Comparison of Power between Adaptive Tests and Other Tests in the Field of Two Sample Scale Problem

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Abstract: ***Usually when we have to deal with an analysis of a particular data set, we often choose a statistical procedure on the reference of the analysis. Now if in this analysis the parametric assumptions are fulfilled we will prefer F test for the two sample scale problem. But if the behaviour of the data set does not fulfil these assumptions then we have to focus on some other tests. In this paper it is tried to throw light on the use of Adaptive tests on the context of this problem. This paper comprises of power comparison between F test, some traditional Non- Parametric tests and proposed Adaptive tests. Under the experiment equal and unequal sample sizes are taken considering different alternatives. This is done by the use of Monte Carlo simulation technique. At the end of the experiment, inspite of many limitations, some satisfactory results are obtained. All those results are displayed in tabular and graphical form.***

Keywords: F test, Mood Test, Sum of Squared Rank Test, Klotz test, Seigel Tukey test, Ansari Bredley Test, Adaptive test, Monte Carlo Simulation, size and power

1. Introduction

To test the hypothesis of equality of two scale parameters, there is always a problem that which test is to use. According to literature if the parent distribution is normal, then F test is preferred. But if the normality assumptions are violated specially when N is large then F test become extremely sensitive. In that situation a researcher has to focus on Non Parametric tests. Various Non Parametric tests are available for using as an alternative of F test such as mood test, Klotz test, Seigel Tukey test, Sum of Squared Rank Test, Ansary Bredly test and so on. But still there is a problem to decide the accurate test for the particular set of data so that to get a valid result and draw a valid inference.. In this paper it is tried to get the solution by using the adaptive procedure. In this procedure at the very beginning we will collect some reasonable information from the data and then we will decide which test will be better for the data for further approach. In this paper power comparisons are made between different parametric and non parametric tests and then it is decided which test can be considered as our Adaptive test.

2. Parametric Test Procedures

2.1 F test

Let $X_1, ..., X_n$ and $Y_1, ..., Y_m$ be independent and identically distributed samples from two populations which each have a normal distribution. The expected values for the two populations can be different, and the hypothesis to be tested is that the variances are equal. Let $\bar{X}=\frac{1}{n}\sum_{i=1}^{n} X_i$ and

$\bar{Y} = \frac{1}{m}\sum_{j=1}^{m} Y_j \bar{Y} = \frac{1}{m}\sum_{j=1}^{m} Y_j$ be the sample means. Let

$S_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ and $S_y^2 = \frac{1}{m-1}\sum_{j=1}^{m}(Y_j - \bar{Y})^2$ be

the sample variances. Then the test statistic

$$F=\frac{S_X^2 S_X^2}{S_y^2 S_y^2}$$

has an F-distribution with $n-1$ and $m-1$ degrees of freedom if the null hypothesis of equality of variances is true. Otherwise it has a non-central F-distribution. The null hypothesis is rejected if F is either too large or too small.

3. Non- Parametric Test Procedures for Two Sample Scale Problem

3.1 Mood test

Mood test is completely analogous to the parametric F-test mentioned above. Let $X_1, X_2, ..., X_n$ be an independent random sample from a distribution F and $Y_1, Y_2, ..., Y_m$ be an independent random sample from a distribution G. Now we combine the two sample and rank them in ascending order. So the array of ordered scores will be $v^1 < v^2 < ... < v^N$. Here our hypothesis (H0) is to test that the two distributions, F and G, are identical with respect to scale, against some ordered alternative.

We Replace the scores in the pooled sample by their corresponding ranks. Knowing that the mean of a set of ranks from 1 to N is (N+1)/2,we determine the sum of squared rank deviations about this mean for the X sample.

Let Zi be the indicator variable. Z_i= 1 if the rank score i belongs to X Sample and 0 otherwise.

The test statistic for N<20 is

$$M = \sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right)^2 Z_i$$

A large value of M implies that the variability of the X sample is significantly greater than the variability of the Y sample. For small values of M, one draws the opposite conclusion. A table of critical values for N<20 is not available at the present time.

When N is greater than 20, values of M approximate a normal distribution with E(M) = $n(N^2-1)/12$ and Var(M)= $nm(N+1)(N^2-4)/180$..In this case the test statistic is

$$Z = \frac{M-E(M)}{SE(M)}$$

3.2 Klotz Test

Let $X_1, X_2, \ldots, X_n$ be an independent random sample from a distribution F and $Y_1, Y_2, \ldots, Y_m$ be an independent random sample from a distribution G. Now we combine the two sample and rank them in ascending order. So the array of ordered scores will be $v^1<v^2<\ldots<v^N$. Here our hypothesis (H0) is to test that the two distributions, F and G, are identical with respect to scale, against some ordered alternative.

We Replace the scores in the pooled sample by their corresponding ranks. Knowing that the mean of a set of ranks from 1 to N is (N+1)/2,we determine the sum of squared rank deviations about this mean for the X sample.

Let Zi be the indicator variable. Z_k= 1 if the rank score k belongs to X Sample and 0 otherwise.

The test statistic is:

$$\text{Klotz}=\sum_{k=1}^{N}\{\emptyset^{-1}(\frac{k}{N+1})^2\}Z_k$$

When N is large then the values of Klotz statistic approximate a normal distribution with E(Kl) = (1/N) $\sum_{k=1}^{N}(Kl_k)$ = mean and Var(Kl)= $\frac{n*m}{N(N-1)}\sum_{k=1}^{N}(Kl_k - mean)^2$..In this case the test statistic is

$$Z = \frac{Kl-mean}{SE(Kl)}$$

3.3Ansari Bradley Test

To compute the Ansari- Bradley statistic special scores are attributed to the observations:
Let $Z=(X_1,X_2,\ldots X_m,Y_1,Y_2,\ldots,Y_n)$ denote the combined sample.

If N= m + n is even then

Sample Z: $Z_{(1)}$, $Z_{(2)}$,... $Z_{(N/2-1)}$, $Z_{(N/2)}$, $Z_{(N/2+1)}$, $Z_{(N/2+2)}$, ..., $Z_{(N-1)}$, $Z_{(N)}$
Scores A_i: 1 2 ... N/2-1 N/2 N/2 N/2-1 ... 2 1

If N= m + n is odd then
Sample Z: $Z_{(1)}$, $Z_{(2)}$, ... $Z_{((N-1)/2)}$, $Z_{((N+1)/2)}$, $Z_{((N+3)/2)}$, ... $Z_{(N-1)}$, $Z_{(N)}$
Scores A_i: 1 2 ... (N-1)/2 (N+1)/2 (N-1)/2 ... 2 1

The Test Statistics is:

$$W=\sum_{i=1}^{n} A_{2i}$$

For large sample the test statistics is :

$$Z=\frac{W-E(W)}{\sqrt{Var(W)}}$$

$$\text{Where } E(W) = \frac{m(m+n+2)}{4} \quad \text{if } m+n \text{ is even.}$$
$$= \frac{m(m+n+1)^2}{4(m+n)} \quad \text{if } m+n \text{ is odd}$$
$$Var(W) = \frac{mn(m+n+2)(m+n-2)}{48(m+n-1)} \quad \text{if } m+n \text{ is even}$$
$$= \frac{mn(m+n+1)(3+(m+n)^2)}{48(m+n)^2} \quad \text{if } m+n \text{ is odd}$$

3.4 Siegel-Tukey Test

This test replaces the pooled data from the two samples with are ordering of the ranks (i) from 1 to N.to illustrate the ranking procedure, note the following table when N is assumed to be an even number.

Ordered score	v^1	v^2	v^3	v^4...	$v^{N/2}$...	v^{N-3}	v^{N-2}	v^{N-1}	v^N
Rank Replacement	1	4	5	8	N......	7	6	3	2

The indicator random variable is Z_i=1, if ith replacement score is associated with the X sample.
=0, otherwise

The test statistics is-

$$ST=\sum_{i=1}^{N} iZ_i$$

In order to determine the significance of ST tables have been developed by Siegel Tukey(1960).

For N>20 the distribution of ST approximates a normal distribution withE(ST)= n (N+1)/2 and Var (ST) = nm (N+1)/12.The test statistic becomes

$$Z=\frac{ST-E(ST)}{\sqrt{Var(ST)}}$$

Sum of Squared Rank Test: let X_{ij} be the random observations, i= 1,2,...,g and j= 1,2,...,n_i. μ_i is the mean and σ_i are the standard deviation. Now the required test statistics is

$$L=\frac{1}{D^2}[\sum_{i=1}^{g}\frac{S_i^2}{n_i} - N\bar{S}^2]$$

Where $Z_{ij} = |y_{ij} - \bar{y}_i|$

$$S_i = \sum_{j=1}^{n_i} R_{ij}^2$$

$$\bar{S} = \frac{1}{N}\sum_{i=1}^{g} S_i \text{ and } D^2 = \frac{1}{N-1}[\sum_{i=1}^{g}\sum_{j=1}^{n_i} R_{ij}^4 - N\bar{S}^2]$$

For large sample size L follows chi square distribution with (g-1) d.f.

4. Aadaptive Procedures

The objective of this paper is to develop an adaptive test for scale for testing independent distributions of the continuous type. It is already mentioned that an adaptive test uses the data first to select a model and then makes an inference based on the model chosen. The models chosen ranges from a light-tailed one (uniform) to a heavy-tailed one (like the Cauchy).

There are many nonparametric tests to deal with comparison tests in two sample cases for scale problems. For example in case of skewed distributions we use the Squared-Ranks test statistics . on the other hand when the distributions are medium-tailed or heavy-tailed such as the double exponential or Cauchy distributions, respectively, then suitable tests would be the Mood and Ansari-Bradley tests, respectively. But if the parent distribution is normal we use F test.

Now let us assume we have two samples from the c.d.f.'s F_x (x) and F_y (x), respectively, in order to test the null hypothesis H_0: $\sigma = 1$. Then we combine the two samples and order the observations in increasing order of magnitude. To measure the skewness we use the statistics

$$Q_1=\frac{\bar{U}_{.05}-\bar{M}_{.5}}{\bar{M}_{.5}-\bar{L}_{.05}}$$

where $\bar{U}_{.05}$,$\bar{M}_{.5}$ and $\bar{L}_{.05}$ the averages of the largest five percent of the Z's, the middle 50 percent of the Z's, and the smallest five percent of the Z's where the Z's denote the order statistics of the combined sample of all N values.

The measure of kurtosis is-

$$Q_2=\frac{\bar{U}_{.05}-\bar{L}_{.05}}{\bar{U}_{.5}-\bar{L}_{.5}}$$

This is to determine whether the tail weight of the underlying distribution is light or heavy. For different distributions, Q_1 and Q_2 have been evaluated in Table 2.1.

This adaptive test will depend on what values of Q_1 and Q_2 are found. On the scheme we propose, we are assuming that the distributions we encounter are not skewed to the left. With this in mind we propose a classification procedure. First, Q_1 and Q_2 are computed using the combined sample of all N values. If $Q_1 < 1.5$ and $Q_2 < 7$, then we will use a medium-tailed model. If $Q_1 > 1.5$ and $Q_2 < 2$ we will use a right-skewed model. If $Q_1 > 1.5$ and $2 < Q_2 < 7$ then we will use a very badly right-skewed model. If $Q_2 > 7$, then we will use a heavier-tailed model.

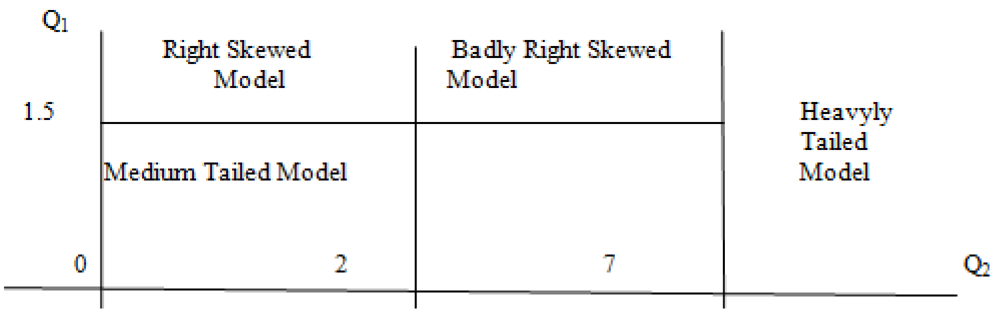

Figure 1.1: Diagram of above division

The adaptive test A will be defined as follows..If the data indicate a skewed model, we will use the Squared-Ranks statistic denoted by L_N .If the data indicate a medium-tailed model, then we can one of the following tests such as Mood statistic or Klotz Statistics. When the data suggest a very heavy-tailed model, then we propose the Ansari-Bradley statistic, A_N .

Table 1.1: Values of Q_1 and Q_2 for some selected Distributions

Distributions	Q_1	Q_2
Uniform	1	1.9
Normal	1	2.563
Exponential	4.5	2.86
Double Exponential	1	3.3
Logistic	1	2.86

5. The Monte Carlo Study

For the simulation study, F- test, Mood test, Ansary Bradley test, Sum of Squared Rank test, Klotz test, Seigel Tukey test and adaptive tests are selected. Here six family of distributions are considered and these are – the Normal, the Cauchy, the Double exponential, the Logistic , the Lognormal .

The study was conducted on computer at the Department of Statistics, Dibrugarh University. To generate the standard normal deviate, the method described in Monte Carlo Method by Hammersly and Handscomb(1964) were used and deviate from the other distributions were generated by using the inverse distribution function on uniform deviates.

In studying the significant levels, we first considered distributions with scale parameter equal to one. For each set of sample $N = \sum_i n_i$, i =1,2 the experiment was repeated 5,000 times and proportion of rejection of the true null hypothesis was recorded and presented in table 1.2 to 2.11.

For the power study of the tests, random deviates were generated as above for each group and added to σ_i . Proportion of rejections based on 5000 replications at the levels .10, .05 and .01 for different combinations of σ_i were recorded and presented in the table 1.2 to table 1.11.

Table 1.2: Empirical Level and Power of Tests under Normal distribution

Sample sizes	*S.D.*	*F test*			*Mood test*			*Klotz Test*		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.0972	.0462	.0068	.1040	.0458	.0212	.1094	.0540	.0054
	(1.2,1)	.2198	.1234	.0292	.2030	.1150	.0546	.2116	.1152	.0176
	(1.2,1)	.3688	.2372	.0716	.3168	.2022	.1154	.3354	.2114	.3084
	(1.8,1)	.6552	.5044	.2306	.5398	.3858	.2506	.5746	.4120	.1098
	(2,1)	.7602	.6286	.3344	.6268	.4710	.3268	.6660	.4990	.1610
	(2.6,1)	.9290	.8600	.6302	.7988	.6714	.5316	.8368	.6950	.3146
(10,15)	(1,1)	.1190	.0588	.0122	.1084	.0548	.0262	.1044	.0498	.0062
	(1.2,1)	.2762	.1720	.0504	.2312	.1386	.0804	.2118	.1102	.0162
	(1.2,1)	.4606	.3232	.1304	.3682	.2448	.1514	.3660	.2112	.0310
	(1.8,1)	.7590	.6360	.3736	.6154	.4666	.3432	.6310	.4502	.0996
	(2,1)	.8408	.7520	.5034	.7142	.5676	.4374	.7334	.5596	.1478
	(2.6,1)	.9618	.9236	.8020	.8718	.7770	.6590	.8944	.7874	.3154
(20,20)	(1,1)	.0912	.0512	.0112	.1050	.0538	.0258	.1006	.0506	.0068
	(1.2,1)	.2854	.1880	.0574	.2670	.1602	.1010	.2856	.1686	.0388
	(1.2,1)	.5360	.4078	.1794	.4760	.3286	.2174	.5124	.3636	.1172
	(1.8,1)	.8800	.8024	.5618	.7960	.6662	.5296	.8376	.7196	.3874
	(2,1)	.9460	.9024	.7254	.8810	.7862	.6660	.9114	.8272	.5284
	(2.6,1)	.9964	.9900	.9582	.9770	.9456	.8934	.9854	.9624	.8226

Table 1.3: Empirical Level and Power of Tests under Normal distribution

Sample sizes	S.D.	AB test			S-T Test			L test			Adaptive Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.0990	.0428	.0222	.1076	.0546	.0282	.1102	.0496	.0072	.0972	.0462	.0068
	(1.2,1)	.1878	.0890	.0463	.1976	.1106	.0610	.1460	.0826	.0146	.2198	.1234	.0292
	(1.2,1)	.2997	.1506	.0878	.3114	.1808	.1096	.2398	.1422	.0322	.3688	.2372	.0716
	(1.8,1)	.4932	.3002	.1980	.5066	.3538	.2368	.4532	.3188	.1036	.6552	.5044	.2306
	(2,1)	.5724	.3834	.2590	.5896	.4286	.3026	.5482	.4100	.1510	.7602	.6286	.3344
	(2.6,1)	.7444	.5644	.4332	.7604	.6148	.4872	.7530	.6328	.3158	.9290	.8600	.6302
(10,15)	(1,1)	.0918	.0480	.0232	.0956	.0506	.0244	.1122	.0564	.0116	.1190	.0588	.0122
	(1.2,1)	.1918	.1062	.0558	.1984	.1134	.0582	.1536	.0864	.0202	.2762	.1720	.0504
	(1.2,1)	.3086	.1966	.1104	.3154	.2098	.1144	.2552	.1520	.0374	.4606	.3232	.1304
	(1.8,1)	.5334	.3886	.2612	.5436	.4002	.2684	.5102	.3602	.1170	.7590	.6360	.3736
	(2,1)	.6288	.4834	.3392	.6370	.4956	.3460	.6178	.4704	.1780	.8408	.7520	.5034
	(2.6,1)	.8162	.6974	.5498	.8204	.7116	.5602	.8430	.7256	.3856	.9618	.9236	.8020
(20,20)	(1,1)	.0988	.0454	.0214	.0986	.0496	.0234	.1066	,0554	,0104	.0912	.0512	.0112
	(1.2,1)	.2554	.1450	.0776	.2514	.1470	.0794	.1872	.1020	.0264	.2854	.1880	.0574
	(1.2,1)	.4382	.2878	.1848	.4316	.2936	.1882	.3592	.2420	.0792	.5360	.4078	.1794
	(1.8,1)	.7334	.5920	.4526	.7304	.5972	.4584	.6986	.5806	.3070	.8800	.8024	.5618
	(2,1)	.8290	.7110	.5852	.8230	.7154	.5896	.8120	.7086	.4478	.9460	.9024	.7254
	(2.6,1)	.9480	.8974	.8256	.9464	.8994	.8284	.9548	.9202	.7576	.9964	.9900	.9582

Table 1.2: Empirical Level and Power of Tests under Cauchy Distribution

Sample sizes	S.D.	F test			Mood test			Klotz Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.3680	.3310	.2682	.1008	.0508	.0220	.1114	.0520	.0078
	(1.2,1)	.4194	.3810	.3116	.1462	.0826	.0396	.1588	.0830	.0134
	(1.2,1)	.4588	.4254	.3512	.1980	.1154	.0594	.2028	.1182	.0206
	(1.8,1)	.5306	.4910	.4226	.3042	.1894	.1080	.3000	.1826	.0404
	(2,1)	.5636	.5222	.4492	.3572	.2258	.1326	.3480	.2140	.0506
	(2.6,1)	.6352	.6050	.5226	.4890	.3402	.2190	.4606	.3156	.0934
(10,15)	(1,1)	.3288	.2986	.2468	.1092	.0514	.0278	.1002	.0430	.0028
	(1.2,1)	.3806	.3466	.2904	.1672	.0924	.0478	.1504	.0708	.0070
	(1.2,1)	.4298	.3932	.3324	.2354	.1336	.0772	.2058	.1070	.0132
	(1.8,1)	.5026	.4730	,4078	.3608	.2364	.1490	.3168	.1862	.0286
	(2,1)	.5334	.5004	.4428	.4210	.2820	.1864	.3694	.2240	.0380
	(2.6,1)	.6132	.5776	.5190	.5628	.4226	.2996	.5004	.3356	.0810
(20,20)	(1,1)	.4058	.3816	.3344	.1088	.0526	.0278	.1038	.0510	.0076
	(1.2,1)	.4648	.4422	.3862	.1886	.1052	.0586	.1694	.0922	.0178
	(1.4,1)	.5184	.4934	.4394	.2820	.1732	.1018	.2460	.1498	.0332
	(1.8,1)	..5992	.5772	.5230	.4660	.3264	.2168	.4030	.2642	.0858
	(2,1)	.6306	.6096	.5600	.5444	.4066	.2828	.4746	.3294	.1158
	(2.6,1)	.7406	.6814	.6390	.7314	.5940	.4648	.6494	.5024	.2266

Table 1.3: Empirical Level and Power of Tests under Cauchy Distribution

Sample sizes	S.D.	AB test			S-T Test			L test			Adaptive Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.1060	.0434	.0218	.1134	.0562	.0312	.4894	.4124	.2942	.4894	.4124	.2942
	(1.2,1)	.1570	.0712	.0368	.1644	.0872	.0494	.5012	.4250	.2996	.5012	.4250	.2996
	(1.2,1)	.2136	.1016	.0564	.2240	.1286	.0720	.5194	.4392	.3084	.5194	.4392	.3084
	(1.8,1)	.3132	.1698	.1028	.3276	.2068	.1284	.5598	.4796	.3332	.5598	.4796	.3332
	(2,1)	.3610	.2090	.1284	.3744	.2438	.1596	.5756	.4974	.3526	.5756	.4974	.3526
	(2.6,1)	.5040	.3110	.2060	.5224	.3546	.2454	.6368	.5556	.4020	.6368	.5556	.4020
(10,15)	(1,1)	.0916	.0474	.0214	.0950	.0500	.0224	.4970	.4206	.3106	.4970	.4206	.3106
	(1.2,1)	.1410	.0814	.0420	.1462	.0834	.0432	.5020	.4312	.3116	.5020	.4312	.3116
	(1.2,1)	.2086	.1186	.0644	.2124	.1268	.0664	.5252	.4484	.3230	.5252	.4484	.3230
	(1.8,1)	.3320	.2138	.1244	.3380	.2262	.1278	.5796	.4942	.3456	.5796	.4942	.3456
	(2,1)	.3872	.2636	.1604	.3954	.2756	.1648	.6004	.5248	.3614	.6004	.5248	.3614
	(2.6,1)	.5388	.3972	.2694	.5424	.4138	.2760	.6714	.5826	.4250	.6714	.5826	.4250
(20,20)	(1,1)	.1024	.0478	.0236	.0972	.0500	.0250	.5390	.4752	.3724	.5390	.4752	.3724
	(1.2,1)	.1842	.0972	.0484	.1820	.1008	.0506	.5476	.4824	.3784	.5476	.4824	.3784
	(1.2,1)	.2832	.1684	.0930	.2796	.1722	.0960	.5760	.5046	.3910	.5760	.5046	.3910
	(1.8,1)	.4706	.3262	.2136	.4664	.3318	.2168	.6336	.5630	.4398	.6336	.5630	.4398
	(2,1)	.5512	.4016	.2808	.5448	.4094	.2866	.6622	.5914	.4632	.6622	.5914	.4632
	(2.6,1)	.7382	.5994	.4666	.7316	.6048	.4726	.7406	.6752	.5428	.7406	.6752	.5428

Table 1.2: Empirical Level and Power of Tests under Logistic Distribution

Sample sizes	*S.D.*	*F test*			*Mood test*			*Klotz Test*		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.0972	.0462	.0068	.1040	.0458	.0212	.1094	.0540	.0054
	(1.2,1)	.2198	.1234	.0292	.2030	.1150	.0546	.2116	.1152	.0176
	(1.2,1)	.3688	.2372	.0716	.3168	.2022	.1154	.3354	.2114	.3084
	(1.8,1)	.6552	.5044	.2306	.5398	.3858	.2506	.5746	.4120	.1098
	(2,1)	.7602	.6286	.3344	.6268	.4710	.3268	.6660	.4990	.1610
	(2.6,1)	.9290	.8600	.6302	.7988	.6714	.5316	.8368	.6950	.3146
(10,15)	(1,1)	.1190	.0588	.0122	.1084	.0548	.0262	.1044	.0498	.0062
	(1.2,1)	.2762	.1720	.0504	.2312	.1386	.0804	.2118	.1102	.0162
	(1.2,1)	.4606	.3232	.1304	.3682	.2448	.1514	.3660	.2112	.0310
	(1.8,1)	.7590	.6360	.3736	.6154	.4666	.3432	.6310	.4502	.0996
	(2,1)	.8408	.7520	.5034	.7142	.5676	.4374	.7334	.5596	.1478
	(2.6,1)	.9618	.9236	.8020	.8718	.7770	.6590	.8944	.7874	.3154
(20,20)	(1,1)	.0912	.0512	.0112	.1050	.0538	.0258	.1006	.0506	.0068
	(1.2,1)	.2854	.1880	.0574	.2670	.1602	.1010	.2856	.1686	.0388
	(1.2,1)	.5360	.4078	.1794	.4760	.3286	.2174	.5124	.3636	.1172
	(1.8,1)	.8800	.8024	.5618	.7960	.6662	.5296	.8376	.7196	.3874
	(2,1)	.9460	.9024	.7254	.8810	.7862	.6660	.9114	.8272	.5284
	(2.6,1)	.9964	.9900	.9582	.9770	.9456	.8934	.9854	.9624	.8226

Table 1.3: Empirical Level and Power of Tests under logistic Distribution

Sample sizes	S.D.	AB test			S-T Test			L test			Adaptive Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.0990	.0428	.0222	.1076	.0546	.0282	.1102	.0496	.0072	.0972	.0462	.0068
	(1.2,1)	.1878	.0890	.0463	.1976	.1106	.0610	.1460	.0826	.0146	.2198	.1234	.0292
	(1.2,1)	.2997	.1506	.0878	.3114	.1808	.1096	.2398	.1422	.0322	.3688	.2372	.0716
	(1.8,1)	.4932	.3002	.1980	.5066	.3538	.2368	.4532	.3188	.1036	.6552	.5044	.2306
	(2,1)	.5724	.3834	.2590	.5896	.4286	.3026	.5482	.4100	.1510	.7602	.6286	.3344
	(2.6,1)	.7444	.5644	.4332	.7604	.6148	.4872	.7530	.6328	.3158	.9290	.8600	.6302
(10,15)	(1,1)	.0918	.0480	.0232	.0956	.0506	.0244	.1122	.0564	.0116	.1190	.0588	.0122
	(1.2,1)	.1918	.1062	.0558	.1984	.1134	.0582	.1536	.0864	.0202	.2762	.1720	.0504
	(1.2,1)	.3086	.1966	.1104	.3154	.2098	.1144	.2552	.1520	.0374	.4606	.3232	.1304
	(1.8,1)	.5334	.3886	.2612	.5436	.4002	.2684	.5102	.3602	.1170	.7590	.6360	.3736
	(2,1)	.6288	.4834	.3392	.6370	.4956	.3460	.6178	.4704	.1780	.8408	.7520	.5034
	(2.6,1)	.8162	.6974	.5498	.8204	.7116	.5602	.8430	.7256	.3856	.9618	.9236	.8020
(20,20)	(1,1)	.0988	.0454	.0214	.0986	.0496	.0234	.1066	,0554	,0104	.0912	.0512	.0112
	(1.2,1)	.2554	.1450	.0776	.2514	.1470	.0794	.1872	.1020	.0264	.2854	.1880	.0574
	(1.2,1)	.4382	.2878	.1848	.4316	.2936	.1882	.3592	.2420	.0792	.5360	.4078	.1794
	(1.8,1)	.7334	.5920	.4526	.7304	.5972	.4584	.6986	.5806	.3070	.8800	.8024	.5618
	(2,1)	.8290	.7110	.5852	.8230	.7154	.5896	.8120	.7086	.4478	.9460	.9024	.7254
	(2.6,1)	.9480	.8974	.8256	.9464	.8994	.8284	.9548	.9202	.7576	.9964	.9900	.9582

Table 1.2: Empirical Level and Power of Tests under Double Exopnential Distribution

Sample sizes	S.D.	F test			Mood test			Klotz Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.3680	.3310	.2682	.1008	.0508	.0220	.1114	.0520	.0078
	(1.2,1)	.4194	.3810	.3116	.1462	.0826	.0396	.1588	.0830	.0134
	(1.2,1)	.4588	.4254	.3512	.1980	.1154	.0594	.2028	.1182	.0206
	(1.8,1)	.5306	.4910	.4226	.3042	.1894	.1080	.3000	.1826	.0404
	(2,1)	.5636	.5222	.4492	.3572	.2258	.1326	.3480	.2140	.0506
	(2.6,1)	.6352	.6050	.5226	.4890	.3402	.2190	.4606	.3156	.0934
(10,15)	(1,1)	.3288	.2986	.2468	.1092	.0514	.0278	.1002	.0430	.0028
	(1.2,1)	.3806	.3466	.2904	.1672	.0924	.0478	.1504	.0708	.0070
	(1.2,1)	.4298	.3932	.3324	.2354	.1336	.0772	.2058	.1070	.0132
	(1.8,1)	.5026	.4730	,4078	.3608	.2364	.1490	.3168	.1862	.0286
	(2,1)	.5334	.5004	.4428	.4210	.2820	.1864	.3694	.2240	.0380
	(2.6,1)	.6132	.5776	.5190	.5628	.4226	.2996	.5004	.3356	.0810
(20,20)	(1,1)	.4058	.3816	.3344	.1088	.0526	.0278	.1038	.0510	.0076
	(1.2,1)	.4648	.4422	.3862	.1886	.1052	.0586	.1694	.0922	.0178
	(1.4,1)	.5184	.4934	.4394	.2820	.1732	.1018	.2460	.1498	.0332
	(1.8,1)	..5992	.5772	.5230	.4660	.3264	.2168	.4030	.2642	.0858
	(2,1)	.6306	.6096	.5600	.5444	.4066	.2828	.4746	.3294	.1158
	(2.6,1)	.7406	.6814	.6390	.7314	.5940	.4648	.6494	.5024	.2266

Table 1.3: Empirical Level and Power of Tests under Double Exponential Distribution:

Sample sizes	S.D.	AB test			S-T Test			L test			Adaptive Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.1060	.0434	.0218	.1134	.0562	.0312	.4894	.4124	.2942	.4894	.4124	.2942
	(1.2,1)	.1570	.0712	.0368	.1644	.0872	.0494	.5012	.4250	.2996	.5012	.4250	.2996
	(1.2,1)	.2136	.1016	.0564	.2240	.1286	.0720	.5194	.4392	.3084	.5194	.4392	.3084
	(1.8,1)	.3132	.1698	.1028	.3276	.2068	.1284	.5598	.4796	.3332	.5598	.4796	.3332
	(2,1)	.3610	.2090	.1284	.3744	.2438	.1596	.5756	.4974	.3526	.5756	.4974	.3526
	(2.6,1)	.5040	.3110	.2060	.5224	.3546	.2454	.6368	.5556	.4020	.6368	.5556	.4020
(10,15)	(1,1)	.0916	.0474	.0214	.0950	.0500	.0224	.4970	.4206	.3106	.4970	.4206	.3106
	(1.2,1)	.1410	.0814	.0420	.1462	.0834	.0432	.5020	.4312	.3116	.5020	.4312	.3116
	(1.2,1)	.2086	.1186	.0644	.2124	.1268	.0664	.5252	.4484	.3230	.5252	.4484	.3230
	(1.8,1)	.3320	.2138	.1244	.3380	.2262	.1278	.5796	.4942	.3456	.5796	.4942	.3456
	(2,1)	.3872	.2636	.1604	.3954	.2756	.1648	.6004	.5248	.3614	.6004	.5248	.3614
	(2.6,1)	.5388	.3972	.2694	.5424	.4138	.2760	.6714	.5826	.4250	.6714	.5826	.4250
(20,20)	(1,1)	.1024	.0478	.0236	.0972	.0500	.0250	.5390	.4752	.3724	.5390	.4752	.3724
	(1.2,1)	.1842	.0972	.0484	.1820	.1008	.0506	.5476	.4824	.3784	.5476	.4824	.3784
	(1.2,1)	.2832	.1684	.0930	.2796	.1722	.0960	.5760	.5046	.3910	.5760	.5046	.3910
	(1.8,1)	.4706	.3262	.2136	.4664	.3318	.2168	.6336	.5630	.4398	.6336	.5630	.4398
	(2,1)	.5512	.4016	.2808	.5448	.4094	.2866	.6622	.5914	.4632	.6622	.5914	.4632
	(2.6,1)	.7382	.5994	.4666	.7316	.6048	.4726	.7406	.6752	.5428	.7406	.6752	.5428

Table 1.3: Empirical Level and Power of Tests under Lognormal Distribution:

Sample sizes	S.D.	F test			Mood test			Klotz Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.1322	.0774	.0214	.0968	.0528	.0244	.1102	.0500	.0060
	(1.2,1)	.2466	.1578	.0522	.1830	.1052	.0544	.2020	.1096	.0136
	(1.2,1)	.3932	.2648	.1064	.2900	.1712	.0974	.3056	.1880	.0324
	(1.8,1)	.6416	.5094	.2580	.5000	.3378	.2128	.5174	.3642	.0936
	(2,1)	.7386	.6150	.3598	.5862	.4256	.2752	.5976	.4442	.1324
	(2.6,1)	.8982	.8294	.6174	.7732	.6422	.4858	.7724	.6416	.2710
(10,15)	(1,1)	.1452	.0860	.0276	.1030	.5080	.0288	.1034	.0478	.0058
	(1.2,1)	.2942	.1962	.0756	.2866	.1224	.0662	.2670	.1122	.0128
	(1.2,1)	.4506	.3338	.1572	.4452	.2186	.1332	.4248	.1884	.0256
	(1.8,1)	.7102	.5988	.3792	.7108	.4302	.3082	.6688	.3880	.0848
	(2,1)	.7988	.7026	.4880	.7962	.5230	.3996	.7690	.4878	.1222
	(2.6,1)	.9388	.8904	.7524	.9364	.7230	.6150	.8760	.7110	.2690
(20,20)	(1,1)	.1294	.0856	.0266	.1012	.0516	.0252	.1010	.0504	.0104
	(1.2,1)	.3094	.2212	.0998	.2476	.1466	.0824	.2550	.1516	.0330
	(1.2,1)	.5264	.4222	.2124	.4276	.2890	.1836	.4576	.3062	.0960
	(1.8,1)	.8314	.7618	.5504	.7490	.6062	.4670	.7726	.6310	.3036
	(2,1)	.9054	.8556	.6930	.8408	.7324	.5970	.8598	.7568	.4322
	(2.6,1)	.9890	.9772	.9210	.9598	.9180	.8534	.9718	.9322	.7358

Table 1.3: Empirical Level and Power of Tests under Lognormal Distribution:

Sample sizes	S.D.	AB test			S-T Test			L test			Adaptive Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.0950	.0360	.0188	.1022	.0496	.0234	.1206	.0584	.0098	.0968	.0528	.0244
	(1.2,1)	.1768	.0822	.0426	.1884	.1008	.0552	.1500	.0822	.0148	.1830	.1052	.0544
	(1.2,1)	.2726	.1336	.0774	.2894	.1644	.0944	.2198	.1332	.0324	.2900	.1712	.0974
	(1.8,1)	.4602	.2830	.1722	.4758	.3268	.2088	.4150	.2830	.0874	.5000	.3378	.2128
	(2,1)	.5446	.3494	.2294	.5614	.3984	.2734	.5126	.3702	.1322	.5862	.4256	.2752
	(2.6,1)	.7114	.5378	.3964	.7218	.5850	.4462	.7184	.5884	.2806	.7732	.6422	.4858
(10,15)	(1,1)	.0942	.0490	.0230	.0984	.0536	.0238	.1126	.0578	.0144	.1030	.5080	.0288
	(1.2,1)	.1832	.1060	.0534	.1890	.1150	.0554	.1584	.0840	.0176	.2866	.1224	.0662
	(1.2,1)	.2844	.1796	.0994	.2912	.1926	.1036	.2462	.1458	.0328	.4452	.2186	.1332
	(1.8,1)	.4892	.3534	.2312	.4982	.3658	.2378	.4646	.3238	.1076	.7108	.4302	.3082
	(2,1)	.5832	.4392	.3030	.5936	.4534	.3088	.5720	.4188	.1612	.7962	.5230	.3996
	(2.6,1)	.7718	.6484	.4996	.7766	.6628	.5090	.7896	.6670	.3386	.9364	.7230	.6150
(20,20)	(1,1)	.0990	.0508	.0246	.0968	.0524	.0260	.1098	.0590	.0120	.1012	.0516	.0252
	(1.2,1)	.2366	.1292	.0778	.2304	.1334	.0806	.1670	.0968	.0282	.2476	.1466	.0824
	(1.2,1)	.4054	.2574	.1620	.4002	.2602	.1666	.3202	.2048	.0704	.4276	.2890	.1836
	(1.8,1)	.6954	.5488	.4070	.6916	.5536	.4134	.6578	.5170	.2482	.7490	.6062	.4670
	(2,1)	.7912	.6636	.5282	.7864	.6702	.5356	.7656	.6582	.3728	.8408	.7324	.5970
	(2.6,1)	.9338	.8706	.7768	.9312	.8720	.7840				.9598	.9180	.8534

Table 1.2: Empirical Level and Power of Tests under Exponential distribution:

Sample sizes	S.D.	F test			Mood test			Klotz Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.1738	.1176	.0418	.0968	.0528	.0244	.1102	.0500	.0600
	(1.2,1)	.1744	.0918	.0904	.1652	.0930	.0482	.1800	.0982	.0118
	(1.2,1)	.2890	.1028	.1460	.2444	.1438	.0806	.2826	.1554	.0244
	(1.8,1)	.4750	.2078	.2954	.4184	.2702	.1668	.4860	.2848	.0650
	(2,1)	.5772	.2914	.3836	.4932	.3320	.2102	.5860	.3548	.0904
	(2.6,1)	.7134	.7810	.5924	.6716	.5230	.3602	.7194	.5256	.1790
(10,15)	(1,1)	.1876	.1258	.0548	.1030	.0508	.0288	.1034	.0478	.0058
	(1.2,1)	.3134	.2348	.1146	.1944	.1054	.0576	.1882	.0984	.0114
	(1.2,1)	.4478	.3504	.1972	.2964	.1772	.1040	.2736	.1616	.0206
	(1.8,1)	.6650	.5680	.3870	.4838	.3448	.2274	.4664	.3024	.0544
	(2,1)	.7472	.6594	.4826	.5672	.4216	.2982	.5578	.3776	.0804
	(2.6,1)	.8918	.8416	.7046	.7386	.6188	.4864	.7552	.5864	.1762
(20,20)	(1,1)	.1754	.1292	.0594	.1012	.0516	.0252	.1010	.0504	.0104
	(1.2,1)	.3422	.2642	.1400	.2160	.1256	.0674	.2218	.1274	.0264
	(1.2,1)	.5238	.4350	.2550	.3596	.2304	.1448	.3816	.2434	.0682
	(1.8,1)	.7842	.7202	.5390	.6376	.4804	.3410	.6632	.5076	.2026
	(2,1)	.8584	.8102	.6586	.7396	.5966	.4560	.7614	.6188	.2920
	(2.6,1)	.9678	.9488	.8758	.9082	.8318	.7214	.9176	.8464	.5698

Table 1.3: Empirical Level and Power of Tests under Exponential distribution:

Sample sizes	S.D.	AB test			S-T Test			L test			Adaptive Test		
(n_1,n_2)	(σ_1,σ_2)	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(10,10)	(1,1)	.0950	.0360	.0188	.1022	.0496	.0234	.1390	.0724	.0136	.1102	.0500	.0600
	(1.2,1)	.1602	.0734	.0360	.1718	.0914	.0478	.1608	.0908	.0190	.1800	.0982	.0118
	(1.2,1)	.2390	.1130	.0654	.2528	.1394	.0812	.2166	.1324	.0338	.2826	.1554	.0244
	(1.8,1)	.3926	.2234	.1296	.4088	.2606	.1592	.3672	.2432	.0812	.4860	.2848	.0650
	(2,1)	.4548	.2830	.1710	.4708	.3234	.2092	.4410	.3176	.1106	.5860	.3548	.0904
	(2.6,1)	.6250	.4318	.3020	.6388	.4804	.3518	.6436	.5024	.2294	.7144	.5256	.1790
(10,15)	(1,1)	.0942	.0490	.0230	.0984	.0536	.0238	.1288	.0702	.0182	.1034	.0478	.0058
	(1.2,1)	.1652	.0942	.0474]	.1704	.1038	.0484	.1598	.0894	.0204	.1882	.0984	.0114
	(1.2,1)	.2466	.1508	.0810	.2522	.1618	.0854	.2282	.1348	.0334	.2736	.1616	.0206
	(1.8,1)	.4156	.2872	.1730	.4226	.2966	.1762	.4034	.2722	.0878	.4664	.3024	.0544
	(2,1)	.4886	.3528	.2296	.4962	.3660	.2358	.4860	.3516	.1294	.5578	.3776	.0804
	(2.6,1)	.6750	.5372	.3860	.6822	.5546	.3970	.6996	.5592	.2678	.7552	.5864	.1762
(20,20)	(1,1)	.0990	.0508	.0246	.0968	.0524	.0260	.1178	.0652	.0146	.1010	.0504	.0104
	(1.2,1)	.2102	.1140	.0648	.2048	.1182	.0668	.1590	.0902	.0290	.2218	.1274	.0264
	(1.2,1)	.3438	.2120	.1258	.3346	.2144	.1280	.2742	.1740	.0586	.3816	.2434	.0682
	(1.8,1)	.6002	.4430	.3018	.5938	.4446	.3062	.5482	.4172	.1840	.6632	.5076	.2026
	(2,1)	.6946	.5484	.4048	.6892	.5524	.4108	.6654	.5360	.2730	.7614	.6188	.2920
	(2.6,1)	.8702	.7676	.6588	.8678	.7710	.6654	.8690	.7954	.5552	.9176	.8464	.5698

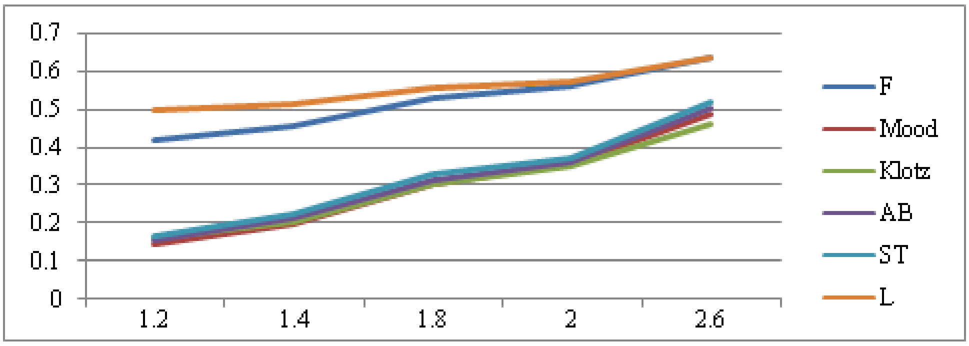

Figure 1.2: Comparison of Power between Different Tests Under Normal Distribution with Sample Sizes (20,20) at 10% Level of Significance

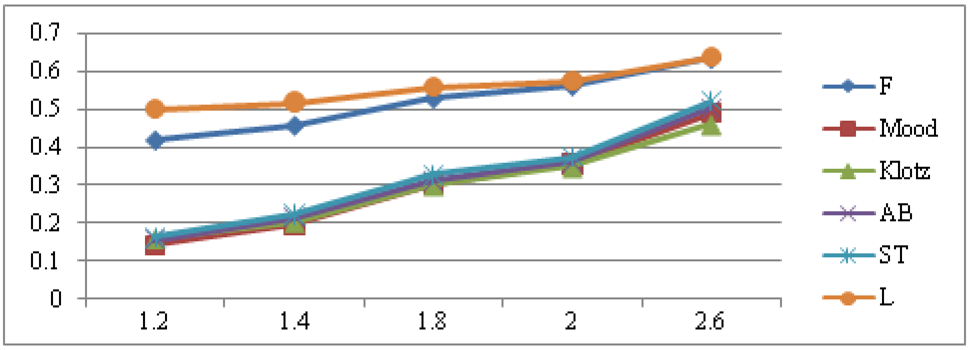

Figure 1.3: Comparison of power Between Different Tests under Cauchy Distribution with Sample Size (10,10) at 10% level Of Significance

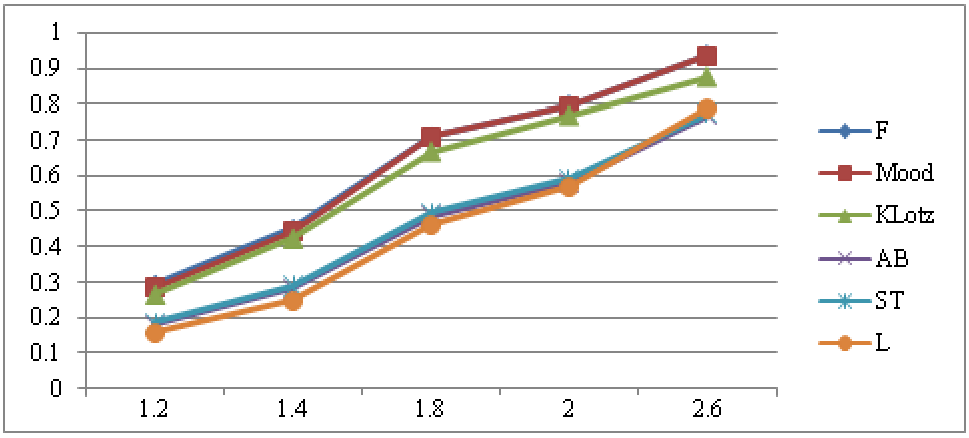

Figure 1.4: Comparison of Power between Different Tests under Logistic Distribution with Sample Sizes (10,15) at 10% Level of Significance

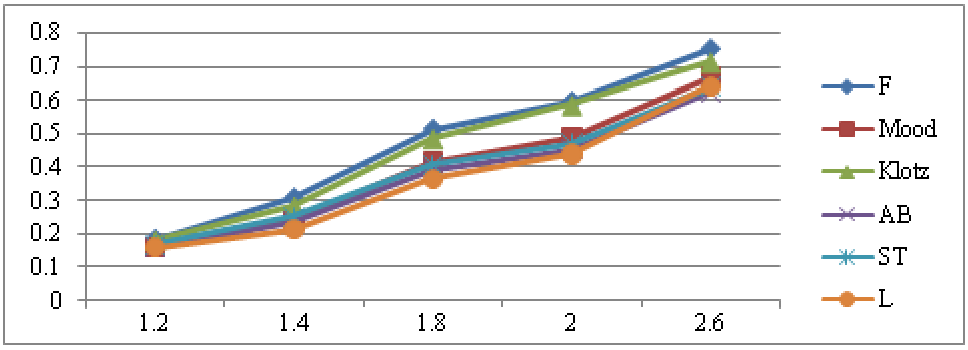

Figure 1.5: Comparison of Power between Different Tests Under Double Exponential distribution with Sample Sizes (10,10) at 10% Level of Significance

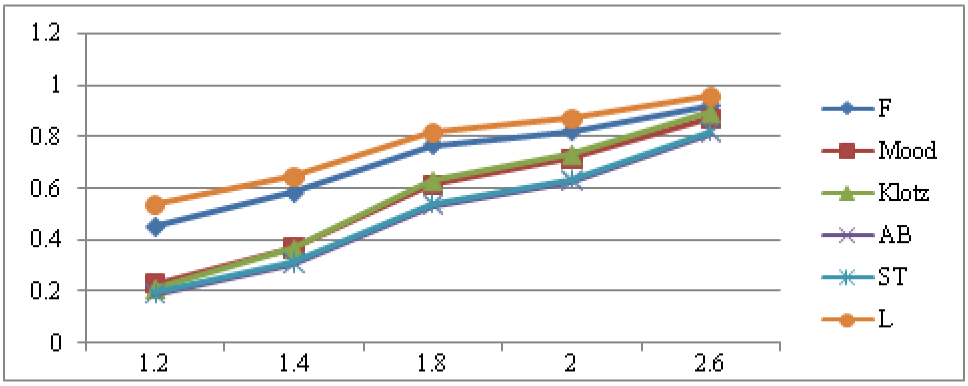

Figure 1.6: Comparison of Power between Different Tests Under Lognormal distribution with Sample Sizes (10,10) at 10% Level of Significance

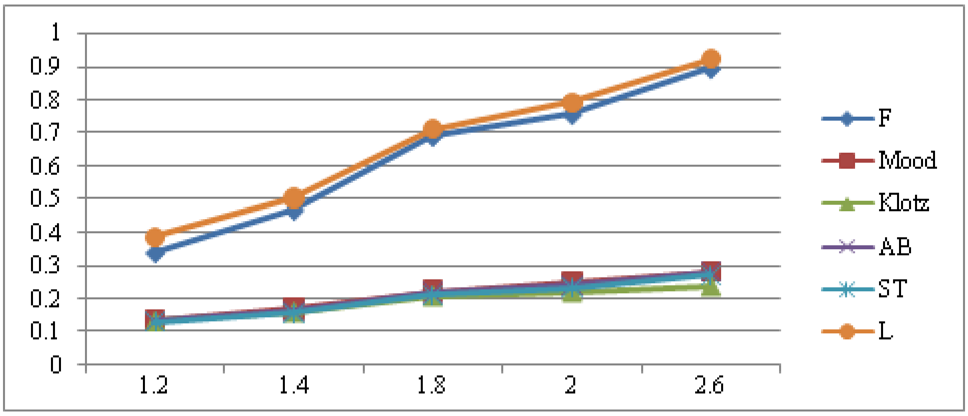

Figure 1.7: Comparison of Power between Different Tests under Exponential Distribution with Sample Sizes (20,20) at 10% Level of Significance

6. Discussion

For comparison purposes we have considered various combinations of sample sizes with equal and unequal sample sizes. We have also considered different sets of σ_i's for the study. From Tables 1.2 – 1.13 it is observed that the parametric F-test maintain the nominal level except the Cauchy and skew distribution lognormal. In these cases F-test seems to be conservatives.

Table 1.2 and table 1.3 shows the power of tests under normal distribution for equal and unequal sample sizes respectively. We have seen that power of F- test is higher than the other tests in this distribution in presence of various combinations of Scale parameters and sample sizes. So here we may conclude that for Normal Distribution F test is our Adaptive test.

Table 1.4 and Table 1.5 gives the power of tests statistics under Cauchy distribution for equal and unequal sample sizes respectively. Here,we observe that Sum of Squared Rank Test is more powerful than other tests both in case of equal and unequal sample sizes at 10%, 5% and 1% level of significance. So if the parent distribution is Cauchy then Sum of Squared Rank test is the Adaptive Test.

Table 1.6 and Table 1.7 displays the power of tests under logistic distribution. We have seen that Mood Test and Klotz test are more powerful than other tests with both equal and unequal sample sizes and at 10%, 5% and 1% level. In this case Mood Test and Klotz test is our Adaptive Test.

Table 1.8 and Table 1.9 shows the empirical power of tests under double exponential distribution. Here it is seen that Klotz test and F test are more powerful than the other tests both in case of equal and unequal sample sizes and at10%, 5% and 1% level.. here our Adaptive Test is the Klotz Test.

Table 1.10 and Table 1.11 shows the power of tests under lognormal distribution. Here,Sum of Squared Rank Test is more powerful than the all other tests both in case of equal and unequal sample sizes at 10%, 5% and 1% level. So in this particular Distribution Sum of Squared Test is our Adaptive test.

Table 1.12 and Table 1.13 shows the power of tests under Exponential distribution. Here,Sum of Squared Rank Test is more powerful than the all other tests both in case of equal and unequal sample sizes at 10%, 5% and 1% level. So in this particular Distribution Sum of Squared Test is our Adaptive test.

References

[1] Ansari, A. R. and Bradley, R. A. (1960). "*Rank-sumtests for dispersion". *Ann. Math. Statist. 31., 1174-89.
[2] Bandyopadhyay,U. and Dutta,D.(2010), *"Adaptive nonparametric tests for the two-sample scale problem under symmetry"* ,Stat Methods Appl,19:153–170
[3] Basu, A.P. and Woodworth,G.(1967), " *A Note on Nonparametric Tests for Scale*" The Annals of Mathematical Statistics, 38, 274-277
[4] Fligner,M.A.(1979), "*A Class of Two-Sample Distribution-Free Tests for Scale*", Journal of the American Statistical Association, 74, 889-893
[5] Fligner,M.A. and Killeen,T.J.,(1976) *"Distribution-Free Two-Sample Tests for Scale"* Journal of the American Statistical Association, 71, 210-213
[6] Hall, P. and Padmanabhan, A.R.(1997), *"Adaptive Inference for the Two-Sample Scale Problem",* Journal of American Statistical Association and American Society for Quality, 39, 412-422
[7] Hogg, R.V.,Randles,R.H. and Fisher,D.M.(1975), "*A Two Sample Adaptive Distribution Free Test,*" Journal of the American Statistical Association, 70,656-661
[8] Klotz. J,(1962), " *Nonparametric Tests for Scale*", The Annals of Mathematical Statistics, 33, 498-512
[9] Moses, L.E.(1963), "*Rank Tests of Dispersion*", The Annals of Mathematical Statistics, 34, 973-983
[10] Puri, M.L.,(1972), "*Some Aspects of Nonparametric Inference*", International Statistical Review / Revue Internationale de Statistique, 40, 99-327
[11] Raghavachari,M(1965), "*The Two-Sample Scale Problem When Locations are Unknown*", The Annals of Mathematical Statistics, 36, 1236-1242