International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Estimating the Shape Parameter for the Power Function Distribution through Shrinkage Technique

Abbas Najim Salman[1], Alaa M. Hamad[2]

[1]Professor, Department of Mathematics, Ibn-Al-Haitham College of Education, University of Baghdad

[2]Lecturer, Department of Mathematics, Ibn-Al-Haitham College of Education, University of Baghdad

Abstract: ***In this paper a recommended method based on shrinkage estimation technique has been implemented to estimate the shape parameter (α) of the power function distribution. The proposed approach will be involving the optimal region of prior estimation with new constant weight factor as well as including all the required statistical process. The statistical analyzing of the pattern behavior of the proposed estimator, equations of bias , mean squared error in addition to the relative efficiency expressions of the mentioned estimator were derived. The obtained outcomes were depending on the suggested data and including all of the analytical expressions.***

Keywords: Power Function Distribution, Single Stage Shrinkage Estimator, Prior Knowledge, Pretest region, Bias Ratio, Mean Squared Error, Relative Efficiency

1. Introduction

In several applications, such as applied mathematics, engineering, natural sciences, and economics, many statistical distributions have been developed and used as a subjective description in the assessment of reliability prediction. "The power function distribution can be considered as a statistical mythology that was used in a sample category for which there is only limited data and in cases that only the relationship between variables is known but data is scare";[1] ."many of researchers concluded that the Power function distribution is preferred over than a lot of statistical distributions such as Exponential, Lognormal and Weibull in the reason that it provides a better fit for failure data and gives more proper information about reliability and hazard rates"; [2] ,[3].Although Bayesian estimates method of parameters have been used by several statisticians and mathematical analysts ,but they have been introduced the estimate for scale parameter named as blue estimate and they have location parameter from Log-gamma distribution. Many of others were presented estimations of normal distribution parameters by using likelihood function; [4]. The power function distribution or it is also common to abbreviate as (PFD) can be considered as a flexible distribution that has the ability to model the different types of data. It is frequently used for the reliability process, life time parameter and income distribution data. "The (PFD) is favored over exponential, lognormal and Weibull because it provides a better fit for failure data and more appropriate information about reliability and hazard rates"; [1]- [5].

Similarly many probability models are also used to assess the pattern of the income distribution but these models are mathematically more complex to handle.

The probability density function of a random variable X which is follows the Power function distribution (PD (α, θ)) is given as below

$$f(x;\Theta,\alpha) = \alpha\Theta^{\alpha}x^{\alpha-1} \qquad 0 \le x \le \Theta^{-1} \quad (1)$$

Here, α refer to the shape parameter and θ refer to the scale parameter.

The following table characterizes some properties for power function distribution and graph of its probability density function.

Table (1): Properties of Power Function Distribution

Notation	P(α,θ)
parameters	α>0 ,θ>0
support	$0 \le x \le \Theta^{-1}$
PDF	$f(x;\Theta,\alpha) = \alpha\Theta^{\alpha}x^{\alpha-1}$
CDF	$F(x;\Theta,\alpha) = \Theta^{\alpha}x^{\alpha}$
Mean=E(x)	$\frac{\alpha}{(\alpha+1)\theta}$
E(x^2)	$\frac{\alpha}{(\alpha+2)\theta}$
Var(x)	$\frac{\alpha}{(\alpha+1)(\alpha+2)\theta^2}$

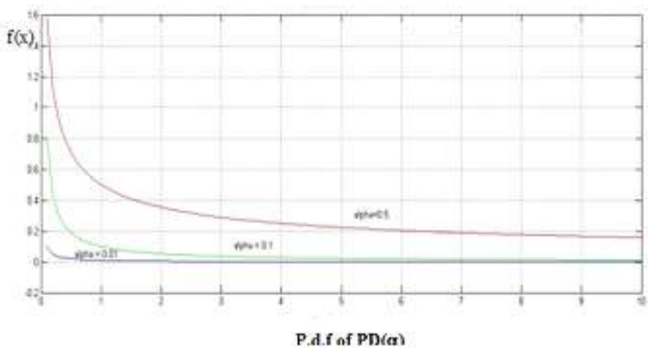

Figure 1: Probability Density Function of the Power Distribution

The problem for estimating the unknown shape parameter (α) of the Power function distribution with known scale parameter (θ=1) has been considered when some prior knowledge (α_0) concerning the real value (α) is on hand

www.ijsr.net

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depends on pretest shrinkage procedure, through suggested an optimal region R and new weight shrinkage factor $\psi(\cdot)$.
As well known in Thompson (1968), shrinkage estimator has the following form

$$\psi_1(\hat{\alpha})\hat{\alpha}+(1-\psi_1(\hat{\alpha}))\alpha_0 \qquad (2)$$

And the pretest shrinkage estimator (PT) is an estimator for test the hypothesesH_0:$\alpha = \alpha0$ against. HA: $\alpha \neq \alpha0$ with of level of significance (Δ).

If the null hypothesis (H0)holds, then we use the shrinkage estimator distinct in (2).

While, when H0 rejected, recall the PT through changed shrinkage weight factor $\psi2(\cdot)$; $0 \le \psi2(\cdot) \le 1$ as below

$$\psi_2(\hat{\alpha})\hat{\alpha}+(1-\psi_2(\hat{\alpha}))\alpha_0 \qquad (3)$$

Thus, the common pretest shrinkage estimator (PT) became as below:

$$\tilde{\alpha}=\begin{cases}\psi_1(\hat{\alpha})\hat{\alpha}+(1-\psi_1(\hat{\alpha}))\alpha_0 & ,\text{if } \hat{\alpha}\in R\\ \psi_2(\hat{\alpha})\hat{\alpha}+(1-\psi_2(\hat{\alpha}))\alpha_0 & ,\text{if } \hat{\alpha}\notin R\end{cases} \qquad (4)$$

Note that $\psi_i(\hat{\alpha}), 0\le\psi_i(\hat{\alpha})\le 1$, i = 1, 2 is a shrinkage weight factor identifying confidence of $\hat{\alpha}$ and $(1-\psi_i(\hat{\alpha}))$ agreeing the confidence of α_0 and $\psi_i(\hat{\alpha})$ may well be a function of $\hat{\alpha}$ or possibly will be fixed , while (R) suggested to be an optimal pretest region for receipt the prior knowledge by means of level of significance Δ.

Numerous of scholars have been considered pretest shrinkage estimator (PT) defined in (4), see for example; [1], [6], [7] and [8].The aim of this paper is to estimation the shape parameter (α) power function distribution with known scale parameter ($\theta = 1$) by proposed pretest shrinkage estimator (PT) which is defined in (5) through study the indicators ; Bias, Mean squared error and Relative Efficiency of this estimator besides display the numerical results for mentioned expressions in annexed tables. Also, study the performance of the consider estimator and make comparisons with the ML estimator as well as with some studies introduced by some authors.

2. Maximum Likelihood Estimator of α

The maximum likelihood estimator (MLE) of Power function distribution (α,θ) has been derived as the following procedure.

Let $x_1, x_2 \ldots x_n$ be random sample of size n follows PD (α,θ), the log- likelihood function be able as below:

$$L = \log L(\alpha,\theta)$$
$$= n\ln(\alpha) + n\,\alpha\ln(\theta) + (\alpha-1)\sum \ln x_i \qquad (5)$$

As we mentioned above, we assume that θ is known ($\theta = 1$). The partial derivative of L in equation (5) and equating the result to the zero

$$\frac{\partial L}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\ln x_i=0 \qquad (6)$$

We obtain the $\hat{\alpha}_{MLE}$ as below

$$\hat{\alpha}_{MLE}=-\frac{n}{\sum_{i=1}^{n}\ln(x_i)}=\frac{n\alpha}{y} \qquad (7)$$

Noted that, $y = -\alpha\sum_{i=1}^{n} \ln x_i \sim G(n,1)$

3. Pretest Shrinkage Estimator (PT) of α ($\tilde{\alpha}$)

Recall the pretest (PT) which is defined in (3) and put$\Psi_1(\hat{\alpha}) = 0$ and $\Psi_2(\hat{\alpha}) = k$ for estimate the shape parameter α of power function distribution.

The equation for Bias of $\tilde{\alpha}$ is

$$\text{Bias}(\tilde{\alpha}/\alpha,R)=E(\tilde{\alpha}-\alpha)$$
$$=\int_R(\alpha_0-\alpha)f(\hat{\alpha})\,d\hat{\alpha}+\int_{\bar{R}}(k(\hat{\alpha}-\alpha_0)+(\alpha_0-\alpha))f(\hat{\alpha})\,d\hat{\alpha}$$

Where, $\bar{R}$ refer to complement of R and f ($\hat{\alpha}$) refer to P.d.f. of $\hat{\alpha}$ with the following form

$$f(\hat{\alpha})=\begin{cases}\dfrac{\left[\dfrac{n}{\hat{\alpha}}\alpha\right]^{n+1}e^{\frac{-n\alpha}{\hat{\alpha}}}}{\Gamma(n)n\alpha} & \text{for } \hat{\alpha}>0,\alpha>0\\ 0 & \text{o.w.}\end{cases} \qquad (8)$$

We conclude,

$$\text{Bias}(\tilde{\alpha}/\alpha,R)=\alpha\{(\zeta-1)J_0(a^*,b^*)+\frac{k}{n-1}+(1-k)(\zeta-1)+nkJ_1(a^*,b^*)$$
$$-kJ_0(a^*,b^*)+(\zeta-1)(1-k)J_0(a^*,b^*)\} \qquad (9)$$

Where

$$J_\ell(a^*,b^*)=\int_{a^*}^{b^*}y^{-\ell}\frac{y^{n-1}e^{-y}}{\Gamma(n)}dy;\ \ell=0,1,2 \qquad (10)$$

Also, $\zeta=\dfrac{\alpha_0}{\alpha}, y=\dfrac{(n-1)\alpha}{\hat{\alpha}}, a^*=\zeta^{-1}\cdot a$

$$\text{And } b^*=\zeta^{-1}\cdot b \qquad (11)$$

Thus, the expression of bias ratio [B $(\cdot)$] of $\tilde{\alpha}$ is well-defined as below:-

$$B(\tilde{\alpha})=\frac{\text{Bias}(\tilde{\alpha}/\alpha,R)}{\alpha} \qquad (12)$$

The equation of Mean squared error (MSE) of $\tilde{\alpha}$ can be deriving as below:

$$\text{MSE}(\tilde{\alpha}/\alpha,R)=E(\tilde{\alpha}-\alpha)^2$$
$$=\alpha^2\Bigg\{k^2\frac{n+2}{(n-1)(n-2)}-\frac{2(\zeta-1)}{n-1}+(\zeta-1)^2$$
$$-n^2J_2(a^*,b^*)-2n\zeta J_1(a^*,b^*)+\zeta^2J_0(a^*,b^*) \qquad (13)$$
$$+2k(\zeta-1)\left[\frac{1}{n-1}-(\zeta-1)\right]+(\zeta-1)^2$$
$$-2kn(\zeta-1)\left[J_1(a^*,b^*)+\frac{1}{n}J_0(a^*,b^*)\right]\Bigg\}$$

The Relative Efficiency of $\tilde{\alpha}$ w.r.t. to the $\hat{\alpha}$ symbolized by R.Eff ($\tilde{\alpha}$/α, R) clear as

$$R.Eff(\tilde{\alpha}/\alpha, R) = \frac{MSE(\hat{\alpha})}{MSE(\tilde{\alpha}/\alpha, R)} \quad (14)$$

See for example; [7] and [8].

4. Discussions Numerical Results

The calculations of [B (·)] and [R.Eff (·)] expressions in equations (12) and (14) were used for the measured estimators $\tilde{\alpha}$. These calculations were achieved for the constants $\Delta = 0.05, 0.01$, n = 4, 8, 16, 20, $k = n.\Delta^4$ and ζ = 0.25(0.25), 2. Some of these calculations are displayed in tables (2), (3), (4) and (5) for some models of these constants.

Table 2: Shown the Ref of $\tilde{\alpha}$ w.r.t.(Δ=0.05), n and (ζ=0.25, 0.5, 0.75 and 1) when $k = n.\Delta^4$

			ζ			
Δ	n	Ref.	0.25	0.5	0.75	1
	4	Ref (.)	1.78	4.005	16.029	$5.865x10^5$
	8	Ref (.)	0.423	0 .952	3.809	$1.562x10^{14}$
	16	Ref (.)	0.153	0.344	1.377	$3.906x10^5$
0.05	20	Ref (.)	0.114	0.257	1.03	$6.4x10^{-9}$
	4	B(·)	0.749	0.5	0.25	$1.304x10^{-4}$
	8	B(·)	0.75	0.5	0.25	$1.142x10^{-8}$
	16	B(·)	0.749	0.499	0.249	$1.067x10^{-4}$
0.05	20	B(·)	0.75	0.5	0.25	$6.579x10^{-6}$

Table 3: Shown the Reef of $\tilde{\alpha}$ w.r.t.(Δ=0.05), n and (ζ=1.25, 1.5, 1.75 and 2) when $k = n.\Delta^4$

			ζ			
Δ	n	Ref.	1.25	1,5	1.75	2
	4	Ref (.)	15.996	4.001	1.779	1.1001
	8	Ref (.)	3.809	0.953	0.423	0.238
	16	Ref (.)	1.375	0.344	0.153	0.086
0.05	20	Ref (.)	0.062	0.25	0.562	0.089
	4	B(·)	0.25	0.5	0.749	0.999
	8	B(·)	0,25	0.5	0.15	1
	16	B(·)	0.75	0.499	0.749	0.999
0.05	20	B(·)	0.25	0.5	0.75	1

Table 4: Shown the Ref of $\tilde{\alpha}$ w.r.t.(Δ=0.01), n and (ζ=0.25, 0.5, 0.75 and 1) when $k = n.\Delta^4$

			ζ			
Δ	n	Ref.	0.25	0.5	0.75	1
	4	Ref (.)	1.778	4.005	16.029	$5.865x10^6$
	8	Ref (.)	0.423	0 .954	3.819	$562x10^6$
	16	Ref (.)	0.152	0.344	1.377	$3.906x10^5$
0.01	20	Ref (.)	0.114	0.258	1.034	$2.5x10^5$
	4	B(·)	0.75	-0.5	-0.25	$11.304x10^{-4}$
	8	B(·)	-0.75	0.499	-0.25	$1.143x10^{-4}$
	16	B(·)	-0.75	0.499	-0.249	$1.0.67x10^{-4}$
0.01	20	B(·)	0.75	0.499	-0.249	$1.053x10^{-4}$

Table 5: Shown the Ref of $\tilde{\alpha}$ w.r.t.(Δ=0.01), n and (ζ=1.25, 1.5, 1.75 and 2)when $k = n.\Delta^4$

			ζ			
Δ	n	Ref.	1.25	1,5	1.75	2
	4	Ref (.)	15.996	4.001	1.779	1.1001
	8	Ref (.)	3.812	0.953	0.424	0.238
0.01	16	Ref (.)	1.375	0.344	0.153	0.086
	20	Ref (.)	1.032	0.258	0.115	0.065
	4	B(·)	0.25	0.5	0.749	0.999
	8	B(·)	0.25	0.5	0.749	0.999
	16	B(·)	0.25	0.499	0.749	0.999
0.01	20	B(·)	0.25	0.499	0.749	0.998

The remark mentioned in the tables leads to the following results:

i. The [R.Eff (·)] of $\tilde{\alpha}$ are adversely proportional with small value ofΔ, i.e. Δ = 0.01 produce maximum efficiency.

ii. The [R.Eff (·)] of $\tilde{\alpha}$ are increasing function with increasing value of k.

iii. [R.Eff (·)] of $\tilde{\alpha}$ determine maximum value when $\alpha=\alpha_0(\zeta=1)$, for all k, n, Δ, and decreasing else ($\zeta\neq1$).

iv. [B(·)] of $\tilde{\alpha}$ increasing when ζ increases.

v. [B(·)] of $\tilde{\alpha}$ are practically small when $\alpha = \alpha_0$ and increases otherwise for all n and Δ .

vi. [R.Eff (·)] of $\tilde{\alpha}$ decreasing function with increases value of k and n, for eachΔ,ζ.

vii. The Effective Interval [the value of ζ that makes R.Eff.(·) greater than one] using the estimator $\tilde{\alpha}$ is [0.5,1.5]. Here the pretest criterion is very important for guarantee that prior information is very closely to the actual value and prevent it far away from it, which get optimal effect of the considered estimator to obtain high efficiency.

viii. An estimator $\tilde{\alpha}$ do better that the MLE exclusively when $\alpha\approx\alpha_0$, which is given the effective of $\tilde{\alpha}$ when given significant weight of α_0. And the augmentation of effectiveness may be reach to ten times.

ix. An estimator $\tilde{\alpha}$ has small MSE relative to some estimators presented by authors, see for instances [1].

5. Conclusions

After the discussions overhead, it's clearly that when using prior knowledge improved the MLE. It can be distinguished that if the prior knowledge α_0 is very close to the true value of the α (i.e.; ζ=1), the estimator $\tilde{\alpha}$ accomplished better than MLE. If one has no assurance of α_0, then proposed pretest shrinkage estimators (PT) will not recommended. We can carefully use the planned estimator for small n at standard Δ and reasonable value of $\psi_i(\hat{\alpha})$.

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