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Applications of Mahgoub Transform to Mechanics, Electrical Circuit Problems

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Abstract: ***This paper sheds light to solve the ordinary differential equations in mechanics and electrical circuit problems with initial conditions by using a new integral transform Mahgoub transform.***

Keywords: Mahgoub transform, Differential Equations

1. Introduction

The linear differential equations with constant coefficients find their most important applications in the study of electrical, mechanical and other linear systems. In fact, such equations play a dominant role in unifying the theory of electrical and mechanical oscillatory systems. Typically, Fourier, Laplace [5], Elzaki [1] , Aboodh [3] and Sumudu transforms [6] are the convenient mathematical tools for solving differential equations. In this paper, we are using a new integral transform Mahgoub transform [2] to solve mechanics and electrical circuit problems. Like Laplace transform, this Mahgoub transform is also suitable to obtain the solution of linear non homogeneous ordinary differential equations with constant coefficients. Mahgoub transform to facilitate the process of solving ordinary and partial differential equations in the time domain. Mahgoub transform is derived from the classical Fourier integral and is widely used in applied mathematics and engineering fields. This transform has deeper connection with Laplace, EL-zaki and Aboodh transforms. Based on the mathematical simplicity of this transform and its fundamental properties, we have to solve ordinary differential equations in mechanics and electrical circuit problems. Tarig M. Elzaki et.al [4] solve the problems of mechanics, electrical circuits, and beams problems using Elzaki transform.

2. Mahgoub Transform

The Mahgoub transform is defined for the function of exponential order. We consider functions in the set A defined by

$$A = \left\{ f(t) : \exists\, M,\ k_1, k_2 > 0,\ \left| f(t) < M\, e^{\frac{|t|}{k_j}} \right| \right\}$$

where the constant M must be finite number, k_1 , k_2 may be finite or infinite.

The Mahgoub transform denoted by the operator M(.) defined by the integral equations

$$\boldsymbol{M[f(t)] = H(v) = v \int_0^\infty f(t) e^{-vt} dt,\ t \geq 0, k_1 \leq v \leq k_2} \quad (1)$$

2.1 Some Standard Functions

For any function $f(t)$, we assume that the integral equation (1) exist and some standard functions are listed in Table – 1.

Table 1: Standard Functions

$f(t)$	1	t	t^2	t^3	t^n	e^{at}	e^{-at}	$sinat$	$cosat$
$M[f(t)]$	1	$\frac{1}{v}$	$\frac{2}{v^2}$	$\frac{3!}{v^3}$	$\frac{n!}{v^n}$	$\frac{v}{v-a}$	$\frac{v}{v+a}$	$\frac{av}{v^2+a^2}$	$\frac{v^2}{v^2+a^2}$

2.2 Transform of Derivatives

Let M [(t)] be the Mahgoub transform. Then
(i) $M\,[\,f'(t)] \;=\; v\,H(v) - v f(0)$
(ii) $M\,[\,f''(t)\,] \;=\; v^2\, H(v) - v\, f'(0) - v^2\, f(0)$
(iii) $M\left[\,f^{(n)}\,(t)\right] = \; v^{(n)}\, H(v) - \sum_{k=0}^{n-1} v^{n-k}\, f^{(k)}\,(0)$

3. Applications to Mechanics

3.1 Example

A particle P of mass 2 grams moves on the X- axis and is attracted towards origin O with a force numerically equal to 8X. If it is initially at rest at X = 10, find its position at any subsequent time assuming
a) No other force acts
b) A damping force numerically equal to 8 times the instantaneous velocity acts.

Solution
(a) From Newton's law, the equation of motion of the particle is

$$2\,\frac{d^2X}{dt^2} = -8X \quad \text{(or)} \quad \frac{d^2X}{dt^2} + 4X = 0 \qquad (2)$$

with the initial conditions X(0) = 10 and $X'(0) = 0$

Taking the Mahgoub transform of both sides of (2), we have

$$M\,[\,X''\,] + 4\,M\,[\,X\,] = 0$$

Using Mahgoub transform of derivatives, we get

$$v^2\,M\,[\,X\,] - v\,X'(0) - v^2\,X(0) + 4\,M\,[\,X\,] = 0$$

Apply the initial conditions, then

$$v^2\,M\,[\,X\,] - 10\,v^2 + 4\,M\,[\,X\,] = 0$$

$$M\,[\,X\,]\,\{v^2 + 4\} = 10\,v^2$$

$$M\,[\,X\,] = 10\left(\frac{v^2}{v^2+4}\right)$$

Take inverse Mahgoub transform, we get

$$\boldsymbol{X} = 10\,M^{-1}\left(\frac{v^2}{v^2+4}\right) = \boldsymbol{10\,cos2t}$$

(b) In this case, the equation of motion of particle is

$$2\,\frac{d^2X}{dt^2} = -8\,X - 8\,\frac{dX}{dt} \quad \text{(or)}$$

$$\frac{d^2X}{dt^2} + 4\,\frac{dX}{dt} + 4X = 0 \qquad (3)$$

with initial conditions $X(0) = 10$ and $X'(0) = 0$

Taking the Mahgoub transform of both sides of (3) , we have

$$M\,[X''] + 4\,M\,[X'] + 4\,M\,[X] = 0$$

Using Mahgoub transform of derivatives, then

$$\{v^2\,M[X] - vX'(0) - v^2X(0)\} + 4\,\{v\,M[X] - vX(0)\} + 4\,M[X] = 0$$

Apply the initial conditions, then

$$M\,[X] = \frac{10\,v^2 + 40\,v}{(v+2)^2} = \frac{10\,v}{v+2} + \frac{20\,v}{(v+2)^2}$$

Now take the inverse Mahgoub transform, then we get

$$X = 10\,M^{-1}\left(\frac{v}{v+2}\right) + 20\,M^{-1}\left(\frac{v}{(v+2)^2}\right)$$

$$\Rightarrow \quad \boldsymbol{X = 10\,e^{-2t} + 20\,t\,e^{-2t}}$$

4. Applications to Electrical Circuits

The Mahgoub transform can also be used to determine the charge on the capacitors and currents as functions of time.

4.1 Example (1)

An alternating *e.m.f.* $E \sin \omega t$ is applied to an inductance L and a capacitance C in series. Find the current in the circuit.

Solution

The differential equation for the determination of the current I in the circuit is given as

$$L\,\frac{dI}{dt} + \frac{Q}{C} = E \sin \omega t \qquad [\text{ since R = 0 }] \qquad (4)$$

where $I = \frac{dQ}{dt}$ (5)

Also at $t = 0,\ I = 0 = Q$

Taking Mahgoub transform of both sides of (4) and (5), we have

$$(4) \Rightarrow M\,[I'] + n^2\,M\,[Q] = \frac{E}{L}\,M\,[\sin\omega t]\ ,$$

where $\frac{1}{LC} = n^2$

Using the Mahgoub transform of derivatives, then

$$\{\,v\,M[I] - v\,I(0)\,\} + n^2\,M\,[Q] = \frac{E}{L}\left(\frac{\omega v}{v^2 + \omega^2}\right)$$

Apply the initial condition, then

$$v\,M\,[I] + n^2\,M\,[Q] = \frac{E}{L}\left(\frac{\omega v}{v^2 + \omega^2}\right) \qquad (6)$$

$$(5) \Rightarrow \quad M\,[I] = M\,[Q'] = v\,M\,[Q] - v\,Q(0)$$

$$M\,[I] = v\,M\,[Q] \qquad (7)$$

From (6) and (7), we get

$$M\,[I] = v^2\,\frac{E\omega}{L}\left(\frac{1}{(v^2+\omega^2)\,(v^2+n^2)}\right)$$

Take inverse Mahgoub transform, we get

$$I = \frac{E\,\omega}{L}\,v^2\,M^{-1}\left(\frac{1}{(v^2+\omega^2)\,(v^2+n^2)}\right)$$

$$\boldsymbol{I = \frac{E\,\omega}{L\,(n^2-\omega^2)}\,\{\cos\omega t - \cos nt\,\}}$$

4.2 Example (2)

Solve $L\,\frac{dx}{dt} + Rx = E\,e^{-at}$, given $x(0) = 0$

Solution:

Given equation is $\frac{dx}{dt} + \frac{R}{L}\,x = \frac{E}{L}\,e^{-at}$ (8)

Take Mahgoub transform of both sides of (8), we get

$$M\,[x'] + \frac{R}{L}\,M\,[x] = \frac{E}{L}\,M\,[e^{-at}]$$

$$v\,M\,[x] - v\,x(0) + \frac{R}{L}\,M\,[x] = \frac{E}{L}\left(\frac{v}{v+a}\right)$$

$$M\,[x] = \frac{E\,v}{(v+a)\,(vL+R)}$$

Take inverse Mahgoub transform, we get

$$x = M^{-1}\left(\frac{E\,v}{(v+a)\,(vL+R)}\right)$$

$$\boldsymbol{x = \frac{E}{R-La}\left[e^{-at} - e^{-\frac{R}{L}t}\right]}$$

5. Conclusion

In this paper we are solving ordinary differential equations in mechanics and electrical circuit problems by using a new integral transform Mahgoub transform. The results are verified.

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