length 2 is adjacent to u_2. Then $S_1 = u_1v_1$ and $S_2 = u_2 - v_2$ path in T and the remaining edges is a star with (2^n - 1) - 3 edges. Hence T admits GSD
.
Theorem 3.4 : If T is a spider tree with diam(T) = t, 3 ≤ t ≤ 5 and d(u) = (2^n - 1) - (t - 2) admits GSD if and only if T - W = P_x where x ≤ 3.

Proof:
Assume T - W = P_x where x ≤ 3.Then by previous theorem T admits GSD. Conversely, the result is obvious.

Result 3.5: If T is a spider tree with diam(T) = 2 and d(u) = 3, then T admits GSD and GPD.

Proof:
Since diam(T) =2 and d(u) = 3. Clearly T is a spider tree with 3 edges. Then we can easily decompose T into paths P_1 and P_2. Therefore, by theorem(3.3) T admits GSD and GPD.

Result 3.6: If T is a spider tree with diam(T) = 4 and d(u) = 5, then T admits GSD and GPD.

Proof:
Since diam(T) = 4 and d(u) = 5, then there is a path of length 4. Therefore, the spider tree can be decomposed into P_1, P_2 and P_4. Also by theorem (3.3) T admits GSD and GPD.

Result 3.7: If T is a spider tree with diam (T) = 5 and d(u) = 4,then T admits GSD and GPD.

Proof:
Sinc Since diam(T) =5, then there is path of length 5. Then P_5 can be decomposed into P_1 and P_4. Also by theorem (3.3) T admits GPD and GSD.

Results 3.8:
(i) If T is a spider tree with $(2^n - 1) - 5 \leq \text{diam(T)} \leq (2^n - 1) - 1$, then T admits GPD but not GSD.
(ii) If T is a spider tree with $6 \leq \text{diam(T)} \leq (2^n - 1) - 6$, then T admits neither GPD nor GSD.

Example 3.9: Consider a spider tree T with $q = 15$.

Diam (T)	GSD	GPD
2	Yes	No
3	Yes	No
4	Yes	No
5	Yes	No
6	No	No
7	No	No
8	No	No
9	No	No
10	No	Yes
11	No	Yes
12	No	Yes
13	No	Yes
14	No	Yes

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