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Nonlinear Spinor Field in Anisotropic Bianchi Type-III Space-Time

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Abstract: ***The role of nonlinear spinor and gravitational field in the evolution of the universe has been studied for the case of anisotropic Bianchi type-III space-time. Assuming that the expansion of the Bianchi-III space-time is proportional to the σ_1^1 component of the shear tensor. The system allows exact solutions only for some special choice of spinor field nonlinearity***.

Keywords: Spinor field, Bianchi type-III space-time,Gravitational Field, Tensor.

1. Introduction

The quantum field theory in curved space-time has been a matter of great interest in recent years because of its applications to cosmology and astrophysics. The evidence of existence of strong gravitational fields in our Universe led to the study of the quantum effects of material fields in external classical gravitational field.

The observe universe is almost Homogeneous and Isotropic at the large scale being usually described by Friedmen–Lemaitre-Robertson-Walker cosmology. However, the observation data revealed some anisotropic in the structure of universe like the anomalies found in a cosmic microwave background (CMB).

The discovery of the cosmic microwave radiation has stimulated a growing interest in anisotropic, general-relativistic cosmological models of the universe. According to Misner [1], the choice of anisotropic cosmological models in the system of Einstein field equation enable us to study the early day universe, which had an anisotropic phase that approaches an isotropic one.

The use of spinor fields in construction of effective theories to explain different phenomena is not new one. Assuming that the affine and the metric properties of the space-time are independent, in 1950 Weyl [2] has shown that the spinor field obeys either a linear equation in a space with torsion, or a nonlinear one in a Riemannian space. Soon after that nonlinear quantum Dirac fields were used by Heisenberg [3, 4] to construct an unified theory of elementary particles.

The problem of an initial singularity still remains at the center of modern day cosmology. Though the Big Bang theory is deeply rooted among the scientists dealing with the cosmology of the early Universe, it is natural to reconsider models of a universe free from initial singularities. Another problem that the modern day cosmology deals with is the accelerated mode of expansion. In order to answer to these questions a number of theories have been proposed by cosmologists. It has been shown that the introduction of a nonlinear spinor field or an interacting spinor and scalar fields depending on some special choice of nonlinearity can give rise to singularity free solutions in one hand [5, 6,7,8], on the other hand they may exploited to explain the late time acceleration [9,10]. Recently some work was done on Bianchi type-I, V, VI and VI_0 [8-13].

In this paper we have study the role of Nonlinear spinor field in the evolution of the universe for Bianchi type-III space-time to find the exact solution.

2. Basic Equations

We choose the Lagrangian density of spinor and gravitational fields in the form:

$$L=\frac{R}{2\chi}+\frac{i}{2}\left[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi-\nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi\right]-m_{sp}\bar{\psi}\psi+L_N \quad (2.1)$$

With R being the scalar curvature and χ being the Einstein's gravitational constant. The nonlinear term L_N described the self-interaction of spinor field and can be presented as some arbitrary function of invariants generated from the linear bilinear forms of spinor field having the form:

$$S=\bar{\psi}\psi \text{ (Scalar) (2.2a)} \quad P=i\bar{\psi}\gamma^5\psi \text{ (Pseudo scalar) (2.2b)} \quad v^{\mu}=\bar{\psi}\gamma^{\mu}\psi \text{ (Vector) (2.2c)}$$

$$A^{\mu}=\bar{\psi}\gamma^5\gamma^{\mu}\psi \text{ (Pseudovector) (2.2d)}$$

$$Q^{\mu\upsilon}=\bar{\psi}\sigma^{\mu\upsilon}\psi \text{ (Antisymmetric tensor) (2.2e)}$$

Where , $\sigma^{\mu\upsilon}=\frac{i}{2}\left[\gamma^{\mu}\gamma^{\upsilon}-\gamma^{\upsilon}\gamma^{\mu}\right]$.

Invariants, corresponding to the bilinear forms, are

$$I=S^2 \quad (2.3a)$$

$$J=P^2 \quad (2.3b)$$

$$I_{\upsilon}=\upsilon_{\mu}\upsilon^{\mu}=(\bar{\psi}\gamma^{\mu}\psi)g_{\mu\upsilon}(\bar{\psi}\gamma^{\mu}\psi) \quad (2.3c)$$

$$I_A=A_{\mu}A^{\mu}=\left(\bar{\psi}\gamma^5\gamma^{\mu}\psi\right)g_{\mu\upsilon}\left(\bar{\psi}\gamma^5\gamma^{\mu}\psi\right) \quad (2.3d)$$

$$I_Q = Q_{\mu\upsilon}Q^{\mu\upsilon} = \left(\overline{\psi}\sigma^{\mu\upsilon}\psi\right)g_{\mu\alpha}\left(\overline{\psi}\sigma^{\alpha\beta}\psi\right) \quad (2.3e)$$

According to the Pauli-Fierz theorem [16] among the five viz.(2.3a,b,c,d,e) invariants only I and J are independent as all other can be expressed by them: $I_\upsilon = -I_A = I + J$ and $I_Q = I - J$. therefore, we choose the nonlinear term $L_N = F(I,\ J)$, thus claiming that it describes the nonlinearity in the most general of its form.

We have choose the Bianchi type -III space-time:

$$ds^2 = dt^2 - a_1^2 e^{-2mz}dx^2 - a_2^2 dy^2 - a_3^2 dz^2 \quad (2.4)$$

with a_1, a_2, a_3 being the function of t only and m being arbitrary constant.

$$i\gamma^\mu \nabla_\mu \psi - m_{sp}\psi + F_S\psi + iF_P\gamma^5\psi = 0 \quad (2.5)$$

$$i\nabla_\mu \overline{\psi}\gamma^\mu + m_{sp}\overline{\psi} - F_S\overline{\psi} - iF_P\overline{\psi}\gamma^5 = 0 \quad (2.6)$$

In equation (2.5) and (2.6) ∇_μ is the covariant derivative of spinor field in which $F_s = \frac{dF}{dS}$, $F_P = \frac{dF}{dP}$.

and

$$\nabla_\mu \psi = \frac{\partial\psi}{\partial x^\mu} - \Gamma_\mu \psi \ , \ \nabla_\mu \overline{\psi} = \frac{\partial\overline{\psi}}{\partial x^\mu} + \overline{\psi}\Gamma_\mu \quad (2.7)$$

With Γ_μ being the spinor affine connections for the metric (2.4) has the form

$$\Gamma_0 = 0 \quad (2.8)$$

$$\Gamma_1 = \frac{1}{2}\left(\dot{a}_1 \overline{\gamma^1}\,\overline{\gamma^0} - m\frac{a_1}{a_3}\overline{\gamma^1}\,\overline{\gamma^3}\right)e^{-mz} \quad (2.9)$$

$$\Gamma_2 = \frac{1}{2}\dot{a}_2 \overline{\gamma^2}\,\overline{\gamma^0} \quad (2.10)$$

$$\Gamma_3 = \frac{1}{2}\dot{a}_3 \overline{\gamma^3}\,\overline{\gamma^0} \quad (2.11)$$

From equation (2.8), (2.9), (2.10) and (2.11) we obtain,

$$\Gamma_u \gamma^\mu = \frac{1}{2}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\overline{\gamma^0} - \frac{m}{2a_3}\overline{\gamma^3} \quad (2.12)$$

$$\gamma^\mu \Gamma\mu = -\frac{1}{2}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\overline{\gamma^0} + \frac{m}{2a_3}\overline{\gamma^3} \quad (2.13)$$

Here, we define

$$V = a_1 a_2 a_3 \quad (2.14)$$

And taking into the the spinor field is a function of t only, we obtain,

$$\overline{\gamma^0}\left(\dot{\psi} + \frac{1}{2}\frac{\dot{V}}{V}\psi\right) - \frac{m}{2a_3}\psi\overline{\gamma^3} + im_{sp}\psi - iF_s\psi + F_p\gamma^5\psi = 0 \quad (2.15)$$

(2.15)

$$\left(\dot{\overline{\psi}} + \frac{1}{2}\frac{\dot{V}}{V}\overline{\psi}\right)\overline{\gamma^0} - \frac{m}{2a_3}\overline{\psi}\,\overline{\gamma^3} - im_{sp}\overline{\psi} + iF_s\overline{\psi} - F_p\overline{\psi}\gamma^5 = 0 \quad (2.16)$$

From equation (2.15) and (2.16) yields,

$$\dot{S} + \frac{\dot{V}}{V}S - 2F_P A^0 = 0 \quad (2.17)$$

$$\dot{P} + \frac{\dot{V}}{V}P - 2m_{sp}A^0 + 2F_S A^0 = 0 \quad (2.18)$$

$$\dot{A}^0 + \frac{\dot{V}}{V}A^0 - mA^3 + 2m_{sp}P - 2F_s P + 2F_p S = 0 \quad (2.19)$$

Where,

$$A^0 = \overline{\psi}\gamma^5\overline{\gamma^0}\psi\ ,\ A^3 = \overline{\psi}\gamma^5\gamma^3\psi \quad (2.20)$$

Equation (2.17), (2.18) and (2.19) are three equations with four unknown. To find the solution of above system of equations, we use the condition

$$A^3 = \alpha A^0 \quad (2.21)$$

From equation (2.17), (2.18) and (2.19) we have,

$$V^2\left(S^2 + P^2 + A^{0^2}\right)e^{-2m\alpha t} = Const. \quad (2.22)$$

From equation (2.17) and (2.18) one find,

$$\frac{1}{2}\frac{\partial}{\partial t}\left(S^2+P^2\right) + \frac{\dot{V}}{V}\left(S^2+P^2\right) - 2\left(F_p S - F_s P\right)A^0 = 0 \quad (2.23)$$

(2.23)

By assuming $F = F\left(S^2 + P^2\right)$ we have,

$$F_p S - F_s P = 0, \quad (2.24)$$

which leads to,

$$V^2\left(S^2 + P^2\right) = C_0^2 \quad (2.25)$$

Now, we write the component of energy momentum tensor for the spinor field as,

$$T_0^0 = m_{sp}S - F \quad (2.26)$$

$$T_1^1 = T_2^2 = T_3^3 = SF_s + PF_p - F \quad (2.27)$$

The Einstein's field equation corresponding to the metric (2.4) and the Lagrangian is of the form:

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} = k(SF_S + PF_P - F) \quad (2.28)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} - \frac{m^2}{a_3^2} = k(SF_S + PF_P - F) \quad (2.29)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1 a_2} = k(SF_S + PF_P - F) \quad (2.30)$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} - \frac{m^2}{a_3^2} = \mathrm{k(m_{sp}S - F)} \quad (2.31)$$

$$m\frac{\dot{a}_3}{a_3} - m\frac{\dot{a}_1}{a_1} = 0 \quad (2.32)$$

From equation (2.32) we get,

$$a_1 = N a_3 \quad (2.33)$$

Where, N being Integration Constant.

Adding equations (2.28), (2.29), (2.30) and 3 times (2.31) yields,

$$\frac{2\ddot{V}}{V}=\frac{4m^2}{a_3^2}+3k[m_{sp}\,S+(SF_S+PF_P-2F)] \quad (2.34)$$

The right hand side of equation (2.34) explicitly depends on a_3.

We need some additional conditions to overcome it. The eminent authors B. Saha et al. (2011) assumes the expansion θ is proportional to any of the components (say σ_1^1) of the shear tensor σ.

We choose a co-moving frame of reference so that

$$\theta=\Gamma^{\mu}_{\mu\upsilon}=\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}=\frac{\dot{V}}{V} \quad (2.35)$$

$$\text{And } \sigma_1^1=\frac{\dot{a}_1}{a_1}-\frac{1}{3}\theta \quad (2.36)$$

The proportionality condition

$$\sigma_1^1=q_1\theta\ ,\ q_1=\text{Con} \quad (2.37)$$

Which leads to,

$$a_1=(a_2a_3)^{\frac{1+3q_1}{2-3q_1}} \quad (2.38)$$

From equation (2.34) , equation (2.14) gives

$$a_1=V^{(\frac{1+3q_1}{3})} \quad (2.39)$$

$$a_2=NV^{(\frac{1-6q_1}{3})} \quad (2.40)$$

$$a_3=\frac{1}{N}V^{(\frac{1+3q_1}{3})} \quad (2.41)$$

The equation (2.20), can be written as,

$$2\ddot{V}=4m^2N^2V^{(\frac{1-6q_1}{3})}+3k[m_{sp}S+k(SF_S+PF_P-2F)]V \quad (2.42)$$

At last, we assume that the spinor field be a massless one and the spinor field nonlinearity is given by $F=F(K)$ with $K=S^2+P^2$.

In this case $F_S=2SF_K$ and $F_P=2PF_K$,

hence $SF_S+PF_P=2(S^2+P^2)F_K=2KF_K$.

The equation (2.42) yields,

$$2\ddot{V}=4m^2N^2V^{(\frac{1-6q_1}{3})}+6k(KF_K-F)V \quad (2.43)$$

For the nonlinearity of the spinor field, we choose the concrete form $F=K^n$. Taking into account that $K=S^2+P^2=\frac{C_0^2}{V^2}$ using this we rewrite (2.43) as,

$$2\ddot{V}=4m^2N^2V^{(\frac{1-6q_1}{3})}+6k(n-1)C_0^{2n}V^{(1-2n)} \quad (2.44)$$

With the solution in quadrature

$$\int\frac{dV}{\sqrt{\frac{6m^2N^2}{(2-3q_1)}V^{(\frac{4-6q_1}{3})}-3kC_0^{2n}V^{2(1-n)}+C_1}}=t+t_0 \quad (2.45)$$

Where C_1 and t_0 being integration constant. Also for given model the Heisenberg-Ivanenko type non-linearity with $n=1$ has no influence on the evolution of the universe

3. Some Properties of the Model

The Non vanishing component of shear tensor $\sigma_{\mu\vartheta}$ are given by,

$$\sigma_{\mu\vartheta}=\frac{1}{2}[u_{\mu:\alpha}P^{\alpha}_{\vartheta}+u_{\vartheta:\alpha}P^{\alpha}_{\vartheta}]-\frac{1}{3}\theta P_{\mu\vartheta} \quad (2.46a)$$

$$\sigma_{11}=[-a_1\dot{a}_1+\frac{a_1^2}{3}(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3})]e^{-2mz} \quad (2.46b)$$

$$\sigma_{22}=[-a_2\dot{a}_2+\frac{a_2^2}{3}(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3})] \quad (2.46c)$$

$$\sigma_{33}=[-a_3\dot{a}_3+\frac{a_3^2}{3}(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3})] \quad (2.46d)$$

$$\sigma_{44}=0 \quad (2.47e)$$

Therefore,

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij}=\frac{1}{2}(\sigma_1^1+\sigma_2^2+\sigma_3^3+\sigma_4^4)=0 \quad (2.48)$$

The model, in general represent shearing universe.

4. Some Special Cases

Case I:

If $m=0$ and $C_1=0$, Equation (2.45) gives,

$$\int\frac{dV}{i\sqrt{3kC_0^{2n}V^{2(1-n)}}}=t+t_0 \quad (2.49)$$

Which represent the imaginary model of the universe.

Case II

If $n=1$ and $C_1=0$, Equation (2.45) gives,

$$\int\frac{dV}{\sqrt{\frac{6N^2m^2}{(2-3q_1)}V^{(\frac{4-6q_1}{3})}-3kC_0^2}}=t+t_0 \quad (2.50)$$

This gives exact solution of the model

5. Conclusion

With the scope of Bianchi type III cosmological model the role of non-linear spinor field on the evoluation of the is studied. It is shown that the model allows exact solution to the NLSF equation for the non-linear term being arbitrary function of the invariant $I = S^2$ and $J = P^2$, where $S = \bar{\psi}\psi$, $P = i\bar{\psi}\gamma^5\psi$ are the real bilinear forms of the spinor field.

Assuming that the expansion of the Bianchi type-III space-time is proportional to the σ_1^1 component of the shear tensor, solution for the metric function $a_i(t)$ are obtained explicitly in terms of volume scale V .

Since $\lim_{t\to 0}(\frac{\sigma}{\theta})^2 = 0$, which means that initially anisotropic space-time becomes isotropic in the process of expansion. Choosing the integration constant C_1 and initial value of V ,it is possible to find the solution (2.45) which is regular everywhere.

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