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Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space Satisfying Integral Type Inequality

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Abstract: ***The aim of this paper is to present some common fixed point theorem in Intuitionistic fuzzy metric space satisfying integral type inequality for E.A. Property.***

Keywords: Common fixed point, Fuzzy metric space, E.A. Property,Semi Compatible maps, Intuitionistic fuzzy metric space.

2000 Mathematics Subject Classification: 47H10, 54H25.

1. Introduction

The theory of fixed point equations in one of the basic tools to handle various physical formulations. Fuzzy set was defined by Zadeh [1]. Kramosil and MIchalek [2]. introduced fuzzy metric space. many authors extend their views. Grorge and Veermanyam [3]. modified the notion of fuzzy metric spaces with the help of continuous t-norms Grabiec[4].

In the recent year, several common fixed point theorems for contractive type mappings have been proved by several authors. Branciari [5],gave a fixed point result for a single mapping satisfying Banach's contraction principle for an integral type inequality.

Aliouche[6] established a common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying a contrative condition of integral type and property (E.A.) introduced by Aamri and El. Moutawakil [7].

Boikanyo and Choudhary [8] prove some common fixed point theorem for pointwise R-weakly commuting mapping s in symmetric space.

K.Atanassov [9] introduced and studied the concept of intuitionistic fuzzy sets. In 2004, J.H.Park [10] define intuitionistic fuzzy metrice space with the help of continuous t –norms and continuous t –conforms.

In this paper, we obtain common fixed point theorem in Intuitionistic fuzzy metric space using E.A. property.

2. Preliminary

Definition 2.1 [11] A binary operation $*:[0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t –norm if it satisfies the following conditions :

(i) *is associative and commutative
(ii) *is continuous,
(iii) $a*1 = a$, for all $a \in [0,1]$
(iv) $a*b \leq c*d$, whenever $a \leq c$ and $b \leq d$, for all $a,b,c,d \in [0,1]$.

Definition 2.2 [11] A binary operation $\Diamond:[0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t –conorm if it satisfies the following conditions :
(i) ◊is associative and commutative
(ii) ◊is continuous,
(iii) $a\Diamond 1 = a$, for all $a \in [0,1]$
(iv) $a\Diamond b \leq c\Diamond d$, whenever $a \leq c$ and $b \leq d$, for all $a,b,c,d \in [0,1]$.

Definition 2.3 [12] A 5-tuple (X, M, N, *, ◊) is said to be an Intuitionistic fuzzy mertic space if X is an arbitrary set, * is a continuous t -norm, ◊ is a continuous t –conorm and M,N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions.
(i) $M(x,y,t) + N(x,y,t) \leq 1$, for all $x,y \in X$ and $t > 0$,
(ii) $M(x,y,0) = 0$, for all $x,y \in X$,
(iii) $M(x,y,t) = 1$, for all $x,y \in X$ and $t > 0$, iff $x = y$,
(iv) $M(x,y,t) = M(y,x,t)$, for all $x,y \in X$ and $t > 0$,
(v) $M(x,y,t)*M(y,z,s) \leq M(x,z,t+s)$, for all $x,y \in X$ and $t ,s > 0$,
(vi) for all $x,y \in x$, $M(x,y,.) : [0,\infty) \rightarrow [0,1]$ is left continuous,
(vii) $\lim_{t\rightarrow\infty} M(x,y,t) = 1$, for all $x,y \in X$ and $t > 0$,
(viii) $N(x,y,0) = 1$, for all $x,y \in X$,
(ix) $N(x,y,t) = 0$, for all $x,y \in X$ and $t > 0$, iff $x = y$,
(x) $N(x,y,t) = N(y,x,t)$, for all $x,y \in X$ and $t > 0$,
(xi) $N(x,y,t)*N(y,z,s) \leq N(x,z,t+s)$, for all $x,y \in X$ and $t ,s > 0$,
(xii) for all $x,y \in x$, $N(x,y,.) : [0,\infty) \rightarrow [0,1]$ is right continuous,
(xiii) $\lim_{t\rightarrow\infty} N(x,y,t) = 0$, for all $x,y \in X$.

Remark 2.1[13]
In intuitionistic metric fuzzy space (X ,M ,*) is an intuitionistic fuzzy space of the form (X ,M ,1- M ,*, ◊) ,

such that t-norm * and t-conorm ◊ are associated as x◊y = 1-((1-x) * (1-y)) for all x,y ∈ X .

Remark 2.2[13]
In intuitionistic fuzzy metric space (X ,M ,N ,*,◊) , M(x,y,*) is non-decreasing and N(x,y,◊) is Non-increasing for all x,y ∈ X .

Example 2.1 – Let (X ,d) be a metric space . Define a*b = ab and a◊ b = min {1,a + b} for all a,b ∈ [0,1] and let M_d and N_d be a fuzzy sets on X^2 x (0,∞) defined as
M_d (x,y,t) = t / t + d(x,y) , N_d(x,y,t) =d(x,y) / t + d(x,y)
Then (X , M_d ,N_d ,*,◊) is an intuitioistic fuzzy metric space.

Definition 2.4[12] Let (X , M , N ,*,◊) be an intuitioistic fuzzy metric space . Then
(1) A sequence {x_n} in X is set to be convergent to a point x in X iff $\lim_{n\to\infty} M(x_n , x,t) = 1$ and
$\lim_{n\to\infty} N(x_n , x,t) = 0$, for all t > 0.

Lemma 2.1[12] Let (X ,M ,N ,*,◊) be an intuitionistic fuzzy metric space. If for all x,y ∈ X and t > 0 with positive number k ∈ (0,1) and M(x,y,kt) ≥ M(x,y,t) and N(x,y,kt) ≤ N(x,y,t) , then x = y.

Definition 2.5[13] A pair of self mappings (P ,Q) of a intuitionistic fuzzy metric space (X , M , N ,*,◊) is said to be compatible if $\lim_{n\to\infty} M(PQx_n , QPx_n , t) = 1$ and $\lim_{n\to\infty} N(PQx_n , QPx_n , t) = 0$, for all t > 0.
When ever {x_n} is a sequence in X such that
$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n$ = z, for some z ∈ X .

Definition 2.6[14]A pair of self mappings (P ,Q) of a intuitionistic fuzzy metric space (X , M , N ,*,◊) is said to be semi compatible if $\lim_{n\to\infty} PQx_n = Qx$, When ever {x_n} is a sequence in X such that
$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n$ = x, for some x ∈ X .

Definition 2.7[7] A pair of self mapping (P,Q) on an intuitionistic fuzzy metric space (X ,M ,N ,*,◊) is said to satisfy the E.A. property if there exist a sequence {x_n} in X such that
$\lim_{n\to\infty} Px_n = z = \lim_{n\to\infty} Qx_n$ for some z ∈ X .

Definition 2.8[15] Mapping A, B, S and T on an intuitionsitic fuzzy metric space (X ,M ,N ,*,◊) are said to satisfy the common E.A. property if there exist a sequence {x_n} and {y_n} in X such that
$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ = z for some z ∈ X .

Theorem3.1 Let (X,M,N,*,◊) be an Intuitionistic fuzzy metric space with continuous t- norm *and continuous t-conorm ◊ . Let P,Q,S and T be self mappings on X, satisfying the following properties ;

1. pair (P,S) and (Q,T) share the common property E.A.
2. S(X) and T(X) are closed subset of X .
3. For any x,y ∈ X and for all t>0 there exist

k∈ (0,1) such that

$$\int_0^{M(Px,Qy,kt)} \emptyset(t)dt \geq \int_0^{\min[M(Sx,Ty,t)*[M(Sx,Px,t).M(Qy,Ty,t)\}*M(Px,Ty,t)]} \emptyset(t)dt$$

And,

$$\int_0^{N(Px,Qy,kt)} \emptyset(t)dt \leq \int_0^{\max[N(Sx,Ty,t)*\{N(Sx,Px,t).N(Qy,Ty,t)\}*N(Px,Ty,t)]} \emptyset(t)dt$$

For all x,y ∈ X, where ϕ : $R^+ \to R^+$ is a Lesbegue integrable mapping which is summable satisfying for each 0 < ε < 1,
$0 < \int_0^{\varepsilon} \emptyset(t)dt < 1, \int_0^{1} \emptyset(t)dt = 1$
Then each of pair (P,S) and (Q,T) have a point of coincidence . If the pairs (P,S) and (Q,T) are semi compatible , then P, Q, S and T have a unique common fixed point .

Proof – Since the pairs (P,S) and (Q,T) Share the common property (E.A.), then there exists two sequences {x_n} and {y_n} in X such that
$\lim_{n\to\infty} P x_n = \lim_{n\to\infty} S_{xn} = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} Ty_n$=z, for some z∈ X .
Since S(X) is closed subset of X ,therefore there exists a point v∈ X such that z=Sv
Now, we prove that Pv=z
By inequality (3), Putting x=v, and y=y_n we get

$$\int_0^{M(Pv,Qy_n,kt)} \emptyset(t)dt \geq \int_0^{\min[M(Sv,Ty_n,t)*\{M(Sv,Pv,t).M(Qy_n,Ty_n,t)\}*M(Pv,Ty_n,t)]} \emptyset(t)dt$$

Taking lim n→ ∞, we get

$$\int_0^{M(Pv,z,kt)} \emptyset(t)dt \geq \int_0^{\min[M(z,z,t)*\{M(z,Pv,t).M(z,z,t)\}*M(Pv,z,t)]} \emptyset(t)dt$$

$$\int_0^{M(Pv,z,kt)} \emptyset(t)dt \geq \int_0^{\min[1*\{M(z,Pv,t).1\}*M(Pv,z,t)]} \emptyset(t)dt$$

$$\int_0^{M(Pv,z,kt)} \emptyset(t)dt \geq \int_0^{M(z,Pv,t)} \emptyset(t)dt$$

Similarly,

$$\int_0^{N(Pv,Qy_n,kt)} \emptyset(t)dt \leq \int_0^{\max[N(Sv,Ty_n,t)\lozenge\{N(Sv,Pv,t).N(Qy_n,Ty_n,t)\}\lozenge N(Pv,Ty_n,t)]} \emptyset(t)dt$$

Taking lin n→∞, we get

$$\int_0^{N(Pv,z,kt)} \emptyset(t)dt \leq \int_0^{\max[N(z,z,t)\lozenge\{N(z,Pv,t).N(z,z,t)\}\lozenge N(Pv,z,t)} \emptyset(t)dt$$

$$\int_0^{N(Pv,z,kt)} \emptyset(t)dt \leq \int_0^{\max[1\lozenge\{N(z,Pv,t).1\}\lozenge N(Pv,z,t)} \emptyset(t)dt$$

$$\int_0^{N(Pv,z,kt)} \emptyset(t)dt \leq \int_0^{N(Pv,z,t)} \emptyset(t)dt$$

By Lemma 2.1, we conclude that Pv=z

Since z=Sv and we proved that z=Pv, then from this we get z=Pv= Sv, which shows that v is a coincidence point of the pair (P,S).

Since T(x) is also closed subset of X. There fore $\lim_{n\to\infty} Ty_n$=z in T(X) and hence there exists w∈ Ssuch that Tw= z = Sv = Pv .

Now, we will prowe that Qw=z.

By using in equality (3) , putting x = v, y = w, we get

$$\int_0^{M(Pv,Qw,kt)} \emptyset(t)dt \geq$$

$$\int_0^{\min\{M(Sv,Tw,t)*\{M(Sv,Pv,t).M(Qw,Tw,t)\}*M(Pv,Tw,t)\}} \phi(t)dt$$

$$\int_0^{M(Pv,Qw,kt)} \phi(t)dt \geq \int_0^{\min\{M(z,z,t)*\{M(z,z,t).M(Qw,z,t)\}*M(z,z,t)\}} \phi(t)dt$$

$$\int_0^{M(z,Qw,kt)} \phi(t)dt \geq \int_0^{\min\{1*\{1.M(Qw,z,t)\}*1\}} \phi(t)dt$$

$$\int_0^{M(z,Qw,kt)} \phi(t)dt \geq \int_0^{M(Qw,z,t)} \phi(t)dt$$

Similarly,

$$\int_0^{N(z,Qw,kt)} \phi(t)dt \leq \int_0^{N(Qw,z,t)} \phi(t)dt$$

Hence, from Lemma 2.1, we get Qw = z.
Combining all results we get Tw = Qw =z.
which shows that w is the coincidence point of the pair (Q ,T).
Now , since the pairs (P ,S) and (Q ,T) are semi compatible and hence we get
$PSx_n \rightarrow Sz$ So , Pz = Sz .
and , $QTy_n \rightarrow Tz$ so , Qz = Tz .
Now , we will prove that Pz = z
Again , using inequality (3), putting x = z , y = w, we get

$$\int_0^{M(Pz,Qw,kt)} \phi(t)dt \geq \int_0^{\min\{M(Sz,Tw,t)*\{M(Sz,Pz,t).M(Qw,Tw,t)\}*M(Pz,Tw,t)\}} \phi(t)dt$$

$$\int_0^{M(Pz,z,kt)} \phi(t)dt \geq \int_0^{\min\{M(Pz,z,t)*\{M(Pz,Pz,t).M(z,z,t)\}*M(Pz,z,t)\}} \phi(t)dt$$

$$\int_0^{M(Pz,z,kt)} \phi(t)dt \geq \int_0^{M(Pz,z,t)} \phi(t)dt$$

Similarly, we get

$$\int_0^{N(Pz,z,kt)} \phi(t)dt \leq \int_0^{N(Pz,z,t)} \phi(t)dt$$

and hence from Lemma 2.1 we get Pz = z.
Since Pz = Sz , there for we get
Pz = z = Sz .
Similarly, we can proved that
Qz = Tz = z.
from this we conclude that
Pz = Qz = Sz = Tz = z ,
which implies that z is a common fixed point of P , Q , S and T .

3. Uniquness

Let u be another common fixed point of P , Q , S and T . Then
z = Pz = Qz = Sz = Tz ,
u = Pu = Qu = Su = Tu .
Now , by using inequality (3) , Putting x = z and y = u, we get

$$\int_0^{M(Pz,Qu,kt)} \phi(t)dt \geq \int_0^{\min\{M(Sz,Tu,t)*\{M(Sz,Pz,t).M(Qu,Tu,t)\}*M(Pz,Tz,t)\}} \phi(t)dt$$

$$\int_0^{M(z,u,k)} \phi(t)dt \geq \int_0^{\min\{M(z,u,t)*\{M(z,z,t).M(u,u,t)\}*M(z,z,t)\}} \phi(t)dt$$

$$\int_0^{M(z,u,kt)} \phi(t)dt \geq \int_0^{\min\{M(z,u,t)*\{1.1\}*1\}} \phi(t)dt$$

$$\int_0^{M(z,u,kt)} \phi(t)dt \geq \int_0^{M(z,u,t)} \phi(t)dt$$

Similarly, we can proved that

$$\int_0^{N(z,u,kt)} \phi(t)dt \leq \int_0^{N(z,u,t)} \phi(t)dt$$

Hence from Lemma 2.1 we get z = u . This completes the proof.

Corollary 3.2 Let (X , M , N ,*,◊) be a complete intuitionistic fuzzy metric apace and let P,Q,S and T be self mappings of X satisfying the conditions of theorem 3.1and there exists k ∈ (0,1) such that for all x,y ∈ X and t > 0,

$$\int_0^{M(Px,Qy,kt)} \phi(t)dt \geq \int_0^{M(Sx,Ty,t)} \phi(t)dt$$

$$\int_0^{N(Px,Qy,kt)} \phi(t)dt \leq \int_0^{N(Sx,Ty,t)} \phi(t)dt$$

Then P,Q,S and T have a unque common fixed point in X .

Corollary 3.3 Let (X , M , N ,*,◊) be a complete intuitionistic fuzzy metric apace and let P,Q,S and T be self mappings of X satisfying the conditions of theorem 3.1and there exists k ∈ (0,1) such that for all x,y ∈ X and t > 0,

$$\int_0^{M(Px,Qy,kt)} \phi(t)dt \geq \int_0^{M(Sx,Ty,t)*M(Sx,Px,t)*M(Px,Ty,t)} \phi(t)dt$$

And

$$\int_0^{N(Px,Qy,kt)} \phi(t)dt \leq \int_0^{N(Sx,Ty,t)*N(Sx,Px,t)*N(Px,Ty,t)} \phi(t)dt$$

Then P,Q,S and T have a unque common fixed point in X .

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