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Unsteady MHD Flow of a Dusty Couple Stress Fluid through a Circular Pipe

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Abstract: ***In this paper, the transient magnetohydrodynamics (MHD) flow of a dusty in compressible electrically conducting couple stress fluid through a circular pipe is studied taking the Hall effect in to consideration. A constant pressure gradient in the axial direction and an uniform magnetic field directed perpendicular to the flow direction are applied. The particle – phase is assumed to behave as a viscous fluid. A numerical solution is obtained for the governing non-linear equation using transform technique.***

Keywords: Dusty fluid , couple stress fluid, magnetohydrodynamics

1. Introduction

The flow of a dusty and electrically fluid conducting fluid through a circular pipe in the presence of transverse magnetic field has important application such as MHD generators, pumps, accelerators, and flow meters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so-called particle phase viscosity. There have been many articles dealing with theoretical modeling and experimental measurements of the particle – phase viscosity in a dusty fluid [1].

The flows of couple stresses fluids have many practical application in modern technology and industries, led various researchers to attempt diverse flow problems related to several Non-Newtonian fluids one such fluid that has attracted the attention of numerous researchers in fluid mechanics during the last five decades in the theory of couple stress fluid proposed by stokes [2] classical theory of viscous Newtonian fluids that allow the sustenance of couple stresses and body couples in the fluid medium. The concept of couple stresses arises due to the way in which the mechanical interactions in the fluid medium are modeled. Singh and Pathak [3] have discussed unsteady flow of a dusty viscous fluid through a uniform pipe with sector of a circle as cross-section, and pulsatile flow of blood with micro-organism through a uniform with sector of a circle as cross-section in the presence of transverse magnetic field has been investigated by Rathod and Parveen [4]. Also unsteady flow of a dusty magnetic conducting couple stress fluid through a pipe and the flow of a conducting fluid in a circular pipe has been investigated by many authors. Gudiraju et. al [5]. Dube and Sharma [6]. Ritter and Peddison [7] Chamkha[ ] and Ritter and Peddieson [8] have reported solutions for unsteady dusty –gas flow in a circular pipe in the absence of a magnetic field and particle – phase viscous stress. Rathod and Baderunnissa [9] have studied by the [8], investigated steady two phase vertical flow in a pipe. Dube and Sharma pulsatile flow of blood in capillaries of small exponential divergence with volume fraction of micro-organism. Rathod et. al, [10] have reported solution for couette flow of a conducting dusty visco-elastic fluid through two that plate under the influence of transverse magnetic field. Rathod and Rasheeda [11, 12] investigated unsteady flow of a dusty magnetic conducting couple stress fluid through a circular pipe and ion slip effect on the unsteady flow of a dusty couple stress fluid through a circular pipe. Rathod and Rasheeda [13] have studied by unsteady MHD couette flow with heat transfer of a couple stress fluid under exponential decaying pressure gradient. The effect of time dependent pressure gradient on steady dusty fluid was studied by Rukmargadachari [14] in a rectangular duct and time dependent pressure gradient effect on unsteady MHD couette flow and heat transfer of a caisson fluid was studied by Attia et. al, [15].

It is of interest in this paper to study the influence of the magnetic fluid as well as the non-Newtonian fluid characteristics on the dusty fluid flow properties in situations where the particle-phase is considered dense enough to include the particulate viscous stresses.

In the present study, a new element is added to the problem studied by Attia [16] by taking the Hall effect in to consideration. Therefore, the unsteady flow of a dusty couple stress fluid through a circular pipe is investigated considering the Hall effect. The carrier fluid is assumed viscous, incompressible and electrically conducting. The particle phase is assumed to be incompressible pressure less and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing nonlinear by momentum equations for both the fluid and particle – phase are solved transform technique. The effect of Hall current, the couple stress parameter and the particle – phase viscosity on the velocity of the fluid and particle phase are reported.

2. Governing Equation

Consider the unsteady, laminar, and ax symmetric horizontal flow of a dusty conducting non-Newtonian couple stress fluid through an infinite long pipe of radius „r" driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The Hall current is taken in to consideration and the magnetic Reynolds number is assumed to be very small, consequently the induced magnetic field is neglected [17] we assume that both phases behave as viscous fluids and that the volume fraction of suspended particles is finite and constant [18] Taking in to account these and the previously mentioned assumptions. The governing momentum equation can be written as

$$\varrho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial t} + \frac{1}{r}\frac{\partial p}{\partial r}\left(\mu r \frac{\partial r}{\partial r}\right) + \frac{\varrho_p \varphi N (V_p - V)}{1-\varphi} - \eta\nabla^2(\nabla^2 V) - \frac{\sigma B_o^2 V}{1+m2} \quad (1)$$

$$\varrho_p \frac{\partial V_p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu_p r \frac{\partial V_p}{\partial r}\right) + \varrho_p N (V - V_p) \quad (2)$$

Where t is the time r is the distance in the radial direction, v is the thud – phase velocity, V_p is the particle – phase velocity, ϱ is the fluid phase density, ϱ_p is the particle – phase density, $\partial\frac{\partial p}{\partial t}$ is the fluid pressure gradient, φ is the particle – phase volume fraction. N is a momentum transfer coefficient (the reciprocal of the relaxation time). The time needed for relative velocity between the phase to reduce e^{-1} of its original value (18). σ is the fluid electrical conductivity m + $\sigma\gamma B_o$ is the Hall parameter, ϒ is the Hall factor (17) B_o is the magnetic induction. μp is the particle – phase viscosity which is assumed constant, and $\boldsymbol{\mu}$ is the apparent viscosity of the fluid. $\propto^{-2} = \frac{a^2\mu}{\eta}$ couple stress parameter. In this work , ϱ_p, $\boldsymbol{\mu}_p$, φ and B_o all are constant. It should be pointed out that the particle phase pressure is assumed negligible and that the particle are being dragged along with the fluid – phase.

The initial and boundary conditions of the problem are given as.

$$v(r, o) = o, \quad v_p(r, o) = o. \quad (3)$$

$$\frac{\partial v(o,t)}{\partial r} = o, \frac{\partial V_p(r,t)}{\partial r} = o, \ v(d, t) = o, \ v_p(d, t) = o. \quad (4)$$

Where „d" is the pipe radius.

equations (1) (2) & (3) constitute a nonlinear initial – value, problem which can be made dimensionless by introducing the following dimensionless variables and parameters.

$$r = \frac{r}{d}, \quad t = \frac{t\mu o}{\varrho t^2}, \quad G_o = \frac{\partial p}{\partial z}, \quad k = \frac{\varrho_p \varphi}{\varrho(1-\varphi)}, \quad \boldsymbol{\mu} = \frac{\mu}{\mu_o}$$

$$v(r, t) = \frac{\mu_o v(r,t)}{G_o d^2}, \quad v_p(r, t) = \frac{\mu_o v(r,t)}{G_o d^2}$$

$\propto = \frac{N d^2 \varrho}{\mu_o}$ is the inverse stokes number

$\beta = \frac{\mu_p}{\mu_o}$ is the viscosity ratio.

$\propto^{-2} = \frac{d^2\mu_o}{\eta}$, is the couple stress parameter.

Equation (1) and (2) after dropping caps for convenience

$$\frac{\partial v}{\partial t} = 1\,\nabla^2 v + K\propto (v_p - v)\ \frac{-1}{\propto^{-2}}\nabla^2(\nabla^2 v) - \frac{H_a^2 v}{1+m^2} \quad (5)$$

$$\frac{\partial V_p}{\partial t} = \beta\,\nabla^2 vp + \propto (v - vp) \quad (6)$$

$$v(r, o) = o, \quad v_p(r, o) = o$$

$$\frac{\partial v(o,t)}{\partial r} = o, \ \frac{\partial V_p(o,t)}{\partial r} = o, v(1, t) = o, \ v_p(1, t) = o.$$

By applying transform techniques

We get

$$v = \frac{4}{\propto}\sum_{m=o}^{\infty}\frac{(-1)^m}{n x_2}\left[1 + \frac{1}{(x_1^2 - 4x_2)^{½}}\{\propto_2 e^{\propto_1 t} - \propto_1 e^{\propto_2 t}\}\ \sum_{i=1}^{\propto}\frac{J_o(r\mathcal{E}_i)}{J_1(\mathcal{E}_i)}\cos\left(\frac{2m+1}{2\propto}\right)\barwedge\theta\right] \quad (7)$$

$$v_p = \frac{4}{\propto}\sum_{m=o}^{\infty}\frac{(-1)^m}{n}\cdot\frac{1}{x_2}\left[\frac{1-e^{-(\propto+\beta)t}}{\propto+\beta} + \frac{1}{\sqrt{x_1^2-4x_2}}\left(\frac{\propto_2(e^{\propto_1 t} - e^{-(\propto+\beta)t})}{\propto_1 + (\propto+\beta)} - \right.\right.$$

$\alpha 1(e\alpha 2t - e - \alpha + \beta t)\alpha 2 + (\alpha + \beta)$

$$\sum_{i=1}^{\propto}\frac{J_o(r\mathcal{E}_i)}{J_1(\mathcal{E}_i)}\cos\left(\frac{2m+1}{2\propto}\right)\barwedge\theta \quad (8)$$

The volumetric flow rates and skin – friction coefficients for both the fluid and particle phases are defined as.

$$Q = 2\barwedge\int_o^1 r v(r,t)dr, \quad Q_p = 2\barwedge\int_o^1 r v_p(r,t)dr,$$

$$C = \frac{\partial v(1,t)}{\partial r}, \quad C_p = -\beta k\frac{\partial V_p(1,t)}{\partial r}$$

Where

$$X_1 = 1 + \beta - \propto + K\propto + \frac{1}{\propto^{-2}} + \frac{H_a^2}{1+m^2}$$

$$X_2 = \beta + (1+K)\propto + \frac{1}{\propto^{-2}}(\propto + \beta) + \frac{H_a^2}{1+m^2}(1+\beta).$$

$$Q = \frac{8}{\propto}\sum_{m=o}^{\infty}\frac{(-1)^n}{n}\cdot\frac{1}{x_2}\left[1 + \frac{1}{(x_1^2-4x_2)^{½}}\ \sum_{i=1}^{\infty}\frac{1}{(\mathcal{E}_i)^2 J_1(\mathcal{E}_i)}\cos\left(\frac{2m+1}{2\propto}\right)\barwedge\theta\right].$$

$$\int_o^{\mathcal{E}_i} J_o(T)dT \quad (9)$$

$$Q_p = \frac{8}{\propto}\sum_{m=o}^{\infty}\frac{(-1)^n}{n}\cdot\frac{1}{x_2}\left[\frac{1-e^{-(\propto+\beta)t}}{\propto+\beta} + \frac{1}{\sqrt{x_1^2-4x_2}}\left\{\frac{\propto_2(e^{\propto_1 t} - e^{-(\propto+\beta)t})}{\propto_1 + (\propto+\beta)} - \right.\right.$$

$\alpha 1e\alpha 2t - e - \alpha 1 + \beta t\alpha 2 + (\alpha + \beta)$.

$$\sum_{i=1}^{\infty}\frac{1}{(\mathcal{E}_i)^2}\int_o^{\mathcal{E}_i}\frac{J_o(T)dT}{J_o(\mathcal{E}_i)}\ \cos\left(\frac{2m+1}{2\propto}\right)\barwedge\theta \quad (10)$$

$$C = \frac{+4}{\propto}\sum_{m=o}^{\infty}\frac{\propto(-1)^n}{n}\cdot\frac{1}{x_2}\left[1 + \frac{1}{\sqrt{x_1^2-4x_2}}(\propto_2 e^{\propto_1 t} - \propto_1 e^{\propto_2 t})\right].\cos\left(\frac{2m+1}{2\propto}\right)\barwedge\theta$$

$$\sum_{i=1}^{\infty}\frac{J_1(r\mathcal{E}_i)}{J_1(\mathcal{E}_i)} \quad (11)$$

$$C_p = 4\,\beta k\sum_{m=o}^{\infty}\propto\frac{(-1)^m}{n}\cdot\frac{1}{x_2}\left[\frac{1-e^{-(\propto+\beta)t}}{\propto+\beta} + \frac{1}{\sqrt{x_1^2-4x_2}}\left(\frac{\propto_2(e^{\propto_1 t} - e^{-(\propto+\beta)t})}{\propto_1 + (\propto+\beta)} - \frac{\propto_1(e^{\propto_2 t} - e^{-(\propto+\beta)t})}{\propto_2 + (\propto+\beta)}\right)\right]$$

$$\sum_{i=1}^{\infty}\frac{J_1(r\mathcal{E}_i)}{J_1(\mathcal{E}_i)}\left[\cos\left(\frac{2m+1}{2\propto}\right)\barwedge\theta\right] \quad (12)$$

3. Results and Discussion

Equations (4), (5) represent a coupled system of nonlinear partial differential equation that are solved by using transform techniques computations have been made for $\propto = 1, k= 10$, Ha = 0.5, B = 0.5, ploted the graph by using mathematic for different values of time parameter.

Imposing of a magnetic field normal to the flow direction gives rise to a drag-like or resistive force and has the tendency to slow down or suppress the movement of the fluid in the pipe, which in turn, reduces the motion of the suspended particle – phase. This is translated in to reduction in the overage velocities of both the fluid and the particle phases and consequently, in their flow rates. In addition, the reduced motion of the particulate suspension in the pipe as a result of increasing the strength of the magnetic field causes lower of increasing gradients at the wall. This has the direct effect of reducing the skin-friction co-efficient of both phases. Including the Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity. Therefore, the Hall parameter leads to an increase in the average velocities of both the fluid and the particle – phases and consequently, in their flow rate and the velocity gradients at the wall.

Fig-1 and 2 present the time evolution of the profiles of the velocity of the fluid v and dust particles v_p respectively, for various values of the couple stress fluid $\propto^{-2}$= q and m = o, Ha = 0.5 and β = 0.5. Both v and v_p increases with the time and v reaches the steady state faster than v_p for all values of $\propto^{-2}$= q. It is clear from Fig-1and 2 that increasing $\propto^{-2}$ which increases the driving force for v. Increase V and, consequently, increase V_p while the effect on their steady – state time can be neglected. Fig-3 and 4 present the time evolution of the profiles of the velocity of the fluid v and dust particles v_p respectively, for various values of the couple stress parameter and m = 1, Ha = 0.5 and B = 0.5. It is indicated in the figures that increasing m increasing v and m turn, v_p due to the decrease in the effective conductivity $\frac{\sigma}{1+m^2}$ which reduces the damping magnetic force on v. It is show that influence of the Hall parameter m on V is more apparent for higher values of $\propto^{-2}$.

Table -1 present the steady state values of the fluid phase volumetric flow rate Q, the particle – phase volumetric flow rate Q_p the fluid – phase skin friction coefficient C, and the particle – phase skin friction coefficient C_p for various values of the parameter $\propto^{-2}$ and m and for Ha = 0.5 and B = 0.5. It is clear that increasing the parameter m increases Q. Q_p, C and C_p for all values of $\propto^{-2}$. This comes from the fact that increasing m increases the velocities and their gradients which increases the average velocities of and their gradients which increases the average velocities of both the fluid and the particle – phases and consequently, increases their flow rates and skin friction coefficient of both phases. It is also shown that increasing $\propto^{-2}$ increases Q, Q_p, C and C_p for all values of m as a result of increasing the velocities of both phases.

Table 1: the steady state values of Q, Qp, C, Cp, for various values of m and q (couple stress)

Q = 0.001	m = o	m = 1	m = 2
Q	0.00421603	0.00140592	0.00084351
Q_p	0.000225079	0.000075046	0.0000450347
C	0.000734619	0.000244939	0.000146987
C_p	0.00279584	0.0018644	0.00111882
Q = 0.05	m = o	m = 1	m2
Q	0.149325	0.050214	0.0302885
Q_p	0.006713	0.00225385	0.0013582
C	0.0260189	0.00874949	0.00527759
C_p	0.198048	0.0665984	0.040714
Q = 0.025	m = o	m = 1	m = 2
Q	0.0889148	0.0298024	0.017941
Q_p	0.00428926	0.00143667	0.000864509
C	0.0154929	0.00519288	0.00312611
C_p	0.117927	0.0395266	0.023795

$\beta = 0$	m = o	m = 1	m = 2
Q	0.00442836	0.00140592	0.000886054
Q_p	0.000240417	0.000075046	0.0000481036
C	0.000771614	0.000244939	0.00015439
C_p	0	0	0
$\beta = 0.5$	m = o	m = 1	m = 2
Q	0.00421603	0.00140572	0.000843571
Q_p	0.000225079	0.000075046	0.0000450347
C	0.000734619	0.000244939	0.000146987
C_p	0.00559169	0.0018644	0.00111882
$\beta = 1$	m = o	m = 1	m2
Q	0.004072	0.00133943	0.000803786
Q_p	0.000210832	0.0000762975	0.0004185
C	0.000699972	0.000233387	0.000140055
C_p	0.00581234	0.00193797	0.00116297

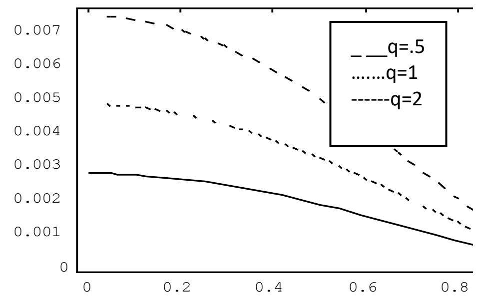
Table 2, the steady state values of Q, Q_p, C, C_p, for various values of m and β.

Table-2 presents the steady state values of the fluid – phase volumetric flow rate Q. the particle–phase volumetric flow rate Q_p. the fluid-phase skin friction coefficient C, and the particle – phase skin friction coefficient C_p for various values of the parameters m and β for Ha = 0.5 and q = 0.001. It is clear that, increasing m increases Q, Q_p, C, C_p for all values of β and its effects becomes more pronounced for smaller values of β. Increasing the parameter β decreases the quantities Q, Q_p and C but C_p for all values of m. This com be attributed to the fact that increasing β increases viscosity and therefore the flow rates of both phases as well as the fluid – phase will friction decreases considerably. However, since C_p in defined as directly proportional to β, it increases as β increases at all times.

4. Conclusion

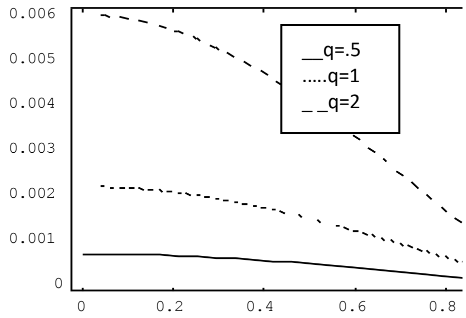
The unsteady MHD flow of a particulate suspension in an electrically conducting couple stress fluid in a circular pipe is studied considering the Hall effect. The governing nonlinear partial differential equation are solved by using transform technique. The effect of the magnetic field parameter Ha, the Hall parameter, couple stress parameter and the particle phase viscosity β on the transient coefficients of both fluid and particle phase is studied. It is shown that increasing the magnetic field decreases the fluid and particle velocities. While increasing the Hall parameter increases both velocities. It is found that increases Q, Q_p, C and C_p for all values of q. The effect of the Hall parameter on the quantities Q, Q_p, C and C_p becomes more pronounced for smaller values of β.

V



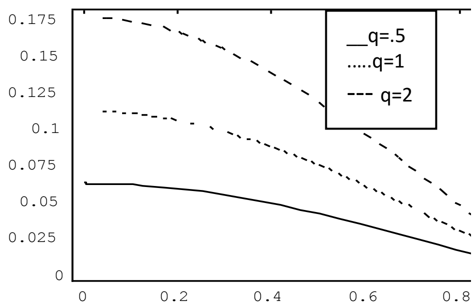
→ r 1(a)

v

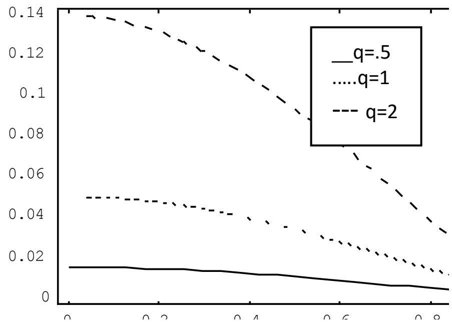


2(a)

v

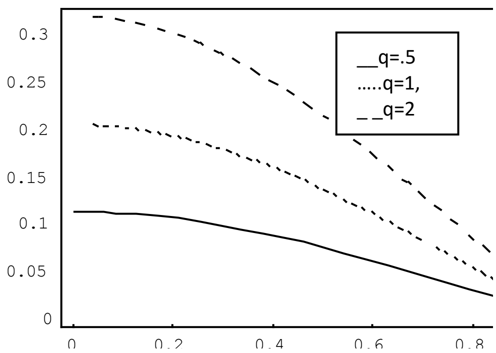


→ r 1(b)

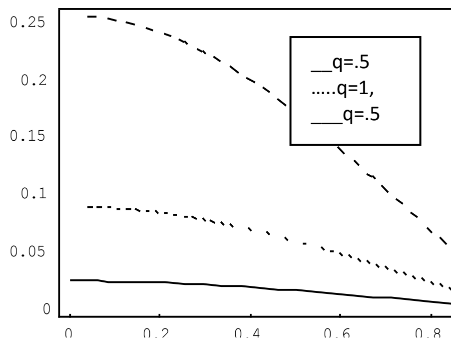


2(b)

v



→ r **1(c)**

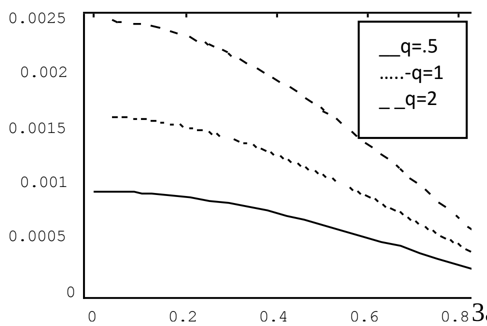


2(c)

Figure 1: Time development of v for various values of q (m=0)

Figure 2: Time development of V_p for various values of q (m=0)

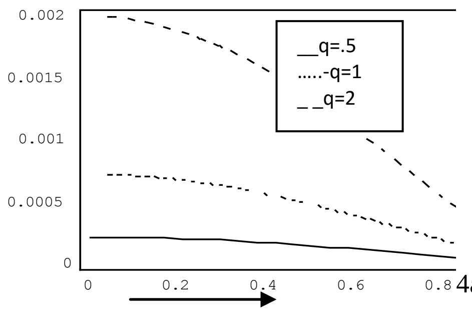
V



3a

r

vp



4a

v

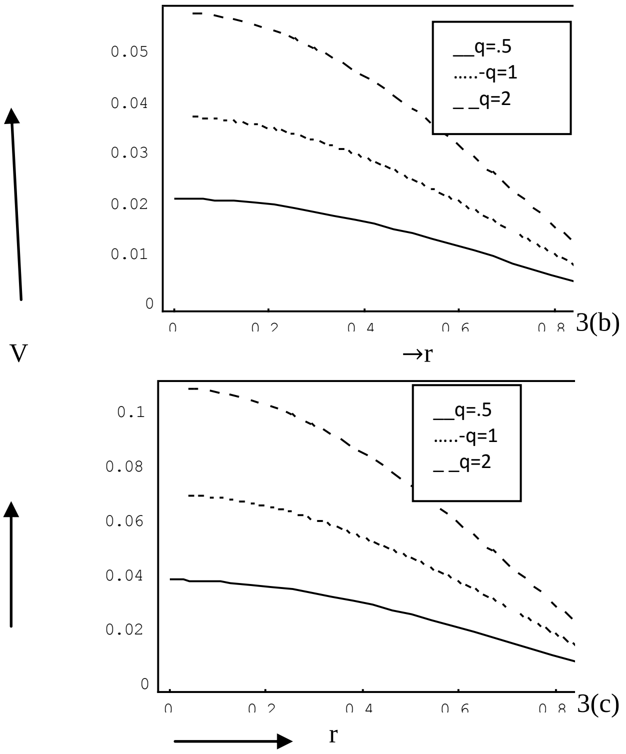


Figure 3: Time development of V for various values of q (m=1)

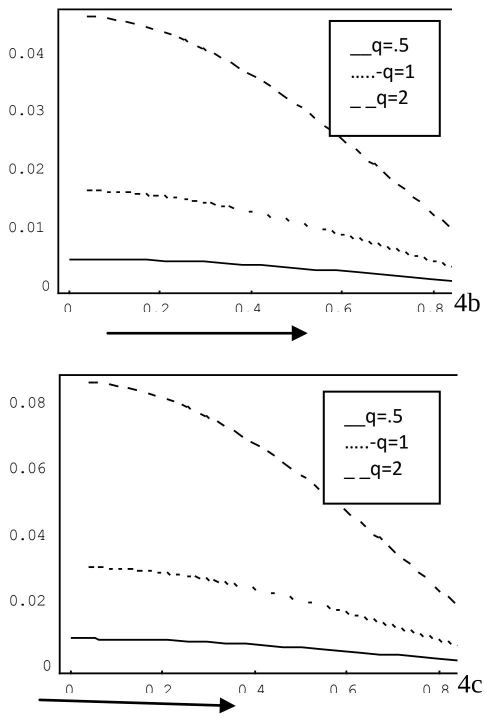


Figure 4: Time development of V_p for various values of q (m=1)

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