Stretch Curvature Tensor of h-Isotropic Non-Riemannian Finsler Manifold

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Abstract: ***Present paper is devoted to the study of stretch curvature tensor of h-isotropic non-Riemannian Finsler manifold. Section 1 is devoted to the study of h-curvature tensor. Section 2 deals to the study of stretch curvature tensor of h-isotropic Finsler manifold.***

Keywords: curvature tensor, scalar curvature, angular metric tensor, stretch curvature tensor, h-isotropic, Finsler manifold.

1. h-Curvature Tensor:

Let F^n be a Finsler space of n-dimension with the fundamental function $L(x,\dot{x})$ and $g_{ij}(x, \dot{x})$ be the fundamental tensor. The angular metric tensor h_{ij} is defined as

(1.1) $h_{ij} = g_{ij} - L^{-2}\dot{x}_i\dot{x}_j$

Wherein

(1.2) $\dot{x}_i = g_{ij}\dot{x}^j$.

A Finsler manifold of scalar curvature is given as follows

(1.3) $R_{ij} = K L^2 h_{ij}$

(1.4) $R_{ijk} = K(\dot{x}_j g_{ik} - \dot{x}_k g_{ij}) + (1/3)L^2(h_{ik}K_{/j} - h_{ij}K_{/k})$

Wherein (/) denotes $(\partial/\partial\dot{x}^i)$.

Contracting equation (1.4) by g^{jk}, we obtain

(1.5) $R_i = (1/3)L^2(h^j_i K_{/j} - h^k_i K_{/k})$.

We have [1]:

(1.6) $H_{hijk} = R_{ijk/h} - 2C_{ilh} R^l_{jk}$

Consequently yields

(1.7) $H_{hijk} = K(g_{hj}g_{ik} - g_{hk}g_{ij}) + B_{hijk} - B_{hikj}$

Wherein

(1.8) $B_{hijk} = (1/3)\{(L^2K_{/h/j} + 3K_{/h}\dot{x}_j)h_{ik} + (2\dot{x}_h h_{ik} - \dot{x}_k h_{hi} - \dot{x}_i h_{hk})K_{/j}\}$

The relation between the h-curvature tensor R_{hijk} of the Cartan connection C□ and the h-curvature tensor H_{hijk} of the Berwald connection B□ is as follows

(1.9) $R_{hijk} = H_{hijk} + C_{hil}R^l_{jk} - P_{hij,k} + P_{hik,j} - Q_{hijk}$

Wherein

(1.10) $Q_{hijk} = P_{hlj}P^l_{ik} - P_{hlk}P^l_{ij}$

Consequently, yields

(1.11) $H_{hijk} = H_{ihjk} + 2(R_{hijk} + Q_{hijk})$

and

(1.12) $H_{hijk} = 2(P_{hij,k} - P_{hik,j}) - H_{ihjk} - 2C^l_{hi}R_{ljk}$

By virtue of equations (1.7) and (1.11), we get

(1.13) $H_{hijk} = H_{ihjk} + (\lambda_{jk}-\lambda_{kj})[h_{hj}\{Kg_{ik} + (1/3)L^2K_{/i/k} + K_{/i}\dot{x}_k + K_{/k}\dot{x}_i + KL^{-2}\dot{x}^i\dot{x}^k\} + h_{ik}\{Kg_{hj} + (1/3)L^2K_{/h/j} + K_{/h}\dot{x}_j + K_{/j}\dot{x}_h + KL^{-2}\dot{x}^h\dot{x}^j\}]$

In view of equation (1.7), the equation (1.12) reduces in the form

(1.14) $H_{hijk} = (1/3)(\lambda_{jk}-\lambda_{kj})[h_{ik}(L^2K_{/h/j} + 3K_{/h}\dot{x}_j + K_{/j}\dot{x}_h) + h_{kh}(L^2K_{/i/j} + 3K_{/i}\dot{x}_j + K_{/j}\dot{x}_i) + h_{hi}(L^2K_{/k/j} + 3K_{/k}\dot{x}_j + K_{/j}\dot{x}_k) - H_{ihjk}$

Contracting equation (1.9) with g^{hi}, we obtain

(1.15) $R_{jk} = H_{jk} + C_l R^l_{jk} + P_{k,j} - P_{j,k} - Q_{jk}$.

In this regard, we have a theorem:

Theorem 1.1:

If the h-curvature tensor R_{hijk} of a Finsler manifold $F^n(n>2)$ of scalar curvature and skew-symmetric tensor Q_{hikj} with respect to j and k are equal, then the condition $H_{hijk} + C_{hil}R^l_{jk} - P_{hij,k} + P_{hik,j} = 0$ holds good.

Proof:

Since the tensor Q_{hijk} is skew-symmetric with respect to j and k then equation (1.9) reduces in the form

(1.16) $R_{hijk} = H_{hijk} + C_{hil}R^l_{jk} - P_{hij,k} + P_{hik,j} + Q_{hikj}$

Inserting $R_{hijk} = Q_{hikj}$ in the equation (1.16), we get

(1.17) $H_{hijk} + C_{hil}R^l_{jk} - P_{hij,k} + P_{hik,j} = 0$.

2. Stretch Curvature Tensor of h-Isotropic Finsler Manifold:

The Finsler manifold F^n is said to be h-isotropic, if the h-curvature tensor R_{hijk} holds the following condition

(2.1) $R_{hijk} = R(g_{hj}g_{ik} - g_{hk}g_{ij})$,

where R is constant.

The components P_{hijk} of hv-curvature tensor P^2 is defined as

(2.2) $P_{hijk} = (C_{ijk,h} - C_{hjk,i}) + C_{hjl}P^l_{ik} - C_{ijl}P^l_{hk}$

Wherein C_{ijk} and

(2.3) $P_{ijk} = P_{hijk}\dot{x}^h$

are components of the (h)hv-torsion tensor C and the (v)hv-torsion tensor P^1 respectively and (,) denotes $(\partial/\partial x^i)$.

h-isotropic non-Riemannian Finsler manifold is given as follows [4]:

(2.4) $R_{hijk} = 0$

(2.5) $C_{hil} R^l_{jk} = R_{hijk}$

(2.6) $P_{hij,k} - P_{hik,j} = C_{hil} R^l_{jk}$

In Cartan's theory, the stretch curvature tensor T_{hijk} is defined as

(2.7) $T_{hijk} = 2(P_{hij,k} - P_{hik,j})$

In this regard, we have theorems:

Theorem 2.1:

In a Finsler manifold $F^n(n>2)$, if the hv-curvature tensor P_{hijk} is symmetric with respect to first two indices then the stretch curvature tensor T_{hijk} is also symmetric with respect to first two indices.

Proof:
Interchanging the indices h and i in equation (2.7), we get

(2.8) $T_{ihjk} = 2(P_{ihj,k} - P_{ihk,j})$

If the hv-curvature tensor P_{hijk} is symmetric with respect to the indices h and i then the equation (2.8) becomes

(2.9) $T_{ihjk} = 2(P_{hij,k} - P_{hik,j})$

From equations (2.7) and (2.9), we obtain

(2.10) $T_{hijk} = T_{ihjk}$

Hence, the stretch curvature tensor T_{hijk} is symmetric with respect to first two indices.

Theorem 2.2:
In the h-isotropic non-Riemannian Finsler manifold F^n, the h-scalar curvature vanishes iff the stretch curvature tensor vanishes.

Proof:
By virtue of equations (2.1), (2.5), (2.6) and (2.7).

References

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T.S. Chauhan (Tarkeshwar Singh Chauhan) received Ph.D. and D.Sc. degrees in Mathematics from M.J.P.R.U., Bareilly in 1992 and 2008 respectively. He has been working in Maths deptt., Bareilly College, Bareilly since 1990 and now he is an Associate Professor. Under his guidance nearly 25 candidates have been awarded Ph.D. degree. Several papers and books are published in different branches in different publications under him.