

# Time-Frequency Analysis of EEG Signal processing for Artifact Detection

Mst. Jannatul Ferdous\*, Md. Sujan Ali

Department of Computer Science and Engineering, Jatiya Kabi Kazi Nazrul Islam University, Mymensingh, Bangladesh  
\*E-mail: mjferdous\_cse@jknui.edu.bd

**Abstract:** EEG is widely used to record the electrical activity of the brain for detecting various kinds of diseases and disorders of the human brain. EEG signals are contaminated with several unwanted artifacts during EEG recording and these artifacts make the analysis of EEG signal difficult by hiding some valuable information. Time-frequency representation of electroencephalogram (EEG) signal provides a source of information that is usually hidden in the Fourier spectrum. The popular methods of short-time Fourier transform, Hilbert Huang transform and the wavelet transform analysis have limitations in representing close frequencies and dealing with fast varying instantaneous frequencies and this is often the nature of EEG signal. The synchrosqueezing transform (SST) is a promising tool to track these resonant frequencies and provide a detailed time-frequency representation. The SST is an extension of the wavelet transform incorporating elements of empirical mode decomposition and frequency reassignment techniques. This new tool produces a well-defined time frequency representation allowing the identification of instantaneous frequencies in EEG signals to highlight individual components. We introduce the SST with applications for EEG signals and produced promising results on synthetic and real examples. In this paper, different time-frequency distributions are compared with each other with respect to their time and frequency resolution. Several examples are given to illustrate the usefulness of time-frequency analysis in electroencephalography.

**Keywords:** EEG signal, STFT, HHT, WPT, SST, time-frequency representation

## 1. Introduction

Time-frequency analysis of electroencephalogram (EEG) during different mental tasks received significant attention. As EEG is non-stationary, time-frequency analysis is essential to analyze brain states during different mental tasks. In the EEG, measured as potential differences on the human head, variations of the amplitude and frequency of certain neurological disorders. Therefore time frequency analysis of the EEG can add valuable information to the traditional visual interpretation, by providing an image of the spectral EEG contents varying with time. Further, the time-frequency information of EEG signal can be used as a feature for classification in brain-computer interface (BCI) applications.

Time-frequency representations provide a powerful tool for the analysis of time series signals. Based on the time-frequency representation (TFR) of EEG signal. Traditional time frequency representations, such as the Short-Time Fourier Transform (STFT) and the Wavelet Transform (WT) and special representations like Empirical Mode Decomposition (EMD), have limitations when signal components are not well separated in the time-frequency plane. Synchrosqueezing, first introduced in the context of speech signals [1] has shown to be an alternative to the EMD method [2] improving spectral resolution.

The recognition, feature extraction and monitoring analysis and other technologies of variety of physiological signals including EEG signal has become a hot topic of research in recent years [3]. All kinds of analysis methods such as Fourier transformation [4-6], Short Time Fourier transformation [7-9], wavelet and wavelet package transformation [10-14], Hilbert-Huang Transformation (HHT) [15-19] and other technologies have been applied to analysis of signal. Fourier transformation is the basic of analysis of time domain and frequency domain for signal,

and is very valid to the analysis of periodic stable signal. But in nature and engineering areas there is a large number of non-periodic non stationary signals, and EEG is typical non periodic signal, so the classic analysis method based on Fourier transformation cannot accurately reflect the local time-varying frequency spectral characteristic of signal, unable to obtain many of the key information.

So a series of new signal time frequency analysis methods are put forward and developed. The basic idea of time frequency analysis is to design the joint function of time and frequency, and at the same time, the density and intensity of energy of signal at different time and frequency is described. Before the analysis and comparison on time-frequency performance of several of methods has been seen, but the research on the comparison and analysis on Short Time Fourier transformation, wavelet transformation, HHT and SST, these four kinds of methods is rare through the designed signal.

A comprehensive survey of time-frequency decomposition methods is beyond the scope of this article, but some basic points about time-frequency transformations can be made that highlight differences among some of the methods and also underscore some more general considerations. Perhaps the most important overarching is that all time-frequency decomposition methods strike some compromise between temporal resolution and frequency resolution in resolving the EEG signals. In general, the larger the time window used to estimate the complex data for a given time point, the greater the frequency resolution but the poorer the temporal resolution. This trade-off between precision in the time domain vs. the frequency domain is formalized in the Heisenberg uncertainty principle [20], discussed again in a later section.

A recent class of time-frequency techniques, referred to as reassignment methods [21–23], aim to improve the “read ability” (localization) of time-frequency representations

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[24]. The synchrosqueezing transform (SST) [25, 1] belongs to this class, it is a post-processing technique based on the continuous wavelet transform that generates highly localized time-frequency representations of nonlinear and non-stationary signals. Synchrosqueezing provides a solution that mitigates the limitations of linear projection based time-frequency algorithms, such as the short-time Fourier transform (STFT) and continuous wavelet transforms (CWT). The synchrosqueezing transform reassigns the energies of these transforms, such that the resulting energies of coefficients are concentrated around the instantaneous frequency curves of the modulated oscillations. As such, synchrosqueezing is an alternative to the recently introduced empirical mode decomposition (EMD) algorithm [26]; it builds upon the EMD, by generating localized time-frequency representations while at the same time providing a well understood theoretical basis.

This study first briefly introduces the principles and characteristics of several methods, short time Fourier transform, HHT, wavelet transform and SST. So, the time frequency graphs are made out through simulation signal, and time frequency performance of four methods is made a contrast and analysis, to explore the application prospect of them in processing and analysis of raw EEG signal. This paper demonstrated the potentiality of the SST in time frequency representation of EEG signals. Firstly, these mentioned methods are tested using synthetic signal and finally, SST and other methods are applied to a real data sample with high amplitude artifact.

## 2. Time Frequency Representation Methods

### A. Short-Term Fourier Transform

A variant of the fast Fourier transform (FFT), known as the STFT, or windowed Fourier transform performs a Fourier transform within a time window that is moved along the time series in order to characterize changes in power and phase of EEG signals over time. Typically, a fixed duration time window is applied to all frequencies. The choice of time window constrains the frequency binsize (i.e., frequency resolution), which is uniform across all frequencies, and also determines the lowest resolvable frequency. The uniformity of the characterization of temporal changes in high-frequency signals requires shorter time windows than those needed to optimally characterize low-frequency signals. A more flexible approach in which window size varies across frequencies to optimize temporal resolution of different frequencies is therefore desirable. The STFT for a non-stationary signal  $s(t)$  is defined as

$$\Psi(t, f) = \int_{-\infty}^{\infty} [s(t) \cdot w^*(t-t')] \cdot e^{-2\pi f t} dt \quad (1)$$

where  $*$  is the complex conjugate,  $w(t)$  is the window function. The STFT of the signal is the Fourier transform of the signal multiplied by a window function. The advantage of STFT is that its physical meaning is clear and there is no cross term appearing [7-9]. It is a time frequency analysis method which application is the most. According to the Heisenberg uncertainty principle, the product of time width and bandwidth of signal should be a constant relevant

to sampling rate. In other words, when time resolution is needed to improve, the resolution of frequency is often needed to sacrifice, and vice versa. When the analyzed signal contains many kinds of types of scale components with large differences, STFT is powerless. Although STFT there are defects such as resolution ratio is not high and so on, its algorithm is simple, and easy to implement, so in a long time it is a powerful tool of non-stationary signal analysis, and still is widely used.

### B. Hilbert-Huang Transform

The Hilbert–Huang transform (HHT) is a way to decompose a signal into so-called intrinsic mode functions (IMF), and obtain instantaneous frequency data. In contrast to other common transforms like the Fourier transform, the HHT is more like an algorithm (an empirical approach) that can be applied to a data set, rather than a theoretical tool. It is the result of the empirical mode decomposition (EMD) and the Hilbert spectral analysis (HSA). The HHT uses the EMD method to decompose a signal into so-called intrinsic mode functions, and uses the HSA method to obtain instantaneous frequency data. The HHT provides a new method of analyzing non stationary and nonlinear time series data. Based on the principle of the empirical mode decomposition technique [27], the signal  $s(t)$  is represented as

$$s(t) = \sum_{m=1}^M C_m(t) + r_M(t) \quad (2)$$

where,  $C_1(t), C_2(t), \dots, C_M(t)$  are all of the intrinsic mode functions included in the signals and  $r_M(t)$  is a negligible residue. Here,  $M$ =total number of intrinsic mode function components. The completeness of the decomposition is given by the Eq. (2).

Instantaneous frequency (IF) represents signal's frequency at an instance, and is defined as the rate of change of the phase angle at the instant of the "analytic" version of the signal. Every IMF is a real valued signal. The discrete Hilbert transform (HT) denoted by  $h_d[\cdot]$  is used to compute the analytic signal for an IMF. HT provides a phase-shift of  $\pm\pi/2$  to all frequency components, whilst leaving the magnitudes unchanged. Then the analytic version of the  $m^{\text{th}}$  IMF  $\hat{c}_m(t)$  is defined as:

$$z_m(t) = \hat{c}_m(t) + jh_d[\hat{c}_m(t)] = a_m(t)e^{j\theta_m(t)} \quad (3)$$

where  $a_m(t)$  and  $\theta_m(t)$  are instantaneous amplitude and phase respectively of the  $m^{\text{th}}$  IMF. The IF of  $m^{\text{th}}$  IMF is then computed by the derivative of the phase  $\theta_m(t)$  as:

$$\omega_m(t) = \frac{d\tilde{\theta}(t)}{dt} \quad \text{where } \tilde{\theta}_m(t) \text{ represents the unwrapped}$$

version of  $\theta_m(t)$ . The median smoothing filter is used to tackle the discontinuities of IF computed by discrete time derivative of the phase vector.

Hilbert spectrum represents the distribution of the signal energy as a function of time and frequency. It is also designated as Hilbert amplitude spectrum  $H(w, t)$  or simply Hilbert spectrum (HS). This process first normalizes the IF vectors of all IMFs between 0 to 0.5. Each IF vector is multiplied by the scaling factor  $\eta=0.5/(IF_{max}-IF_{min})$ ,

where  $IF_{max} = \text{Max}(f_1, f_2, \dots, f_m, \dots, f_M)$  and  $IF_{min} = \text{Min}(f_1, f_2, \dots, f_m, \dots, f_M)$ . The bin spacing of the HS is  $0.5/B$ , where  $B$  is the number of desired frequency bins selected arbitrarily. Each element  $H(\omega, t)$  is defined as the weighted sum of the instantaneous amplitudes of all the IMFs at  $w^{\text{th}}$  frequency bin,

$$H(\omega, t) = \sum_{m=1}^M a_m(t) w_m^{\omega}(t) \quad (4)$$

where the weight factor  $w_m^{\omega}(t)$  takes 1 if  $\eta x f_m(t)$  falls within  $w^{\text{th}}$  band, otherwise is 0. After computing the elements over the frequency bins,  $H$  represents the instantaneous signal spectrum in TF space as a 2D table. The time resolution of  $H$  is equal to the sampling rate and the frequency resolution can be chosen up to Nyquist limit [15-19, 27].

HHT is an empirical approach, and has been tested and validated exhaustively but only empirically. In almost all the cases studied, HHT gives results much sharper than any of the traditional analysis methods in time-frequency-energy representation. Additionally, it reveals true physical meanings in many of the data examined.

### C. Wavelet Packet Transform

The wavelet packet transform (WPT) is a generalization of the wavelet decomposition process that offers a better performance compared to the ordinary wavelet methods. In the wavelet analysis, a signal is split into an approximation and a detail. The approximation is then itself split into a second-level approximation and detail and the process is repeated. On the other hand, WPT is applied in both the detail and the approximation coefficients are divided to get all nodes for the decomposed levels and generates the full decomposition tree. A low (l) and high (h) pass filter is frequently applied to generate a complete subband tree to some desired depth. The low-pass and high-pass filters are generated using orthogonal basis functions. The wavelet packet coefficients  $C_{j,k}^i$  succeeding to the signal  $s(t)$  can be obtain as,

$$C_{j,k}^i = \int_{-\infty}^{\infty} s(t) W_{j,k}^i(t) dt \quad (5)$$

where,  $i$  is the modulation parameter,  $j$  is the dilation parameter and  $k$  is the translation parameter.  $i = 1, 2, \dots, j^L$ , and  $L$  is the level of decomposition in wavelet packet tree.

By applying WPT on each channel, it produce  $2^L$  sub band wavelet packets, where  $L$  is the number of levels. The structure of WPT decomposition, the lower and the higher frequency bands are decomposed giving a balanced binary tree structure. In this present work, five levels is generated 32 subspaces ( $2^L = 2^5$ ) and wavelet frequency interval of each subspace is calculated by

$$\left[ \frac{(b-1)f_s}{2^{L+1}}, \frac{bf_s}{2^{L+1}} \right] \text{ where, the frequency factor, } b=1, 2,$$

3, 4, 5, ..... $2^L$ ,  $f_s$  is the sampling frequency of the EEG signal. In this study  $f_s = 256\text{Hz}$ .  $s(t)$  is the original signal with the frequency  $[0 \sim f_s/2]$  [28]. Because wavelet packets divide the frequency axis into finer intervals than the

DWT, wavelet packets are superior at time-frequency analysis.

Wavelet packet decomposition algorithm decomposes gradually not only low frequency band of the signal, but also the high frequency band of the signal. Furthermore, wavelet packet decomposition can choose frequency band adaptively in accordance with the feature of the analyzed signal, and make a good match with the spectrum of the signal, which improve time frequency resolution.

### D. Synchrosqueezing Transform

The synchrosqueezing transform (SST) is a post processing technique applied to the continuous wavelet transform in order to generate localized time-frequency representation of non-stationary signals. The continuous wavelet transform is a projection based algorithm that identifies oscillatory components of interest through a series of time-frequency filters known as wavelets. The SST was originally introduced in the context of audio signal analysis and is shown to be an alternative to EMD. SST aims to decompose a signal into constituent components with time-varying harmonic behavior. These signals are assumed to be the addition of individual time-varying harmonic components yielding

$$s(t) = \sum_{k=1}^K A_k(t) \cos(\theta_k(t)) + \eta(t) \quad (6)$$

where  $A_k(t)$  is the instantaneous amplitude,  $\eta(t)$  represents additive noise,  $K$  stands for the maximum number of components in one signal, and  $\theta_k(t)$  is the instantaneous phase of the  $k^{\text{th}}$  component. The instantaneous frequency  $f_k(t)$  of the  $k^{\text{th}}$  component is estimated from the instantaneous phase as

$$f_k(t) = \frac{1}{2\pi} \frac{d}{dt} \theta_k(t) \quad (7)$$

In seismic signals, the number  $K$  of harmonics or components in the signal is infinite. They can appear at different time slots, with different amplitudes  $A_k(t)$ , instantaneous frequencies  $f_k(t)$ , and they may be separated by their spectral bandwidths  $\Delta f_k(t)$ .

The spectral bandwidth defines the spreading around the central frequency, which in our case is the instantaneous frequency; for a completed disentangling of concepts. This magnitude is a constraint for traditional time frequency representation methods. The STFT and the CWT tend to smear the energy of the superimposed instantaneous frequencies around their center frequencies [1]. The smearing equals the standard deviation around the central frequency, which is the spectral bandwidth. SST is able to decompose signals into constituent components with time-varying oscillatory characteristics. Thus, by using SST we can recover the amplitude  $A_k(t)$  and the instantaneous frequency  $f_k(t)$  for each component.

**From CWT to SST:** The CWT of a signal  $s(t)$  is

$$w_s(a,b) = \frac{1}{\sqrt{a}} \int s(t) \psi^* \left( \frac{t-b}{a} \right) dt \quad (8)$$

where  $\psi^*$  is the complex conjugate of the mother wavelet and  $b$  is the time shift applied to the mother wavelet, which is also scaled by  $a$ . The CWT is the cross correlation of the signal  $s(t)$  with several wavelets that are scaled and translated versions of the original mother wavelet. The symbols  $w_s(a,b)$  are the coefficients representing a concentrated time-frequency picture, which is used to extract the instantaneous frequencies. It is observe that there is a limit to reduce the smearing effect in the time-frequency representation using the CWT. This smearing mainly occurs in the scale dimension  $a$ , for constant time offset  $b$  show that if smearing along the time axis can be neglected, then the instantaneous frequency  $w_s(a,b)$  can be computed as the derivative of the WT at any point  $(a,b)$  with respect to  $b$ , for all  $w_s(a,b) \neq 0$ :

$$w_s(a,b) = \frac{-j}{2\pi W_s(a,b)} \frac{\partial W_s(a,b)}{\partial b}$$

The final step in the new time-frequency representation is to map the information from the time-scale plane to the time-frequency plane. Every point  $(b, a)$  is converted to  $(b, w_s(a,b))$ , and this operation is called synchrosqueezing. Because  $a$  and  $b$  are discrete values, we can have a scaling step  $\Delta a_k = a_{k-1} - a_k$  for any  $a_k$  where  $W_s(a,b)$  is computed. Likewise, when mapping from the time-scale plane to the time-frequency plan  $(b, a) \rightarrow (b, w_{inst}(a,b))$  the SST  $T_s(w,b)$  is determined only at the centers  $\omega_l$  of

the frequency range  $[\omega_l - \Delta\omega/2, \omega_l + \Delta\omega/2]$  with  $\Delta\omega = \omega_l - \omega_{l-1}$ :

$$T_s(\omega_l, b) = \frac{1}{\Delta\omega} \sum_{a_k: |\omega(a_k, b) - \omega_l| \leq \Delta\omega/2} W_s(a_k, b) a^{-3/2} \Delta a_k.$$

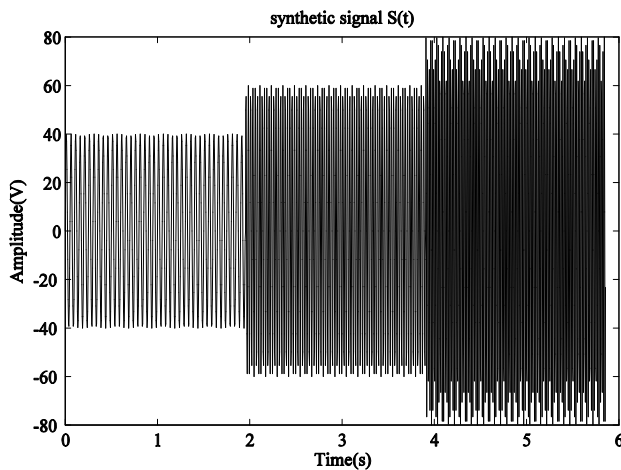
The above equation shows that the time-frequency representation of the signal  $s(t)$  is synchrosqueezed [29] along the frequency (or scale) axis only. The synchrosqueezing transform reallocates the coefficients of the continuous wavelet transform to get a concentrated image over the time frequency plane, from which the instantaneous frequencies are then extracted; this is an ultimate goal in EEG signal analysis. The identified frequencies are used to describe their source EEG and eventually gain a better understanding of the artifact detection.

### 3. Results And Discussion

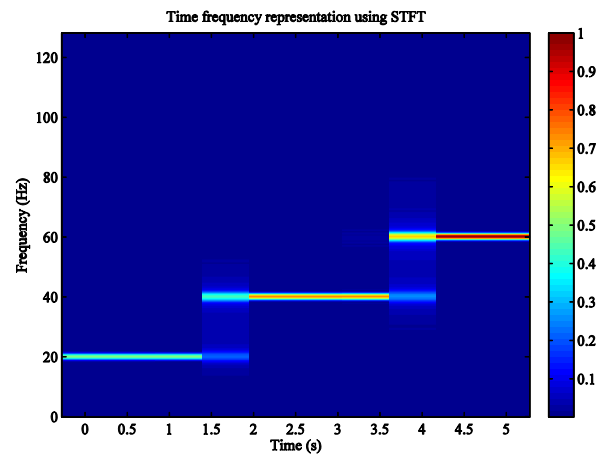
#### A. Synthetic data

In this section, the different TFR method is tested with a (9) challenging synthetic signal. A noiseless synthetic signal is created as the concatenation of the following components (the IF is in brackets):

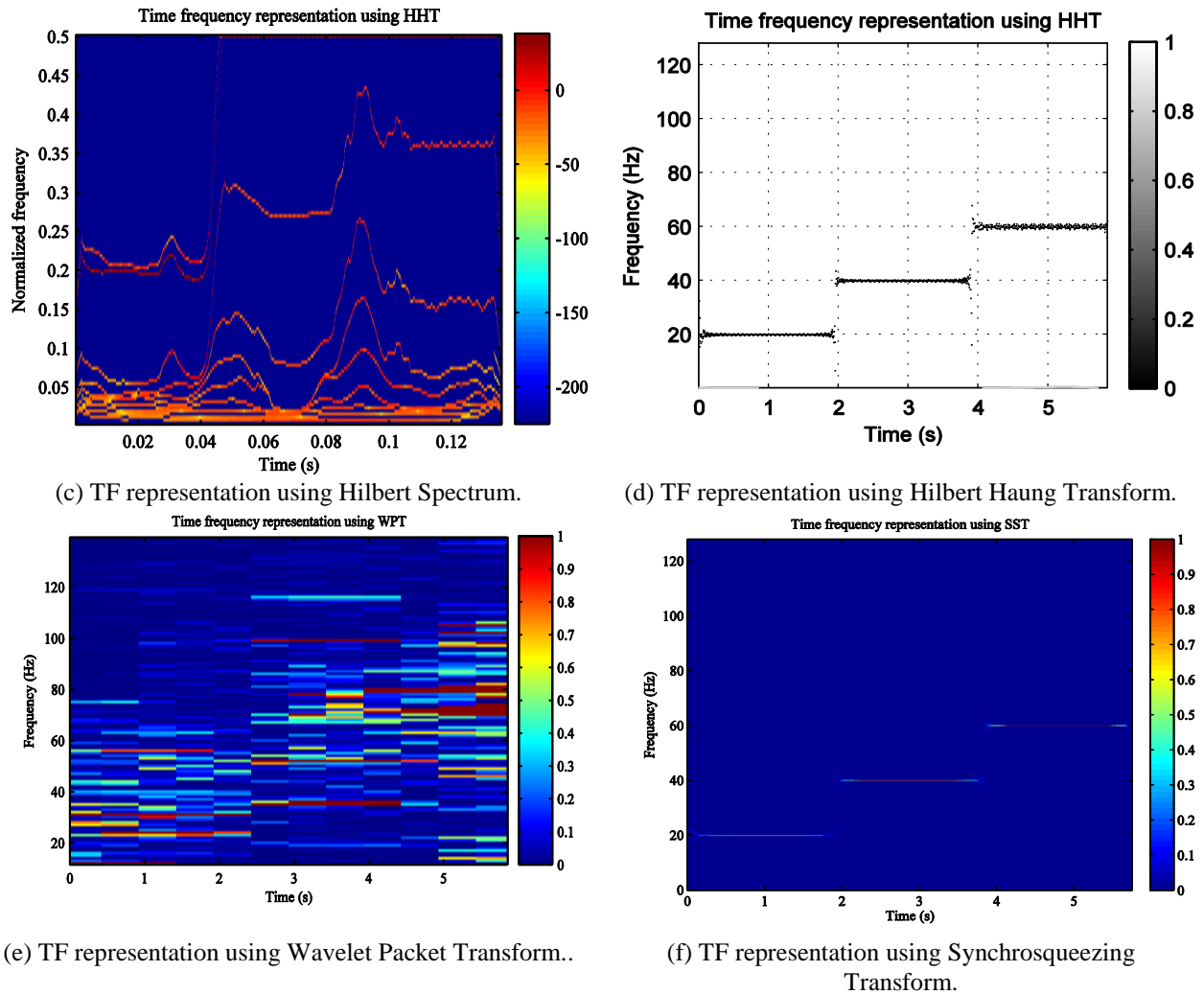
$$\begin{aligned} s_1(t) &= 40 \sin(40\pi t); [IF = 20]; \\ s_2(t) &= 60 \sin(80\pi t); [IF = 40]; \\ s_3(t) &= 80 \sin(120\pi t); [IF = 60]; \end{aligned}$$



(a) The synthetic signals  $S(t)$  with three sinusoidal components.



(b) TF representation using Short Term Fourier Transform.



**Figure 1:** Synthetic signal  $S(t)$  and its TF representation

The synthetic signal  $s(t)$  (Figure 1a) has three constant harmonics of 20 Hz ( $s_1(t)$ ) and 40 Hz ( $s_2(t)$ ) from time 0 to 4 s, the 60 Hz component ( $s_3(t)$ ) appears at time 4 s and vanishes at 6 s. Comparison of Synchrosqueezing with the STFT, HHT and WPT are illustrated in Figure 1. The STFT is able to identify the three components but with a low resolution (Figure 1b), especially when they overlap at 4 s. Figure 1c) shows the instantaneous frequencies for each component using HHT. It looks better time resolution than STFT but lower frequency resolution for wide frequency bandwidth. Figure 1e) shows the instantaneous frequencies for each component using decimated WPT. The WPT is able to identify the three components but with a low resolution, it is good for nearest frequency resolution. On the other hand, the SST (Figure 1f) is able to perfectly delineate each individual component and to resolve the instantaneous frequencies close to the theoretical value, which are presented good time and frequency resolution. For comparison purposes, we include the HHT result, because the SST is an extension of the CWT and EMD. The instantaneous frequencies obtained as the derivative of each one of the independent components. In the SST shows a sharper representation of the instantaneous frequencies.

Figure 1 shows the STFT, HHT, WPT and SST time-frequency representations. Four methods all can distinguish the change of main frequency. From Figure 1, spectrum

graph of short time Fourier transformation it can be seen that, although this figure can distinguish the moment of three main frequency components and mutation occurrence, the resolution is not high. According to the Heisenberg uncertainty principle, the product of time width and bandwidth of signal should be a constant relevant to sampling rate. In other words, when time resolution is needed to improve, the resolution of frequency is often needed to sacrifice, and vice versa. For EEG signal, when the frequency resolution is required, the time window length is likely to appear too large, and at this time FFT may be difficult to meet the requirement. The treatment effect is not satisfactory for non-stationary nonlinear signal such as EEG. Figure 1(d) is the HHT spectrum. It can be seen from the figure that processing effect of HHT on nonlinear unstable signal has a unique advantage, because this method does not exist the steps of partition and translation of windows when using, adopting that separately making analysis for the short time series. Therefore, it makes a good balance between the requirements of time resolution and frequency resolution, that is to say, as long as it meets the Shannon theorem (i.e., sampling law), this method can improve time domain resolution as much as possible under the situation that without sacrificing frequency domain resolution. Despite the high-amplitude noise, the resonance frequencies as well as their variations are all clearly visible on the SST representation (Figure 1f). The SST is able to map smooth

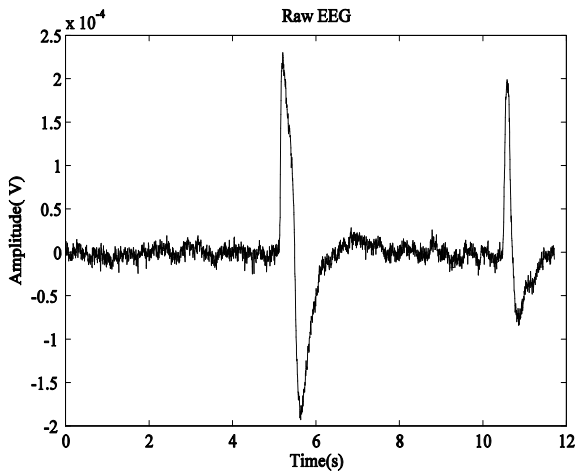
and sharp changes in the frequency lines. Compared with the STFT, the SST brings out the resonance frequencies more sharply, improving significantly the frequency resolution. The SST is then able to distinguish between the different lines constituting the resonance frequency at 2, 4 and 8 Hz while the STFT cannot. Last but not least, the SST determines the IFs at all times. Therefore, the time resolution of this method is not limited by the size of any window.

**B. Application to real EEG data**

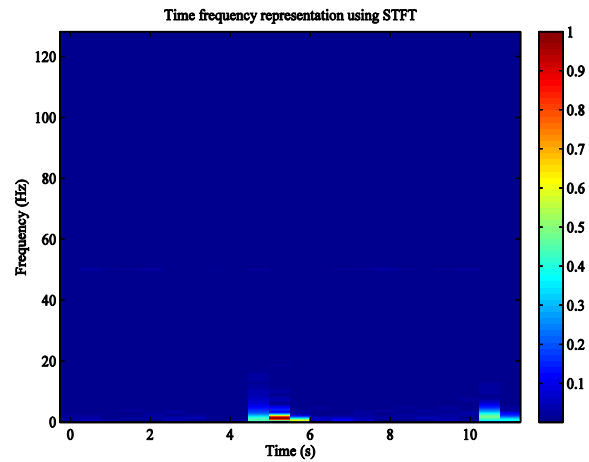
**Dataset description:**

In this section, the real EEG data used to evaluate the TFR method is collected from well-known publicly available real EEG data set (downloaded from <http://www.commsp.ee.ic.ac.uk/~mandic/EEG-256Hz.zip>).

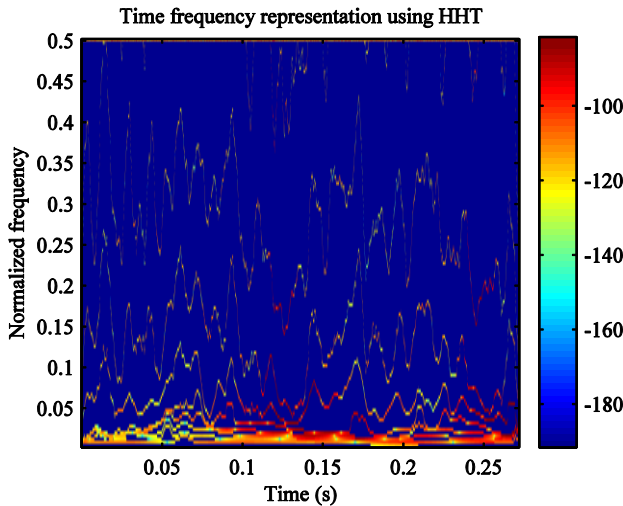
The data was recorded from 8 electrodes for 30 seconds and sampled at 256Hz. The labels for the recorded electrodes are provided in each mat file. The ground electrode was placed at position Cz, according to the 10-20 system. The electrode positions used were Fp1, Fp2, C3, C4, O1, O2, vEOG, hEOG and correspond respectively to the columns of the data matrix. For instance data(:, 1) gives the recordings corresponding to Fp1. This data set also contains EOG artifacts from blink of the eyes. In addition, the data set was sampled at 256 Hz and notch filtered at 50 Hz. It shows the real EEG signals, the EOG artifacts were present on all six EEG channels, while the artifacts are much stronger on the frontal lobe electrodes (Fp1, Fp2) and highly non-stationary (its amplitude differs between successive eye movements). Unlike the “artificially” contaminated EEG recordings, there is no ground truth “pure” EEG (pre-contaminated EEG).



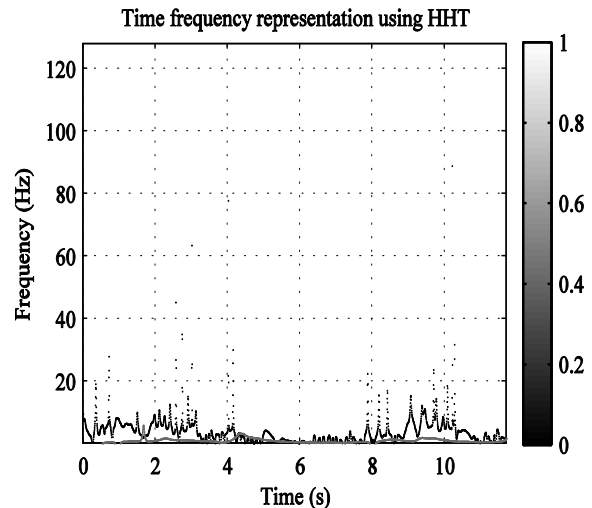
a) Time domain representation of raw EEG signal.



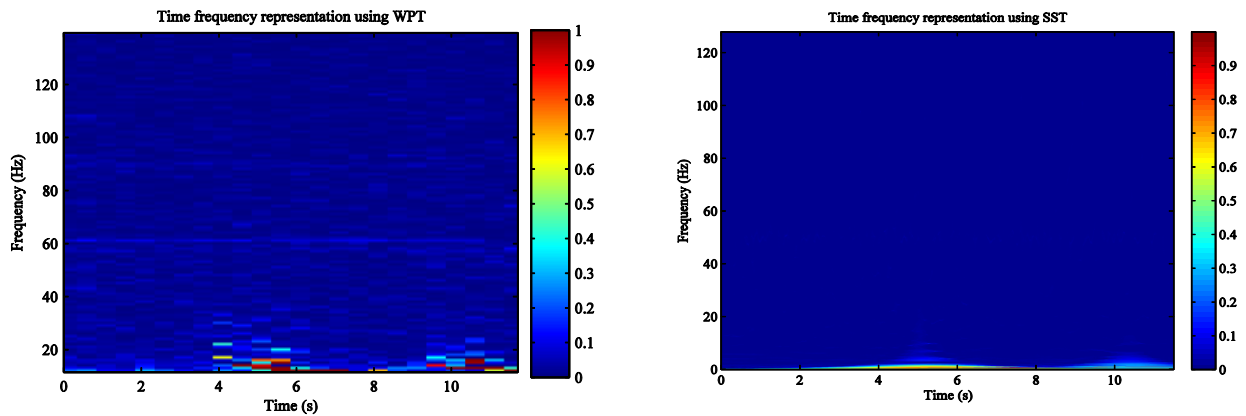
(b) Short Term Fourier Transform TFR for raw EEG signal.



(c) Hilbert Spectrum for raw EEG signal.



(d) TFR using raw EEG for HHT.



e) TFR using raw EEG for WPT.

f) Synchrosqueezing TF representation using raw EEG signal.

**Figure 2:** Time-frequency analysis of raw EEG signal. Upper plot shows the STFT and the lower plot the SST. The SST is able to delineate the spectral components some of them were missing in the STFT representation.

### Experimental results

In this section we apply the different time frequency representation method as like as SST, HHT, WT and SST to a real dataset. Figure 2. illustrated different time frequency representation method using raw EEG. From the TFR representation it is visualized the presence of artifact in EEG signal. Typically 0.5 to 100  $\mu\text{V}$  are the range of EEG signal and it lies between 0 to 50 Hz. Generally the ocular artifacts are occur in the range between 0 to 8 or 10 hz, and muscle artifacts present over 20 Hz. The spectral analysis of wide-sense stationary signals using the Fourier transform is well-established. For nonstationary signals, there exist local Fourier methods such as the short-time Fourier transforms (STFT). Because wavelets are localized in time and frequency, it is possible to use wavelet-based counterparts to the STFT for the time-frequency analysis of nonstationary signals. For example, it is possible to construct the scalogram based on the continuous wavelet transform (CWT). However, a potential drawback of using the CWT is that it is computationally expensive. The discrete wavelet transform (DWT) permits a time-frequency decomposition of the input signal, but the degree of frequency resolution in the DWT is typically considered too coarse for practical time-frequency analysis. As a compromise between the DWT- and CWT-based techniques, wavelet packets provide a computationally-efficient alternative with sufficient frequency resolution. a time-frequency analysis of EEG signal using wavelet packets is presented in this section.

In the STFT a Hamming window of length 256 and 50% overlap is used. The STFT is able to recognize the three components but with a poor resolution. In this present work, in WPT the frequency bands corresponding to five decomposition levels for Daubechies 4 (db4) mother wavelet which is chosen for this filter. Hilbert spectral analysis (HSA) is a method for examining each IMF's instantaneous frequency as functions of time. The final result is a frequency-time distribution of signal amplitude (or energy), designated as the Hilbert spectrum, which permits the identification of localized features. The individual IMFs are completely disjoint at any temporal position. The overall effect of IF of all IMFs is used in time-frequency (TF) representation of the time domain signal. For the SST a

bump wavelet is used with a ratio central frequency to bandwidth of 50. The discretization of the scales of CWT is 64. The SST produces a sharper time-frequency representation showing frequency components that were hidden in the STFT representation. From Figure 2, it is clear that the three methods (STFT, HHT and SST) clearly shown the EEG signal lies below the 20 Hz except WPT. The TFR of WPT shows the EEG signal lies all over the frequency 0 to 128 Hz but below 20 Hz it exhibits high energy. The high energy indicate the artifacts of EEG signal and obviously the artifact is ocular artifact which is found in similarity in Figure 2(a). From Figure 2 (a), it is seen that the raw EEG is mixed with EOG( eye blink) artifact. So, from above TFR, it is clear that the frequency localization of WPT is good for real EEG than SST but the time and frequency resolution is better of SST than other three methods namely STFT, HHT and WPT.

### 4. Conclusions

The examples given in this text lead to the conclusion that among the tested TFRs, SST are really valuable tools in analyzing the frequency content of EEG. The proposed method is computationally fast and is suitable for real-time BCI applications. To evaluate the TFR, a comparison with short-time Fourier transform (STFT), Hilbert–Huang transform (HHT) and wavelet packet transform (WPT) for both synthesized and real EEG data is performed in this paper. The popular methods of STFT has limitations in representing close frequencies and dealing with fast varying instantaneous frequencies and this is often the nature of EEG signals. HHT spectrum displays the distribution of signal energy in time frequency domain, and can help more clearly understand the distribution situation of low and high frequency part in signal. Moreover, HHT method structures basis function from the signal itself, directly reflects the characteristic of signal itself, and eliminate the generated virtual volume due to the mismatch of base function and signal in transformation process. so the obtained HHT spectrum can not only locate the position of frequency mutation point, but also can accurately distinguish two main frequency components. The time frequency resolution of wavelet transformation are both very good, for the smaller frequency component, the time point of frequency hopping

is described not enough accurately. The synchrosqueezing transform (SST) is a promising tool to track these resonant frequencies and provide a detailed time-frequency representation. The synchrosqueezing transform to EEG signals and also show its superior precision, in both time and frequency, at identifying the components of complicated oscillatory signals. Synchrosqueezing can be used to analyze spectrally and decompose a wide variety of signals with high precision in time and frequency.

## 5. Acknowledgement

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## 6. Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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