A New Ranking Approach for Solving Fuzzy Transportation Problems with Trapezoidal Fuzzy Numbers

M. K. Purushothkumar[1], Dr. M. Ananthanarayanan[2]

[1]Assistant Professor, Department of Mathematics, Dr. Ambedkar Govt. Arts College, Chennai - 600 039, Tamilnadu, India

[2]Associate professor, Department of Mathematics, A.M.Jain college, Chennai - 600 114, Tamilnadu, India

Abstract: ***In this paper, we consider the fuzzy transportation problem (FTP) where cost, availability and demand of the product are represented Trapezoidal fuzzy numbers. We develop fuzzy version of Vogel's Algorithm for finding fuzzy optimal solution of fuzzy transportation problem and Fuzzy transportation problem can be converted into a crisp valued Transportation problem using fuzzy ranking techniques.***

Keywords: Fuzzy set, fuzzy transportation problem, trapezoidal fuzzy number, fuzzy ranking techniques.

1. Introduction

A fuzzy transportation problem is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. Very first basic transportation problem was developed by Hitchcock [2].In 1965, Zadeh [5] introduced the notation of fuzziness that was reinforced by Bellman and Zadeh [6].Zimmermann [7,8] has discussed about the effective solutions of fuzzy set theory, Fuzzy linear programming with several objective functions. In 1981 R.R. Yager [9] procedure for ordering fuzzy subsets of the unit interval, S.H. Chen [3] (1985) Ranking fuzzy numbers with maximizing set and minimizing set,S. Chanas, D. Kuchta [4] (1996) solved Fuzzy integer transportation problem.P. Fortemps and M. Roubens [10] (1996) work on Ranking and defuzzification methods based on area compensation. S.Abbasbandy and T.Hajjari [1], A new approach for ranking of trapezoidal Fuzzy numbers (2009). A.Nagoor Gani andK. Abdul Razak [11] (2006) have solved fuzzy transportation problem in two stages.Then Pandian and Natrajan [12] (2010) has solved fuzzy transportation problem of trapezoidal numbers with algorithms and zero point method. A new method on ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations by Chen and Chen [13]was derived. N. RaviShankar and P. Phani Bushan Rao [14,15], have given a review on Ranking Fuzzy Numbers Using the Centroid Point Method.

In this paper we investigate more realistic problems, namely the transportation problem with fuzzy costs. Since the objective is to minimize the total cost or to maximize the total profit, subject to some fuzzy constraints, the objective function is also considered as a fuzzy number. First we transform the fuzzy quantities as the cost, supply and demands, into crisp quantities by centriod Ranking method, and then by classical algorithms, obtain the optimum solution of the problem. This method is a systematic procedure, easy to apply and can be utilized for the all the type of transportation problem.

This paper is organised as follows: In section 2 deals with some basic definitions ,In section 3 new ranking function are discussed, In section4 provides the mathematical formulation of fuzzy transportation problem, section 5,MODI methods is adopted to solve Fuzzy transportation problems. To illustrate the proposed method a numerical example is solved. Finally the paper ends with a conclusion.

2. Basic Definitions

2.1 Definition

Let A be a classical set and μ_A (x) be a function from A to [0,1]. A fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}}$ (x) is defined by

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)): x \in A \text{ and } \mu_{\tilde{A}}(x) \in [0,1]\}$$

2.2 Definition

A fuzzy set A defined on the set of real numbers R is said to be fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following characteristic

1) A is normal .Its means that there exists an $x \in R$ such that $\mu_A(x)=1$.
2) A is convex .Its means that for every $x_1, x_2 \in R$,
3) $\mu_A(\lambda x_1+(1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, \lambda \in [0,1]$.
4) μ_A is upper semi-continuous.

2.3 Definition

A real fuzzy number $\tilde{A} = (a_1,a_2,a_3,a_4)$ is a fuzzy subset from the real line R with the membership function $\mu_{\tilde{A}}$ (x) satisfying the following conditions:

(i) $\mu_{\tilde{A}}$ (x) is a continuous mapping from R to the closed interval [0,1].

(ii) $\mu_{\tilde{A}}(x) = 0$ for every $\in(-\infty, a_1]$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing and continuous on $[a_1,a_2]$

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(iv) $\mu_{\tilde{A}}(x)$ = 1 for every $\epsilon[a_2, a_3]$
(v) $\mu_{\tilde{A}}(x)$ is strictly decreasing and continuous on $[a_3,a_4]$
(vi) $\mu_{\tilde{A}}(x)$ = 0 for every $\in[a_4, \infty)$

2.4 Definition

A generalise fuzzy number $\tilde{A} = (a_1,a_2,a_3,a_4;\omega)$ is said to be a generalised trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{\omega(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ \omega & for\ a_2 \le x \le a_3 \\ \dfrac{\omega(a_4-x)}{(a_4-a_3)} & for\ a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

If $\omega = 1$, then $\tilde{A} = (a_1,a_2,a_3,a_4;1)$is a normalized trapezoidal fuzzy number and $\tilde{A}$ is a generalized or non normal trapezoidal fuzzy number if $0 < \omega < 1$.

In particular case if $a_2=a_3$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A}=(a_1,a_2,a_4;\omega)$. If $\omega=1$, then$\tilde{(A)}=(a_1,a_2,a_4;1)$ is normalized triangular fuzzy number and $\tilde{A}$ is a generalized triangular fuzzy number if $0 < \omega < 1$.

2.5 Definition

Let $A_1=(a_1,a_2,a_3,a_4;\omega_1)$ and $A_2=(b_1,b_2,b_3,b_4;\omega_2)$ be generalized trapezoidal fuzzy numbers then
(1) $A_1+A_2=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4;\min(\omega_1,\omega_2))$
(2) $A_1-A_2=(a_1-b_4,a_2-b_3,a_3-b_2,a_4+b_1;\min(\omega_1,\omega_2))$

3. New Proposed Ranking Method

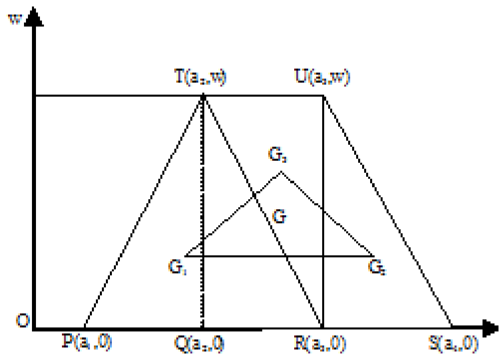

Figure 1

The centroid of a trapezoid is considered as the balancing point of the trapezoid (Figure 1). Divide the trapezoid into three triangles. These three triangles are PTR, RUS and TRU respectively. Let the centroids of the three triangles be G_1,G_2 and G_3 respectively. The centroid of centroids G_1,G_2 and G_3 is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point is balancing point of each individual triangle, and the centroid of these centroid points is a much more balancing point for a generalized trapezoidal fuzzy number. Therefore this point is a better reference point than the centroid point of the trapezoidal.

Consider a generalized trapezoidal fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4;\omega)$. The centroids of the three triangles are

$$G_1 = \left(\frac{a_1+a_2+a_3}{3}, \frac{\omega}{3}\right), G_2 = \left(\frac{2a_3+a_4}{3}, \frac{\omega}{3}\right) \text{ and } G_3 = \left(\frac{a_2+2a_3}{3}, \frac{2\omega}{3}\right)$$

respectively. Equation of the line $\overline{G_1 G_2}$ is $y=\omega/3$ and G_3 does not lie on the line$\overline{G_1 G_2}$.
Therefore G_1,G_2 and G_3 are non-collinear and they form triangle.

We define the centroid $G_{\tilde{A}}(x_0,y_0)$ of the triangle with vertices G_1,G_2 and G_3 of the generalized trapezoidal fuzzy number
$\tilde{A} = (a_1,a_2,a_3,a_4;\omega)$ as

$$G_{\tilde{A}}(x_0,y_0) = \left(\frac{a_1+2a_2+5a_3+a_4}{9}, \frac{4\omega}{9}\right)$$

The ranking function of the generalized trapezoidal fuzzy number $\tilde{A} = (a_1,a_2,a_3,a_4;\omega)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as

$$R(\tilde{A}) = x_0 y_0 = \frac{a_1+2a_2+5a_3+a_4}{9}\ X\ \frac{4\omega}{9}$$

4. Mathematical Formulation of Fuzzy Transformation Problem

The fuzzy transportation problems, in which a decision maker is uncertain about the precise value of transportation cost, availability and demand, can be formulated as follows

$$\text{minimize} \quad \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \quad i=1,2,3,\ldots,m.$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \quad j=1,2,3,\ldots,n.$$

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j, \quad i=1,2,3,\ldots,m,\ j=1,2,3,\ldots,n \quad \text{and} \quad \tilde{x}_{ij} \ge 0.$$

Where m = total number of sources
n = total number of destinations
$\tilde{a}_i$ = the fuzzy availability of the product at ith source
$\tilde{b}_j$ = the fuzzy demand of the product at jth destination
$\tilde{c}_{ij}$ = the fuzzy transportation cost for unit quantity of the product from ith source to jth destination
$\tilde{x}_{ij}$ = the fuzzy quantity of the product that should be transported from ith source to jth destination to minimize the total fuzzy transportation cost
$\sum_{i=1}^{m} \tilde{a}_i$ = total fuzzy availability of the product
$\sum_{j=1}^{n} \tilde{b}_j$ = total fuzzy demand of the product
$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$ = total fuzzy transportation cost

If $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem. This problem can be represented as follows:

Table 1

	1	…….	n	Supply
1	$\widetilde{c_{11}}$		$\widetilde{c_{1n}}$	$\widetilde{a_1}$
. . . .				
m	$\widetilde{c_{m1}}$		$\widetilde{c_{mn}}$	$\widetilde{a_m}$
Demand	$\widetilde{b_1}$	……	$\widetilde{b_n}$	

5. Numerical Example

A factory has three origins O_1,O_2,O_3 four destinations D_1,D_2,D_3,D_4 .The fuzzy transportation cost for unit quantity of the product from ith source to jth destination is $\tilde{c}_{ij}$ where

Table 2

Origins/ Destinations	*D_1*	*D_2*	*D_3*	*D_4*
O_1	(-4,0,4,16)	(-4,0,4,16)	(-4,0,4,16)	(-2,0,2,8)
O_2	(8,16,24,32)	(8,14,18,24)	(4,8,12,16)	(2,6,10,14)
O_3	(4,8,18,26)	(0,12,16,20)	(0,12,16,20)	(8,14,18,24)

Fuzzy supply of the product at sources are (0,4,8,12), (4,8,18,26), (4,8,12,16) and the fuzzy demand of the product at destinations are (2,6,10,14), (2,2,8,12), (2,6,10,14), (2,6,10,14) respectively.

The Fuzzy transportation problem is given by

Table 3

Origins/ destinations	D_1	D_2	D_3	D_4	Fuzzy Supply
O_1	(-4,0,4,16)	(-4,0,4,16)	(-4,0,4,16)	(-2,0,2,8)	(0,4,8,12)
O_2	(8,16,24,32)	(8,14,18,24)	(4,8,12,16)	(2,6,10,14)	(4,8,18,26)
O_3	(4,8,18,26)	(0,12,16,20)	(0,12,16,20)	(8,14,18,24)	(4,8,12,16)
Fuzzy demand	(2,6,10,14)	(2,2,8,12)	(2,6,10,14)	(2,6,10,14)	

In conformation to model the fuzzy transportation problem can be formulated in the following mathematical form

Min z=R(-4,0,4,16) x_{11} + R(-4,0,4,16) x_{12} + R(-4,0,4,16) x_{13} + R(-2,0,2,8) x_{14} + R(8,16,24,32) x_{21}+ R(8,14,18,24) x_{22} + R(4,8,12,16) x_{23} + R(2,6,10,14) x_{24} + R(4,8,18,26) x_{31} + R(0,12,16,20) x_{32} + R(0,12,16,20) x_{33} + R(8,14,18,24) x_{34}

For ranking the fuzzy numbers, we use centroid ranking method,

$$R(\tilde{A}) = \left(\frac{a_1 + 2a_2 + 5a_3 + a_4}{9}\right)\left(\frac{4w}{9}\right)$$

Now for ranking we take ω=1

$$R(-4,0,4,16) = \left(\frac{-4 + 2(0) + 5(4) + 16}{9}\right)\left(\frac{4}{9}\right) = 1.58$$

Proceeding similarly, the ranking indices for the cost are calculated

R(-4,0,4,16)=1.58 R(8,16,24,32)=9.48
R(2,6,10,14)=3.85
R(-4,0,4,16)=1.58 R(8,14,18,24)=7.41
R(4,8,18,26)=6.72
R(-2,0,2,8)=0.79 R(4,8,12,16)=4.74
R(0,12,16,20)=6.12
R(0,12,16,20)=6.12 R(8,14,18,24)=7.41

Rank of all supply
R(0,4,8,12)=2.96 R(4,8,18,26)=6.72 R(4,8,12,16)=4.74

Rank of all Demand
R(2,6,10,14)=3.85 R(2,2,8,12)=2.87 R(2,6,10,14)=3.85
R(2,6,10,14)=3.85

Now by using the centroid ranking method, we convert the given fuzzy problem into a crisp valued problem.

Table 4

Origins/ Destinations	*D_1*	*D_2*	*D_3*	*D_4*	*Supply*
O_1	1.58	1.58	1.58	0.79	2.96
O_2	9.48	7.41	4.74	3.85	6.72
O_3	6.72	6.12	6.12	7.41	4.74
Fuzzy demand	3.85	2.87	3.85	3.85	14.42

Using VAM procedure we obtain the initial solution as

Table 5

2.96			
		2.87	3.85
0.89	2.87	0.98	

Since the number of non negative allocations at independent positions is (m+n-1)=6,

We apply MODI method for optimal solution

Table 6

Origins/ Destination	*D_1*	*D_2*	*D_3*	*D_4*	*Supply*
O_1	$\boxed{2.96}$ 1.58	1.58	1.58	0.79	2.96
O_2	9.48	7.41	$\boxed{2.87}$ 4.74	$\boxed{3.85}$ 3.85	6.72
O_3	$\boxed{0.89}$ 6.72	$\boxed{2.87}$ 6.12	$\boxed{0.98}$ 6.12	7.41	4.74
Fuzzy demand	3.85	2.87	3.85	3.85	14.42

The crisp value of the fuzzy transportation problem is 62.6459.

6. Conclusion

In this paper, the transportation costs are considered as imprecise numbers by fuzzy numbers which are more realistic and general in nature .More over fuzzy transportation problem of trapezoidal numbers has been transformed into crsip transportation problem by centroid ranking method. Numerical examples show that by this method we can have the fuzzy optimal solution .This technique can also be used for solving other types of problems like, assignment problems and network flow problems.

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Authors Profile

M. K. Purushothkumar is working as Assistant Professor in Department of Mathematics, Dr. Ambedkar Government Arts College, Chennai-600 039, Tamilnadu, India.

Dr. M. Ananthanarayanan is working as associate professor in Department of Mathematics, A.M.Jain College, Meenambakkam, Chennai 600 114, Tamilnadu, India.