International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Checkmate or Not-: Validation and Application of Zermelo's Theorem in Chess

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Abstract: ***In this paper author, has tried to prove Zermelo's theorem by using mathematical induction and by using Zermelo's theorem author has proved that "Chess has a solution". This would be done by analysing few games played between two opponents (Viswanathan Anand from India and Garry Kasprov from Russia) and then analysing the defects made by each of the player per move and would try to prove that a person inducing more defects losses the game and if both the person induces equal defects or no defects then the game eventually ends in a draw. In first 3 matches, first match was won by a player playing with white pieces (Anand), one match was won by a player playing with black pieces (Garry) and one match was drawn between both the opponents, By the use of fraction defectives [1] author have shown that the first two matches (match won by white and match won by black) were won by the player playing less defective moves compared to his opponent while one match which was drawn between two players have moves exhibiting more or less equal defects or no defects(s) and the fourth match is a dummy match which will serve as a proving of the 3rd match between Anand and Garry. Through this it is concluded that all player loses a game if they themselves induces defects in games otherwise the match would end in a draw. Moreover, author has also rectified the defects and provided correct moves in place of defective moves thereby proving that match would end in a draw.***

Keywords: Chess, dummy, fraction defectives, Zermelo's theorem.

1. Introduction

In 1913 Zermelo's paper named as "Uber eine Anwendung der Mengenlehre auf die Theorie des Schachpiels" is regarded as the first ever work done to analyse mathematically the strategies in games. Which gave birth to a theorem called "Zermelo's theorem" which states that-: assuming in a game where two opponents play against each other, the game is a game of perfect information **[2]**, the game has finite number of moves and the game had 3 possible outcomes i.e. win for player 1 or loss for player two, win for player two or loss for player 2 and tie then either player one or player two has a winning strategy or both can force a draw. But the limitation of this theorem is it does not tell us that what would be the outcome of the game i.e. either the game could be won by player 1 or by player 2 or it could be a tie, moreover this theorem does not tell us what that solution is?

[1] The proportion or fraction nonconforming (defective) in a population is defined as the ratio of the number of nonconforming items in the population to the total number of items in that population.

[2] Games of perfect information are the games in which both the players know what had happened prior in the game (sequential games) for e.g. chess, checkers, tic-tac-toe etc.

Therefore, any game which satisfies the above assumptions of Zermelo's theorem can lead to three possible outcomes and if the third outcome is the solution (i.e. a tie) of the game played between two opponents then such games are called as games of perfect competition and such games do have a solution only thing we need to find out is the solution.

Let's take an example of tic-tac-toe, it's a two-player game, it's a game of perfect information, has 5 moves for player 1 and 4 moves for player 2 i.e. game having finite number of moves and has three possible outcomes and the conclusion is that tic-tac-toe can lead to one of the three outcomes-:

1. Either player 1 will win.
2. Either player 2 will win.
3. Or a tie i.e. it will lead to a draw if both players does not commit mistake(s)

Let's become more ambitious and look into chess, chess is a two-player game and both players know what happened prior in the game i.e. it's a game of perfect information, it has a finite number of moves although it is satisfies the assumptions of Zermelo's theorem therefore according to Zermelo's theorem chess has three possible outcome-:

1. Either white can force a win on black.
2. Either black can force a win on white.
3. Or a tie.

But if someone says that chess has only one possible outcome i.e. a tie it means chess is a game of perfect competition and chess has a solution. But now the question arises if you look into the stats of any chess player he/she has also lost few games in his life. Even the greatest chess player in the history of chess i.e. Garry Kasprov of Russia has also witnessed some defeats in his chess career then how could one say that chess has a solution or a game of chess should always end in a draw. Answer to the above question is that in chess both the opponent does not commit mistakes or commit less mistakes than their opponent then the game will end in a draw. It means that a quality full game of chess will lead to a draw. It means by implementing Total Quality Management (TQM) in chess we can prove "chess has a solution".

2. Literature Review

Chess is defined as a board game of mental skill for two players, played on a chessboard, a checkered game board with 64 squares arranged in an eight-by-eight grid(generally green and white) on which each playing piece is moved according to precise rules. The object is to put the opponent's king under a direct attack from which escape is not possible (checkmate).

Let us assume two persons' A and B are playing chess. Now suppose that if there is no perfect strategy for a player A and let him always win/draw. This implies that no matter what A does he is going to win, it means there is a strategy B can follow to win/draw and that is don't allow A what he is trying to do but the only problem is to identify what A is trying to do. Wait a minute - this means there is a perfect strategy for B!

This tells us that at least one of the two players do have a perfect strategy which lets that player always win or draw.

1) White can always win if he plays perfectly.
2) Black can always win if he plays perfectly.
3) and if both players play perfectly then they always draw.

It means there must be a perfect algorithm for chess, at least for one of the two players. This also means till date all the chess games that shares a result of the win of any player is because of mistake(s) committed by the opponent of that particular player.

Regan (2012) helps to find that how frequently a human can find out the best possible move out of the various available moves choices in chess which would help in skill assessment of a chess player, also this work helps in detecting fraud and in accessing trader's aptitude.

Duclos & Voirin (2010) provides information about how to formulate and interpret a P-chart, thereby utilizing and reporting a study on P-chart, the outcome shows that reduction of adverse events is not possible by measurement but by trying out quality improvement initiatives or Total Quality Management.

Steven et al. (2009) states by analysing the play of world class chess players they concluded that even world class players are unable to induct properly when it comes down to centipede games while they are able to induct backward in the race to 100 concluding that centipede game is not useful to test Backward Induction. This is because of the fact that there are a lot of variations present in chess to reach to a particular position for e.g. Hippopotamus Defense but when it comes to sequential openings then almost all world class players are aware of opening lines or ECO (Encyclopedia of chess openings) but are very less equipped with ECE (Encyclopedia of chess endings) as most of the games at high level ends by resignation.But this paper does not tell about when a player is not able to induct backward also whether or not there is really a need to induct backward w.r.t the game.

Larson (2008) depicts that in chess, either white or black has a winning strategy, or both can force a draw.But this paper uses mathematical proving to prove the above statement without the use of diagrams like most of the research work done on chess which makes understanding a bit difficult for chess players in absence of diagrams.

Pierre et al. (2007) recommends a new rating system called as Bayesian Skill Rating System in place of Statistical rating system (for e.g. ELO rating system) because they do not take into account the skill development of a player over time.

This paper shows-:

a) Overall playing strength increases over past 150 years.
b) Players ability to force a draw proves player has better predictive power.

Hurd (2005) provides a method to construct formally verified databases to store end games, using HOL 4 theorem and buddy BDD engine. This database helps you to play perfect chess during pawn less endgames it also helps you to access the games you have played as this database stores the number of moves required to force checkmate. But they didn't take consideration of all end games but only pawn less end games which are only 29.74% of total end games and to formulate the database, higher order logic with a four tuple was used which is understandable by only computer experts or students of Computer Science whereas chess players are familiar with algebraic chess notation or diagrammatic representation of chess pieces.

Strejczek (2004) explains how to create a chess playing programme of (ELO 2100) and to provide an overview about modern chess programming including methods, algorithms and techniques also testing programme on opening book.

Bart & Atherton (2003) shows that which part of brain is involved and gets activated while playing chess differentiating on the basis of class of chess player Amateur and Expert Chess Player this helps in understanding the neuroscientific changes that accompany the development of talent (both behavioural and neuro cognitive).

Schwalbe and Walker (2001) attempts to derive a proof of Zermelo's theorem and throw some light on related work of Konig and Kalmar.

Glickman and jones (1998) attempts to depict some failures in the rating system used by the U.S. Chess Federation (USCF) called as the "Elo System" developed by Aprad Elo (1903-1992). They also criticized the attenuation factor K used in the formula for updating Elo Rating.

$r_{post} = r_{pre} + K (S - S_{exp})$, where r_{post} is player's updated post tournament rating, r_{pre} is player's pre-tournament rating, S is player's total score in the tournament, S_{exp} is the expected total score estimated from the player's pre-tournament ratings, and K is attenuation factor that determines the weight that should be given to the player's performance relative to his or her pre-tournament rating and gave their own ideology for improving rating system based on time

variation, Isolated rating pools and variation in attenuation factor.

Chase & Simon (1973) develops a technique for studying what and how a chess player perceives while playing chess using two techniques.

Perception task by viewing a chess position in plain view
Recalling a chess position after viewing it for 5 seconds.

Implementing it on 3 chess players of varying strengths
1) National Master or NM having a FIDE rating of 2000-2100
2) International Master or IM having a FIDE rating of 2100-2499
3) Grand Master or GM having a FIDE rating of 2500 and above.

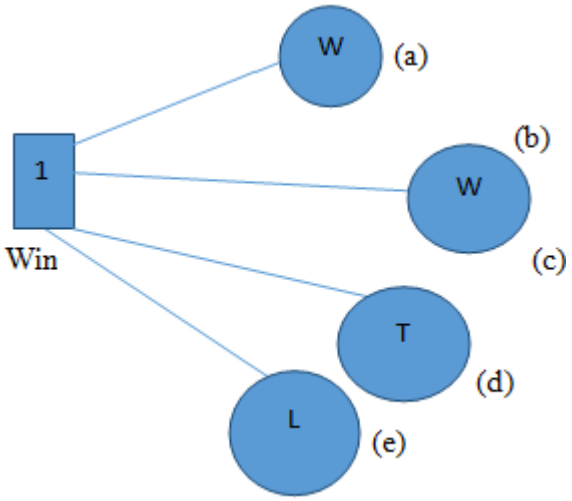
All the above past research done on Zermelo's theorem and chess.

3. Research Methodology

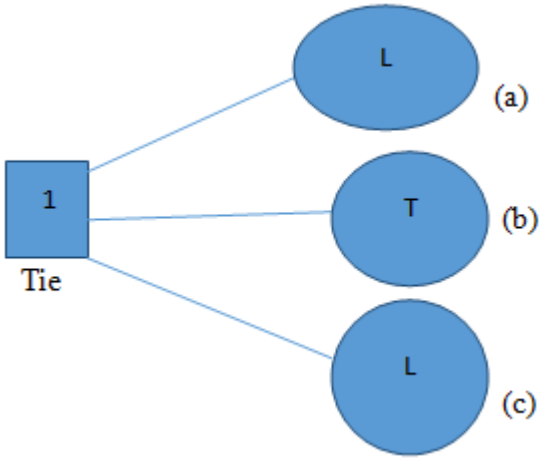
This paper is practical in nature. In this paper an attempt has been made to find out the validity of Zermelo's theorem and also to find out the relationship between the win or loss of a particular player and the effort defectives of the two players while playing chess.

Proof of Zermelo's theorem by using mathematical induction-:
Proof (By induction) on maximum game of length N

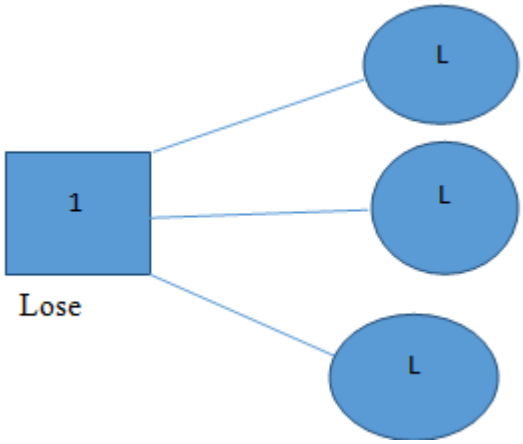
1. If N=1



Where 1= player 1 playing with white pieces.
W= a strategy where player 1 could force a win i.e. checkmate by white.
L=a strategy where player 2 could force a win i.e. checkmate by black.
T= a strategy where 1 could force a tie i.e. draw.
Where it is obvious that player 1 will either choose strategy (a) or strategy (c).



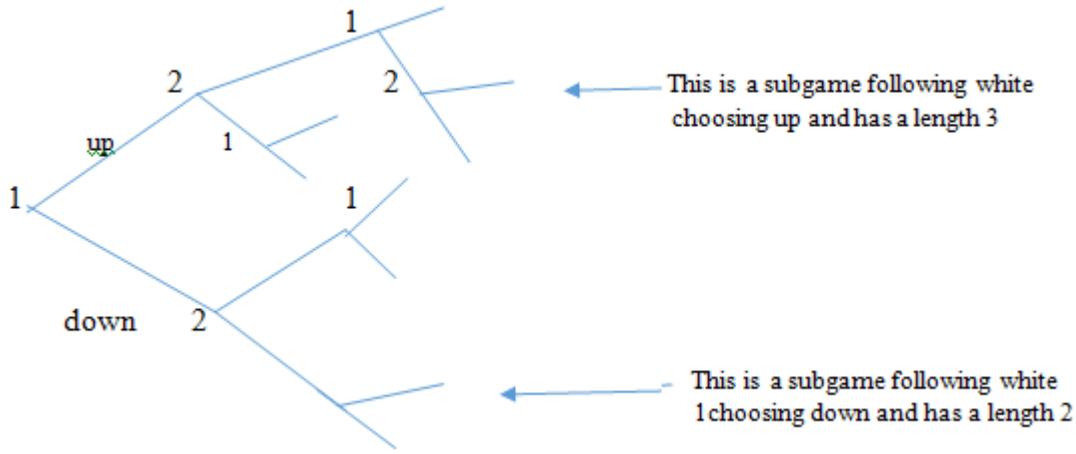
Where 1= player 1 playing with white pieces.
L=a strategy where player 2 could force a win i.e. checkmate by black.
T= a strategy where 1 could force a tie i.e. draw.
Where it is obvious that player 1 will choose strategy (b).



Where 1= player 1 playing with white pieces.
L=a strategy where player 2 could force a win i.e. checkmate by black.
Where it is obvious that no matter which strategy player 1 uses he/she is going to lose the game.
This shows that there is all possible outcome of a chess game of length equals to 1.

2. Induction Hypothesis-: Suppose the claim is true for all games of length equals and less than N

3. Therefore we claim that it will be true for all games of length N+1

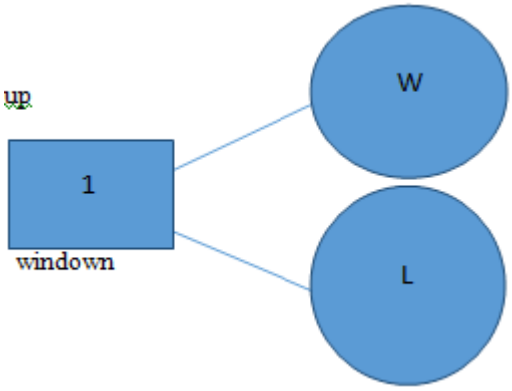


In this example, N=3 i.e. N+1=4

In the above given example let's take that if player 1 follows a strategy named as up or down consist of a game of length 3 or 2 and already holds true due to our assumption of induction hypothesis moreover these are themselves a sub game of a game of the initial game.

So, by induction hypothesis we can say choosing strategy up or down by player 1 gives you a game of solution let say win (up) and lose (down).

Therefore, the above game can be transformed as



But this game has a solution and is a game of length 1.
Hence proved.

Mathematical Induction

It is a technique used in mathematics to prove any valid statement using pre-defined steps. Basically, it is a three-step rule.
Step 1-: Firstly, you prove that the given statement is valid for unity or one.
Step 2-: Then we assume that the statement is true for the value N.
Step 3-: Then with the use of the above statement we try to prove that it is also valid for the value N+1.
Thus, the statement becomes valid for all values of all set of natural numbers.

Zermelo's Theorem

If a game is a two-person game, a game of finite moves and a game of perfect information then the game has three possible outcomes-:

1. Player 1 will win the game.
2. Player 2 will win the game.
3. It will be a draw.

When Zermelo's theorem is applied to chess, Zermelo's Theorem states "either white can force a win, or black can force a win, or both sides can force at least a draw". It means that one of the two opponents have a way of winning or losing the game. Which states that chess holds a Solution.

4. Tools and Techniques

In this paper author, has analyzed 4 games using a chess software named "Rybka" used to convert moves of player in quantitative forms and comparing them with the move played before. Using the quantitative data given by Rybka other techniques used are Fraction defective i.e. Standard deviation and Formula of P-chart.

5. Results and Conclusion

In this paper author, has analyzed 4 games out of which 3 matches are played between two similar opponents at different location, in different years and in different tournaments to randomize the samples and found that-:

Game 1 which was played between GM Viswanathan Anand (playing with white) and GM Gary Kasprov (playing with black) at blitz chess final in the year 1996 in this match Anand induced total fraction defectives of 6.27 and Gary induced total fraction defectives of 10.01 which was higher than Anand therefore Anand (playing with white) won the match.

Game 2 was played between GM Viswanathan Anand (playing with white) and GM Gary Kasprov (playing with black) at PCA tournament in the year 1996 in this match Anand induced total fraction defectives of 45.66 and Gary

induced total fraction defectives of 26.35 which was lesser than Anand therefore Gary (playing with black) won the match.

Game 3 was played between GM Gary Kasprov (playing with white) and GM Viswanathan Anand (playing with black) at classic tournament in the year 1995 in this match Gary induced total fraction defectives of 2.65 and Anand induced total fraction defectives of 2.37 which was more or less equal having a difference of 0.28 hence match was a draw.

Game 4 acts as a proof for game 3 where the author has taken the moves from ECO (Encyclopedia of chess openings) and attempted to draw the game using three fold repetition rule and analyzed the total fraction defectives and the difference between the defects of the two side which came out to be 1.62 for white and 1.42 for black having a difference of 0.2 which shows that the difference of defects in game 3 was insignificant as game 4 was also a draw.

6. Conclusion

In a game of chess, a player inducing lesser defects is a winner and if both the player induces more of less equal defects then the game eventually ends in a draw.

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Note: The Proof for the paper and research work is given in the appendix below but not to be published.