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Non Linear Radiation and Dissipation Effects on Natural Convection Flow of Viscoelastic Fluids between Vertical Plates Filled with Forchiemer-Darcy Medium

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Abstract: ***A fourth order accurate finite difference method is introduced to solve the governing equations of natural convection of viscoelastic fluid flow and heat transfer under the influences of and nonlinear radiation and dissipation. The fluid flows through non-Darcy porous medium which lies between two heated vertical plates that are kept at constant, but different, temperatures. The coupled nonlinear differential equations are linearized and iterations are used to approximate that linearized terms. The finite difference method transforms the coupled linearized differential (momentum and energy) equations to a linear system of algebraic equations. Some comparisons are made to study the convergence and stability of the present results. Effects of parameters of fluid and heat on the velocity field, temperature, skin friction factor and Nusselt number are illustrated and discussed. The present results and their comparisons with available results are listed and shown in tables. The present results show that the numerical solution is of excellent agreement with previous analytical and numerical solution.***

Keywords: Nonlinear, radiation, Natural convection, viscoelastic fluid, non-Darcy medium, skin friction, heat transfer, Nusselt number, fourth order schemes, finite difference method

1. Introduction

Numerical methods with high accuracy are very important tools to solve highly non-linear differential equations. Iterative techniques are required to solve the linearized differential equations to achieve an appropriate accuracy. The finite difference method (FDM) is widely used to solve the linear and non-linear differential equations because of simplicity of this method. The natural convection of non-Newtonian fluids in porous medium has many engineering applications such as heat exchangers, fiber insulation, cooling of electronic equipments, nuclear reactors, solar devices, in polymer processing industries, food industries, and petroleum reservoirs.

The natural convection of non-Newtonian fluids has been studied by many authors [1-11]. Rajagopal and Na [2] introduced a numerical solution for natural convection flow of Rivlin-Ericksen fluid and heat transfer between parallel plates. They studied and computed the skin friction and Nusselt number. Zibabakhsh and Domairy [3] used the homotopy analysis method for solving the natural convection flow of a non-Newtonian fluid between two vertical flat plates. Kargar, and Akbarzade [6] used the homotopy perturbation method (HPM) for the study of natural convection flow of a non-Newtonian fluid between two vertical flat plates. Rashidi et al. [7] used the differential transformation method (*DTM*) to solve the governing equations of natural convection flow of third grade non-Newtonian fluids. Murar [9] studied the natural convection flow in a vertical channel in the presence of non-linear radiation and viscous dissipation. He used the finite difference method (*FDM*) to solve the governing coupled equations. Siddiqa et al. [10] studied the natural convection flow of a two-phase dusty non-Newtonian power law fluid along a vertical surface. The continuity, momentum and energy equations are solved numerically with the aid of implicit finite difference method (*FDM*). They studied and computed the skin friction and Nusselt number. The natural convection flow in non-Darcy porous media past a vertical surface has been studied by Khani et al. [4]. They presented an analytic solution of governing equations of third grade viscoelastic fluid with Darcy-Forchheimer model. Jyoti [11] used the homotopy analysis method (*HAM*) to study the third grade fluid with natural heat convection between two vertical plates.

The nonlinear radiation effect on Newtonian and non-Newtonian fluids has been studied [12-13]. Mushtaq et al. [12] introduced a numerical of non-linear radiation heat transfer for the flow of an electrically conducting second grade fluid. Shooting method with fourth and fifth Runge-Kutta integration has been used to solve the governing momentum and energy equations. Ahmed et al. [13] introduced a finite element investigation of the flow of a Newtonian fluid in dilating and squeezing porous channel under the influence of non-linear thermal radiation.

The aim of present work is to study and compute the effects of nonlinear radiation and Forchiemer-Darcy resistance force on natural convection of viscoelastic (Rivlin-Ericksen) fluid and heat transfer between vertical plates. Fourth order accurate finite difference schemes are used to solve the coupled non-linear differential (momentum and energy) equations. Linearization technique is applied to transform the non-linear terms linearized ones. Iterations are used up required accuracy. An error analysis is made to achieve accuracy, convergence and stability of present results and their agreement with available previous works. Skin friction and Nusselt number are computed and tabulated.

2. Formulation of the Problem

Consider a non-Newtonian fluid flows in a non-Darcy porous medium between two vertically parallel plates as shown in Fig. 1. The two stationary plates are kept at constant but different temperatures T_1 (for left plate) and T_2 (for right plate) with $T_1 > T_2$. The fluid particles, frequently, rise near left plate but they fall near right plate due to their difference in temperatures [2]. The flow is steady and laminar and viscous dissipation and radiation effects are taken into consideration.

According to above assumptions the governing (momentum and energy) equations are written, respectively, as [2, 4, 14].

$$\mu\frac{d^2V}{dX^2}+6\beta_3\left(\frac{dV}{dX}\right)^2\frac{d^2V}{dX^2}+\rho_0\,\gamma(T-T_m)\,g=\frac{\mu}{k_p}V+\frac{\rho_0 B}{k_p}V^2 \quad (1)$$

$$k\frac{d^2T}{dX^2}+2\beta_3\left(\frac{dV}{dX}\right)^4+\mu\left(\frac{dV}{dX}\right)^2=\frac{dq_r}{dX} \quad (2)$$

The boundary conditions are shown in Fig. 1 and they are written as

$$V(-h)=V(h)=0\,,T(-h)=T_1 \text{ and } T(h)=T_2 \quad (3)$$

The radiative heat flux is approximated using Rosseland approximation [11] as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{dT^4}{dX} \quad (4)$$

To introduce a general solution for any case of dimensions and scales, the following quantities are chosen [2, 4, 6].

$$x=\frac{X}{h},\ v=\frac{V}{V_0},\ \theta=\frac{T-T_m}{T_1-T_2},\ T_r=\frac{T_1}{T_2}\ E_c=\frac{V_0^2}{c(T_1-T_2)},$$

$$P_r=\frac{\mu\,c}{k},\ B_r=E_cP_r,\ \delta=\frac{\beta_3 V_0^2}{\mu\,h^2},\ M=\frac{h^2}{k_p},$$

$$F_s=\frac{\rho_0 BV_0 h^2}{\mu\,k_p},\ R_e=\frac{\rho_0\,V_0\,h}{\mu},\ R_d=\frac{k\,k^*}{4\sigma^*T_1^3},$$

$$V_0=\frac{\rho_0\,h^2(T_1-T_2)}{\gamma\,g},\text{ where,}$$

$$T_m=\frac{T_1+T_2}{2} \quad (5)$$

Under the above assumptions (eqns. 4 and 5) and quantities, the dimensionless forms of governing equations (1 and 2) with boundary conditions (3) are rewritten as

$$\left(1+6\,\delta\left(\frac{dv}{dx}\right)^2\right)\frac{d^2v}{dx^2}-M\,v-F_s\,v^2+\theta=0 \quad (6)$$

$$\left(1+\frac{4}{3R_d}\left\{(T_r-1)\theta+\frac{T_r+1}{2}\right\}^3\right)\frac{d^2\theta}{dx^2}+B_r\left(1+2\delta\left(\frac{d\,v}{dx}\right)^2\right)\left(\frac{dv}{dx}\right)^2+\frac{4(T_r-1)}{R_d}\left\{(T_r-1)\theta+\frac{T_r+1}{2}\right\}^2\left(\frac{d\theta}{dx}\right)^2=0 \quad (7)$$

$$v(-1)=v(1)=0\,,\ \theta(-1)=-\theta(1)=1/2 \quad (8)$$

3. Numerical Solution

The system of coupled non-linear ordinary differential equations (6, 7), with boundary conditions (8) are solved for the flow velocity and temperature using the finite difference method (*FDM*). The following linearized form should be applied because of nonlinearity in this system,

$$\left(1+6\delta\left(\frac{d\bar v}{dx}\right)^2\right)\frac{d^2v}{dx^2}-\left(M+F_s\,\bar v\right)v+\theta=0 \quad (9)$$

$$\left[1+\frac{4}{3R_d}\left((T_r-1)\bar\theta+\frac{T_r+1}{2}\right)^3\right]\frac{d^2\theta}{dx^2}+B_r\left(\frac{d\,\bar v}{dx}+2\delta\left(\frac{d\,\bar v}{dx}\right)^3\right)\frac{dv}{dx}+\frac{4(T_r-1)}{R_d}\left[\left((T_r-1)\bar\theta+\frac{T_r+1}{2}\right)^2\frac{d\bar\theta}{dx}\right]\frac{d\theta}{dx}=0 \quad (10)$$

where, bar notation refers to the iterated terms which transform the system (6, 7) to a linearized one.

The finite domain of solution (-1 < x < 1) is divided into m-subintervals such that the mesh size is $\Delta=2/m$, with counter i=1, 2, 3, …, m+1. The linearized system of coupled non-linear ordinary differential equations (9 and 10) is transformed to system algebraic equations using the fourth order difference schemes. The following fourth order schemes are obtained by Taylor's expansions of the variable $f(x)$ about point $x_i=(i-1)\Delta-1$.

$$\frac{df_i}{dx}=\frac{f_{i-2}-8f_{i-1}+8f_{i+1}-f_{i+2}}{12\Delta}+O(\Delta^4) \quad (11)$$

$$\frac{d^2f_i}{dx^2}=\frac{-f_{i-2}+16f_{i-1}-30f_i+16f_{i+1}-f_{i+2}}{12\Delta^2}+O(\Delta^4) \quad (12)$$

$$\frac{df_2}{dx}=\frac{-3f_1-10f_2+18f_3-6f_4+f_5}{12\Delta}+O(\Delta^4) \quad (13)$$

$$\frac{d^2f_2}{dx^2}=\frac{10f_1-15f_2-4f_3+14f_4-6f_5+f_6}{12\Delta^2}+O(\Delta^4) \quad (14)$$

$$\frac{df_{n-1}}{dx}=\frac{-f_{n-4}+6f_{n-3}-18f_{n-2}+10f_{n-1}+3f_n}{12\Delta}+O(\Delta^4) \quad (15)$$

$$\frac{d^2f_{n-1}}{dx^2}=\frac{f_{n-5}-6f_{n-4}+14f_{n-3}-4f_{n-2}+15f_{n-1}+10f_n}{12\Delta^2}+O(\Delta^4) \quad (16)$$

The skin friction factor and Nusselt number factor are two important fluid flow and heat transfer parameters because of their very importance in the engineering applications, since, they can be used to improve the shape and efficiency of many equipments in aerodynamics. These quantities are computed after solution the governing equations.

The skin friction factor at left plate is defined as [15]

$$\hat{C}_{fL} = \frac{2\,\tau_w}{\rho_0 V_0^2} \tag{17}$$

Nusselt number is defined as the ratio of the convective conduction to the pure molecular thermal conductance [16]. Thus the Nusselt number at left plate may be written as

$$N_{uL} = \frac{4h}{k(T_1 - T_m)}\left[k + \frac{16\sigma^* T_1^3}{3k^*}\right]\left(-\frac{\partial T}{\partial X}\right)\Bigg|_{X=-h} \tag{18}$$

The dimensionless form of these factors are written as

$$C_{fL} = \frac{R_e \hat{C}_{fL}}{2} = \left(\frac{dv}{dx}\right)_{x=-1} \tag{19}$$

$$N_{uL} = 4\left(1 + \frac{4\theta_r^3}{3R_d}\right)\left[\frac{-\partial\theta/\partial x}{\theta}\right]_{x=-1} \tag{20}$$

Fourth order difference schemes should be applied on equations (19 and 20) to minimize round off errors in computations. These schemes can be deduced by Taylor's expansion of independent variables (v and θ) about x=-1. Thus the dimensionless skin friction factor and Nusselt number are discretized as

$$C_{fL} = \frac{-25v_1 + 48v_2 - 36v_3 + 16v_4 - 3v_5}{12\Delta} + o(\Delta^4) \tag{21}$$

$$N_{uL} = \frac{-25\theta_1 + 48\theta_2 - 36\theta_3 + 16\theta_4 - 3\theta_5}{12\Delta} + o(\Delta^4) \tag{22}$$

4. Error Analysis

The linearized terms in governing equations need iterations to achieve convergence of the present work. Thus, a good initial guess is required to reach, fast and accurate results. The trend of previous works is used as an initial guess for linearized terms. For number of subintervals m, we find that, the fourth order truncation error of the solution is $O(2/m)^4$. Thus, the *FDM* is a good method to verify the convergence and stability of the analytical and experimental solutions. It is observed that, (5 to 80) iterations are required to achieve (10^{-8} to 10^{-12}) round off error such that number of subintervals ($20 \le m \le 2000$).

Tables (1 and 2) illustrate convergence of present solution depending on the influences of v and θ by number of sub-intervals (m=20, 200 and 2000) which give orders of truncation error (Δ^4=10^{-4}, 10^{-8}, 10^{-12}), respectively. Relatively small and large fluid and heat parameters are used (from 1 to 1000) to illustrate the power of present method to solve the non-linear differential equations. It is observed that the present solution is convergent and accurate.

Tables (3-6) illustrate good agreements of present results with earlier literature works ([6-8] and [11]). It is observed that the absolute difference between present results and differential transformation method, *DTM* [7] and homotopy analysis method, *HAM* [11] is less than 5.09*10^{-7}.

5. Results and Discussion

Computations of dimensionless velocity v, temperature θ, skin friction factor C_{fL} and Nusselt number N_{uL} are made for different values of flow and heat parameters (Tr, M, F_s, δ, R_d and B_r) to illustrate their effects on dimensionless quantities. Certain values of these parameters are chosen to show variation and convergence of present results as they are plotted tabulated and compared with analytical and numerical available results.

The effect of Brinkman number (B_r=1, 50 and 100) on the variations of v and θ profiles are shown in Fig. 2. It is observed that increasing Brinkman number (B_r) increases v and θ because of dissipation.

The effect of temperature ratio (T_r=1.5, 2 and 2.5) on the variations of v and θ profiles are shown in Fig. 3. It is observed that increasing T_r increases v and θ because of radiation. It also is observed that velocity v is relatively affected by T_r more than θ.
The effect of radiation parameter (R_d=1, 2 and 3) on the variations of v and θ profiles are shown in Fig. 4. It is observed that increasing $1/R_d$ increases v and θ because of radiation. It also is observed that velocity v is relatively affected by R_d more than θ.

The effect of viscoelastic parameter δ on the variations of v and θ profiles is shown in Fig. 5. It is observed that increasing δ decreases v and θ. It also is observed that velocity v is relatively affected by δ more than θ.

Tables (7 and 8) show effects of some fluid flow and heat transfer parameters (T_r, δ and B_r) on the friction factor (C_{fL}) and Nusselt number (N_{uL}) when M=F_s=1 and R_d=100. It is observed that C_{fL} increases with increasing T_r and B_r but, C_{fL} decreases with increasing δ, It is also observed that N_{uL} increases with increasing T_r and δ, but, N_{uL} decreases with increasing B_r.

6. Conclusions

The finite difference method with fourth order accurate is used to solve the nonlinear momentum and energy equations between heated vertical plates. The effects of: nonlinear radiation, dissipation and Forchiemer-Darcy resistance force on viscoelastic (Rivlin-Ericksen) fluid and heat transfer are taken into consideration. An error analysis is made to achieve accuracy, convergence and stability of present results and their agreement with available previous works. The fourth order difference schemes reduce the required number of sub-intervals of the domain of solution. Hence, the storage memory and processing time are reduced in computers. The effects of fluid and heat parameters on velocity, temperature, skin friction factor and Nusselt number are studied and discussed. Samples of present results are listed and shown in tables and figures. It is observed that, increasing Brinkman number and temperature ratio increase velocity, skin friction factor and temperature because of dissipation. It is also observed that, increasing viscoelastic parameter decreases velocity, skin friction factor and temperature because resistance to the flow. The Nusselt number decreases with increasing Brinkman number, but, it increase with increasing both viscoelastic parameter and temperature ratio. The results which are introduced in tables are very useful in engineering design and comparisons with future analytical and experimental works.

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Table 1: Convergence of present results with relatively small parameters: $M=F_s=R_d=B_r=\delta=1$, $Tr=1.5$

	$v(x)$			$\theta(x)$		
m / x	20 ($\Delta^4=10^{-4}$)	200 ($\Delta^4=10^{-8}$)	2000 ($\Delta^4=10^{-12}$)	20 ($\Delta^4=10^{-4}$)	200 ($\Delta^4=10^{-8}$)	2000 ($\Delta^4=10^{-12}$)
-1	0	0	0	0.5	0.5	0.5
-0.8	0.0163777112	0.0305706543	0.0306171305	0.4299799420	0.4307198914	0.4308045172
-0.6	0.0216158119	0.0457975622	0.0458915209	0.3555444845	0.3571143442	0.3572938117
-0.4	0.0206248826	0.0487315918	0.0488707276	0.2761503680	0.2786628267	0.2789499164
-0.2	0.0160765142	0.0425595109	0.0427383069	0.1910468433	0.1946435806	0.1950543403
0	0.0094877557	0.0304154193	0.0306248834	0.0992574877	0.1041180250	0.1046727584
0.2	0.0019485842	0.0154152899	0.0156420281	-0.0004615359	0.0058929016	0.0066175859
0.4	-0.0053448137	0.0008325258	0.0010570365	-0.109695473	-0.1015477090	-0.1006193412
0.6	-0.0105730848	-0.0096499014	-0.0094557831	-0.230492616	-0.2201547774	-0.2189780952
0.8	-0.0106169347	-0.0116645499	-0.0115407593	-0.365483573	-0.3524997559	-0.3510143280
1	0	0	0	-0.5	-0.5	-0.5

Table 2: Convergence of present results with relatively large parameters: M=10, F_s=10, R_d=1000, B_r=10, δ=30, Tr=10

	$v(x)$			$\theta(x)$		
m / x	20 (Δ^4=10^{-4})	200 (Δ^4=10^{-8})	2000 (Δ^4=10^{-12})	20 (Δ^4=10^{-4})	200 (Δ^4=10^{-8})	2000 (Δ^4=10^{-12})
-1	0	0	0	0.5	0.5	0.5
-0.8	0.0140675932	0.0140767801	0.0140771269	0.4394277338	0.4395086246	0.4395125925
-0.6	0.0200916466	0.0201110886	0.0201119796	0.3720398947	0.3722182261	0.3722269398
-0.4	0.0196959988	0.0197289568	0.0197305020	0.2971954479	0.2974928529	0.2975073466
-0.2	0.0154694903	0.0155145565	0.0155167656	0.2136113091	0.2140552428	0.2140768387
0	0.0091984395	0.0092551410	0.0092579953	0.1198014480	0.1204245426	0.1204548177
0.2	0.0020365902	0.0021027664	0.0021061918	0.0145472519	0.0153835416	0.0154241349
0.4	-0.0048912329	-0.0048217027	-0.0048179141	-0.1024140462	-0.1013381930	-0.1012860332
0.6	-0.0098678881	-0.0098089637	-0.0098053259	-0.2296223620	-0.2283005685	-0.2282365764
0.8	-0.0095969901	-0.0095716177	-0.0095692732	-0.3638345357	-0.3622932829	-0.3622183346
1	0	0	0	-0.50	-0.5	-0.50

Table 3: Comparison of velocity v with earlier literature works at δ=0.5: M=F_s=0, R_d=∞, B_r=1, m=500

x	HPM [6]	RVIM [8]	HAM [11]	Present results (Δ^4=2.56*10^{-10})	Absolute difference [11]
-1	0	0	0	0	0
-0.8	0.0239	0.02356863	0.02392391	0.023919349	4.56*10^{-6}
-0.6	0.0322	0.03153540	0.03217724	0.032172691	4.55*10^{-6}
-0.4	0.0284	0.02756369	0.02841114	0.028406873	4.27*10^{-6}
-0.2	0.0166	0.01565187	0.01662161	0.016617647	3.96*10^{-6}
0	0.0008	0.00019888	0.00081131	0.000807629	3.68*10^{-6}
0.2	-0.0151	-0.01604876	-0.01507910	-0.015082460	3.36*10^{-6}
0.4	-0.0271	-0.02794788	-0.0271006	-0.027103713	3.11*10^{-6}
0.6	-0.0312	-0.03186690	-0.0312274	-0.031230148	2.75*10^{-6}
0.8	-0.0234	-0.02378185	-0.0234270	-0.023429061	2.06*10^{-6}
1	0	0	0	0	0

Table 4: Comparison of temperature θ with earlier literature works at δ=0.5: M=F_s=0, R_d=∞, B_r=1, m=500

x	HPM [6]	RVIM [8]	HAM [11]	Present results (Δ^4=2.56*10^{-10})	Absolute difference [11]
-1	0.5	0.49794410	0.5	0.5	0
-0.8	0.4008	0.39866980	0.4007343	0.400735882	1.56*10^{-6}
-0.6	0.3012	0.29911352	0.30117607	0.301177385	1.32*10^{-6}
-0.4	0.2016	0.19953058	0.20158997	0.201590907	9.37*10^{-7}
-0.2	0.1019	0.09986820	0.1019269	0.101927502	6.02*10^{-7}
0	0.0021	0.00126049	0.00206022	0.002060513	2.93*10^{-7}
0.2	-0.0981	-0.1001317	-0.09807	-0.098070049	4.93*10^{-8}
0.4	-0.1984	-0.2004692	-0.1984082	-0.19840851	3.10*10^{-7}
0.6	-0.2988	-0.3008857	-0.2988279	-0.298828518	6.18*10^{-7}
0.8	-0.3993	-0.4013296	-0.399274	-0.399274732	7.32*10^{-7}
1	-0.5	-0.5	-0.5	-0.5	0

Table 5: Comparison of velocity with earlier literature works at δ=1 and 10: M=F_s=0, R_d=∞, B_r=1, m=500

	δ=1			δ=10		
x	MDTM [7]	Present results (Δ^4=2.56*10^{-10})	Absolute difference	MDTM [7]	Present results (Δ^4=2.56*10^{-10})	Absolute difference
-1	0	0	0	0	0	0
-0.8	0.0236092	0.023609239	3.89*10^{-8}	0.020283	0.020283005	5.09*10^{-9}
-0.6	0.0318811	0.031881105	5.42*10^{-9}	0.0285755	0.028575363	1.37*10^{-7}
-0.4	0.0281619	0.028161945	4.49*10^{-8}	0.0253301	0.025330107	6.58*10^{-9}
-0.2	0.0164583	0.016458249	5.07*10^{-8}	0.014624	0.014624039	3.88*10^{-8}
0	0.000786885	0.000786881	4.48*10^{-9}	0.000588807	0.000588806	6.20*10^{-10}
0.2	-0.0149624	-0.014962397	3.07*10^{-9}	-0.0135018	-0.013501849	4.88*10^{-8}
0.4	-0.0268935	-0.026893492	8.30*10^{-9}	-0.0243835	-0.02438353	3.01*10^{-8}
0.6	-0.030969	-0.030969047	4.71*10^{-8}	-0.0279372	-0.027937167	3.28*10^{-8}
0.8	-0.0231422	-0.02314216	3.98*10^{-8}	-0.0200027	-0.020002731	3.14*10^{-8}
1	0	0	0	0	0	0

Table 6: Comparison of temperature with earlier literature works at δ=1 and 10:
M=F_s=0, R_d=∞, B_r=1, m=500

	δ=1			δ=10		
x	MDTM [7]	Present results (Δ^4=2.56*10^{-10})	Absolute difference	MDTM [7]	Present results (Δ^4=2.56*10^{-10})	Absolute difference
-1	0.5	0.5	0	0.5	0.5	0
-0.8	0.400729	0.400729468	4.68*10^{-7}	0.400654	0.4006544381	4.38*10^{-7}
-0.6	0.301166	0.301166314	3.14*10^{-7}	0.30104	0.3010401445	1.45*10^{-7}
-0.4	0.201575	0.201575071	7.10*10^{-8}	0.201397	0.2013969491	5.09*10^{-8}
-0.2	0.101908	0.101907808	1.92*10^{-7}	0.101687	0.1016871657	1.66*10^{-7}
0	0.00203924	0.002039242	1.63*10^{-9}	0.00180116	0.0018011602	2.37*10^{-10}
0.2	-0.0980898	-0.098089808	7.65*10^{-9}	-0.0983111	-0.0983111117	1.17*10^{-8}
0.4	-0.198424	-0.198424379	3.79*10^{-7}	-0.198603	-0.1986028017	1.98*10^{-7}
0.6	-0.298840	-0.298839497	5.03*10^{-7}	-0.298965	-0.2989646242	3.76*10^{-7}
0.8	-0.399281	-0.399280916	8.35*10^{-8}	-0.399354	-0.3993535644	4.36*10^{-7}
1	-0.5	-0.5	0	-0.5	-0.5	0

Table 7: Effects of flow and heat parameters on the friction factor C_{fL}:
M=F_s=1, R_d=100, m=2000 (Δ^4=10^{-12})

T_r	δ=1			δ=5		
	B_r =1	5	10	1	5	10
1.5	0.152336641	0.154962100	0.158282977	0.135990374	0.137878375	0.140253316
3.0	0.166702886	0.169107640	0.172183969	0.146714922	0.148403910	0.150552516
6.0	0.217015267	0.218673157	0.220818815	0.182725509	0.183816722	0.185223145
9.0	0.245931562	0.246780633	0.247861169	0.202630062	0.203172941	0.203862450

Table 8: Effects of flow and heat parameters on the Nusselt number N_{uL}:
M=F_s=1, R_d=100, m=2000 (Δ^4=10^{-12})

T_r	δ=1			δ=5		
	B_r =1	5	10	1	5	10
1.5	4.071065139	3.91663411	3.710515086	4.072953482	3.926727634	3.732534428
3.0	4.489192193	4.305931409	4.061065274	4.491639876	4.318989697	4.089461579
6.0	7.368059791	7.016670110	6.555116975	7.373888607	7.047033018	6.619141710
9.0	14.80883288	14.30101864	13.65079190	14.81762304	14.34582897	13.74264531

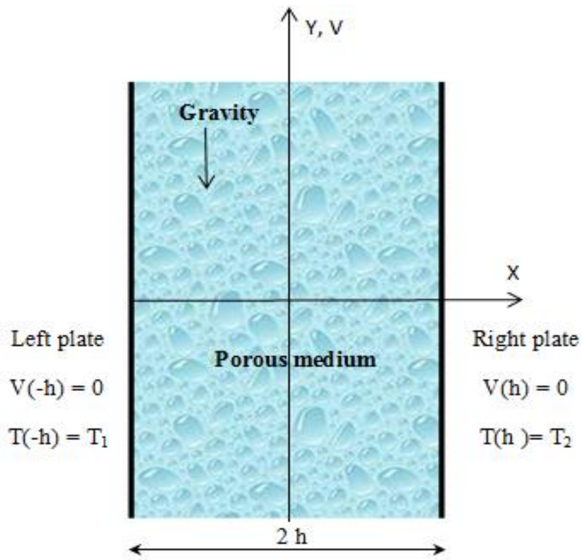

Figure 1: Channel Geometry and boundary conditions.

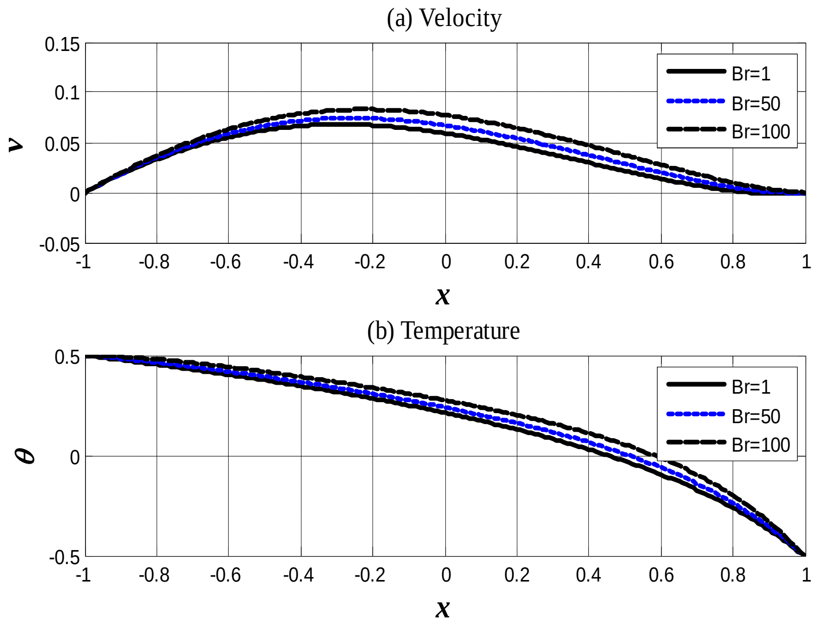

Figure 2: Variation of v and θ profiles with Brinkman number (B_r) when M=1, F_s=1, δ=5, R_d =5 and T_r=3

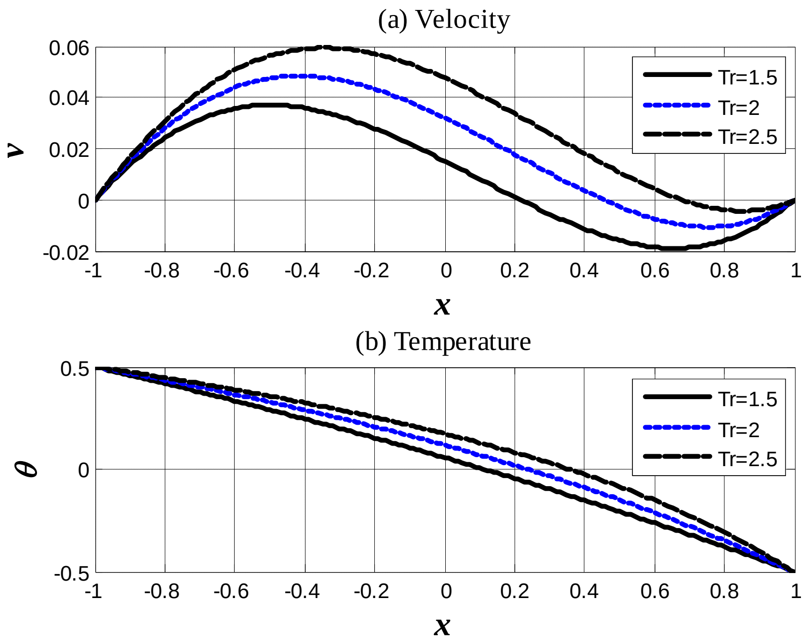

Figure 3: Variation of v and θ profiles with relative temperature (Tr) when M=1, F_s=1, δ=5, R_d =5 and B_r=5.

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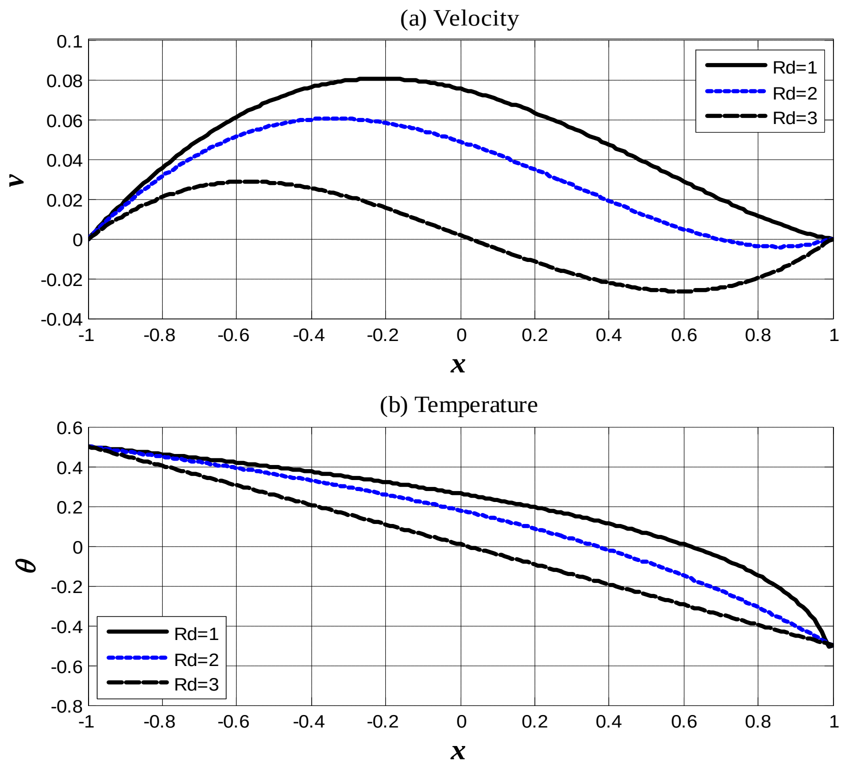

Figure 4: Variation of v and θ profiles with radiation parameter (R_d) when M=1, F_s=1, δ=5, and Br=5, T_r=3.

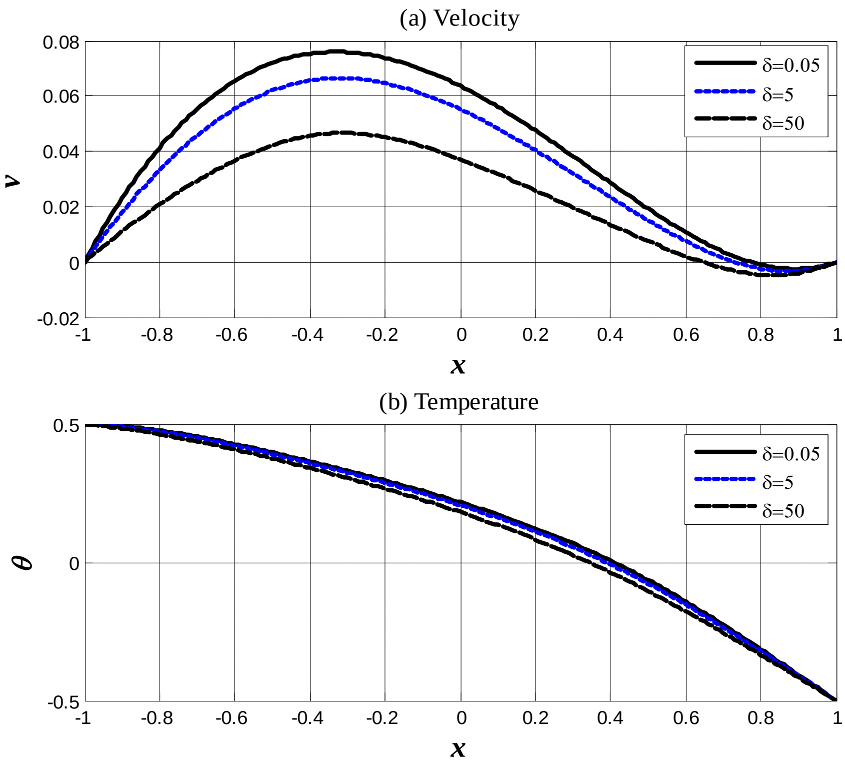

Figure 5: Variation of v and θ profiles with viscoelastic parameter (δ) when M=1, F_s=1, R_d =1000 and Br=50, T_r=10.