A New Ranking Approach for Solving Fuzzy Transportation Problems with Trapezoidal Fuzzy Numbers

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Abstract: In this paper, we consider the fuzzy transportation problem (FTP) where cost, availability and demand of the product are represented Trapezoidal fuzzy numbers. We develop fuzzy version of Vogel’s Algorithm for finding fuzzy optimal solution of fuzzy transportation problem and Fuzzy transportation problem can be converted into a crisp valued Transportation problem using fuzzy ranking techniques.

Keywords: Fuzzy set, fuzzy transportation problem, trapezoidal fuzzy number, fuzzy ranking techniques.

1. Introduction


In this paper we investigate more realistic problems, namely the transportation problem with fuzzy costs. Since the objective is to minimize the total cost or to maximize the total profit, subject to some fuzzy constraints, the objective function is also considered as a fuzzy number. First we transform the fuzzy quantities as the cost, supply and demands, into crisp quantities by centroid Ranking method, and then by classical algorithms, obtain the optimum solution of the problem. This method is a systematic procedure, easy to apply and can be utilized for all the type of transportation problem.

This paper is organised as follows: In section 2 deals with some basic definitions. In section 3 new ranking function are discussed. In section 4 provides the mathematical formulation of fuzzy transportation problem, section 5, MODI methods is adopted to solve Fuzzy transportation problems. To illustrate the proposed method a numerical example is solved. Finally the paper ends with a conclusion.

2. Basic Definitions

2.1 Definition

Let A be a classical set and μ_A (x) be a function from A to [0,1]. A fuzzy set A with membership function μ_A (x) is defined by

\[ A = \{ (x, μ_A(x)) : x ∈ A \text{ and } μ_A(x) ∈ [0,1] \} \]

2.2 Definition

A fuzzy set A defined on the set of real numbers R is said to be fuzzy number if its membership function μ_A : R → [0,1] has the following characteristic

1) A is normal. Its means that there exists an x ∈ R such that μ_A(x) = 1.
2) A is convex. Its means that for every x_1, x_2 ∈ R,
3) \[ μ_A(λx_1 + (1-λ)x_2) ≥ \min\{μ_A(x_1), μ_A(x_2)\}, λ ∈ [0,1]. \]
4) μ_A is upper semi-continuous.

2.3 Definition

A real fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4) \) is a fuzzy subset from the real line R with the membership function μ_{\tilde{A}} (x) satisfying the following conditions:

(i) μ_{\tilde{A}} (x) is a continuous mapping from R to the closed interval [0,1].
(ii) μ_{\tilde{A}} (x) = 0 for every x ∈ (−∞, a_1]
(iii) μ_{\tilde{A}} (x) is strictly increasing and continuous on [a_1, a_2]
(iv) $\mu_A(x) = 1$ for every $c[a_2, a_3]$
(v) $\mu_A(x)$ is strictly decreasing and continuous on $[a_3, a_4]$
(vi) $\mu_A(x) = 0$ for every $x \in [a_4, \infty)$

2.4 Definition

A generalise fuzzy number $A^\approx=(a_1, a_2, a_3, a_4 ; \omega)$ is said to be a generalised trapezoidal fuzzy number if its membership function is given by

$$
\mu_A(x) = \begin{cases} 
\frac{\omega(x - a_1)}{(a_2 - a_1)} & \text{for } a_1 \leq x \leq a_2 \\
\frac{\omega(a_3 - x)}{(a_4 - a_3)} & \text{for } a_2 \leq x \leq a_3 \\
\frac{\omega(x - a_3)}{(a_4 - a_3)} & \text{for } a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
$$

If $\omega = 1$, then $A^\approx=(a_1, a_2, a_3, a_4 ; 1)$ is a normalized trapezoidal fuzzy number and $A^\approx$ is a generalised or non normal trapezoidal fuzzy number if $0 < \omega < 1$.

In particular case if $a_2 = a_3$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $A^\approx=(a_1, a_2, a_3 ; \omega)$. If $\omega=1$, then $A=(a_1, a_2, a_3, 1)$ is a normalized triangular fuzzy number and $A$ is a generalised triangular fuzzy number if $0 < \omega < 1$.

2.5 Definition

Let $A_1=(a_1, a_2, a_3, a_4 ; \omega_1)$ and $A_2=(b_1, b_2, b_3, b_4 ; \omega_2)$ be generalised trapezoidal fuzzy numbers then

1. $A_1+A_2=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4;\min(\omega_1, \omega_2))$
2. $A_1-A_2=(a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4;\min(\omega_1, \omega_2))$

3. New Proposed Ranking Method

The centroid of a trapezoid is considered as the balancing point of the trapezoid (Figure 1). Divide the trapezoid into three triangles. These three triangles are PTR, RUS and TRU respectively. Let the centroids of the three triangles be $G_1, G_2$ and $G_3$ respectively. The centroid of centroids $G_1, G_2$ and $G_3$ is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point is balancing point of each individual triangle, and the centroid of these centroid points is a much more balancing point for a generalised trapezoidal fuzzy number.

Therefore this point is a better reference point than the centroid point of the trapezoidal.

Consider a general trapezoidal fuzzy number $A=(a_1, a_2, a_3, a_4;\omega)$. The centroids of the three triangles are

$$
\Gamma_1 = \left( \frac{a_1+2a_2+5a_3+a_4}{9}, \omega \right), \quad \Gamma_2 = \left( \frac{2a_3+a_4}{3}, \frac{2\omega}{3} \right), \quad \Gamma_3 = \left( \frac{a_2+2a_3}{3}, \frac{2\omega}{3} \right)
$$

The ranking function of the general trapezoidal fuzzy number $A=(a_1, a_2, a_3, a_4;\omega)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as

$$
R(A) = x_0y_0 = \frac{a_1+2a_2+5a_3+a_4}{9} X \frac{4\omega}{9}
$$

4. Mathematical Formulation of Fuzzy Transformation Problem

The fuzzy transportation problems, in which a decision maker is uncertain about the precise value of transportation cost, availability and demand, can be formulated as follows

minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \quad i = 1, 2, 3, ..., m.$

$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \quad j = 1, 2, 3, ..., n.$

$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j, \quad i = 1, 2, 3, ..., m, j = 1, 2, 3, ..., n$ and $\tilde{x}_{ij} \geq 0$.

Where $m =$ total number of sources
$n =$ total number of destinations
$\tilde{a}_i =$ the fuzzy availability of the product at $i$th source
$\tilde{b}_j =$ the fuzzy demand of the product at $j$th destination
$\tilde{c}_{ij} =$ the fuzzy transportation cost for unit quantity of the product from $i$th source to $j$th destination
$\tilde{x}_{ij} =$ the fuzzy quantity of the product that should be transported from $i$th source to $j$th destination to minimize the total fuzzy transportation cost

$$
\sum_{i=1}^{m} \tilde{a}_i = \text{total fuzzy availability of the product}
\sum_{j=1}^{n} \tilde{b}_j = \text{total fuzzy demand of the product}
\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} = \text{total fuzzy transportation cost}
$$
If \( \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j \) then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem. This problem can be presented as follows:

\[
R(i,j) = (\tilde{a}_i + 2\tilde{a}_j + 5\tilde{a}_3 + \tilde{a}_4) + \frac{4w}{9}
\]

5. Numerical Example
A factory has three origins \( O_1, O_2, O_3 \) four destinations \( D_1, D_2, D_3, D_4 \). The fuzzy transportation cost for unit quantity of the product from ith source to jth destination is \( \tilde{c}_{ij} \) where

\[
\tilde{c}_{ij} = (a_i + 2a_j + 5a_3 + a_4)\times \frac{4w}{9}
\]

Fuzzy supply of the product at sources are \((0,4,8,12), (4,8,12,16)\) and the fuzzy demand of the product at destinations are \((2,6,10,14), (2,2,8,12), (2,6,10,14), (2,6,10,14)\) respectively.

The Fuzzy transportation problem is given by

\[
\begin{align*}
\text{Supply} & \quad \tilde{a}_1 & \quad \tilde{a}_2 & \quad \tilde{a}_3 & \quad \tilde{a}_4 \\
\text{Demand} & \quad \tilde{b}_1 & \quad \tilde{b}_2 & \quad \tilde{b}_3 & \quad \tilde{b}_4 \\
D_1 & \quad (0,4,8,12) & \quad (0,4,8,12) & \quad (0,4,8,12) & \quad (0,4,8,12) \\
D_2 & \quad (8,12,16,20) & \quad (8,12,16,20) & \quad (8,12,16,20) & \quad (8,12,16,20) \\
D_3 & \quad (8,12,16,20) & \quad (8,12,16,20) & \quad (8,12,16,20) & \quad (8,12,16,20) \\
D_4 & \quad (8,12,16,20) & \quad (8,12,16,20) & \quad (8,12,16,20) & \quad (8,12,16,20) \\
\end{align*}
\]

In conformation to model the fuzzy transportation problem can be formulated in the following mathematical form

Min \( z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} + \sum_{j=1}^{n} \tilde{b}_j (\tilde{x}_{1j} + \tilde{x}_{2j} + \tilde{x}_{3j} + \tilde{x}_{4j}) \)

For ranking the fuzzy numbers, we use centroid ranking method,

\[
R(\tilde{a}) = \left( a_1 + 2a_j + 5a_3 + a_4 \right) \times \frac{4w}{9}
\]

Now for ranking we take \( \omega = 1 \)

\[
R(-4,0,4,16) = \left( -4 + 2(0) + 5(4) + 16 \right) \times \frac{4w}{9} = 1.58
\]

Proceeding similarly, the ranking indices for the cost are calculated

\[
\begin{align*}
R(0,4,8,12) &= 14.42 \\
R(4,8,12,16) &= 14.42 \\
R(2,6,10,14) &= 14.42 \\
R(4,8,18,26) &= 14.42 \\
R(4,8,12,16) &= 14.42 \\
R(0,12,16,20) &= 14.42 \\
R(8,14,18,24) &= 14.42
\end{align*}
\]

Since the number of non negative allocations at independent positions is \( m+n-1 = 6 \),

We apply MODI method for optimal solution

<table>
<thead>
<tr>
<th>Origins/ Destinations</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
<th>( D_4 )</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_1 )</td>
<td>2.96</td>
<td>1.58</td>
<td>1.58</td>
<td>0.79</td>
<td>2.96</td>
</tr>
<tr>
<td>( O_2 )</td>
<td>9.48</td>
<td>7.41</td>
<td>2.87</td>
<td>4.74</td>
<td>3.85</td>
</tr>
<tr>
<td>( O_3 )</td>
<td>6.72</td>
<td>6.12</td>
<td>7.41</td>
<td>4.74</td>
<td>3.85</td>
</tr>
</tbody>
</table>

The crisp value of the fuzzy transportation problem is 62.6459.

6. Conclusion
In this paper, the transportation costs are considered as imprecise numbers by fuzzy numbers which are more realistic and general in nature. More over fuzzy transportation problem of trapezoidal numbers has been transformed into crisp transportation problem by centroid ranking method. Numerical examples show that by this method we can have the fuzzy optimal solution. This technique can also be used for solving other types of problems.
problems like, assignment problems and network flow problems.

References


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