

Comparison of Result Analysis in Weibull Distribution Based on Increasing Right Type Censorship and Hybrid Censorship

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Abstract: In the area of certainty, the pressure-resistance model describes the life of a component that has random resistance X and is subjected to random pressure Y . The component is degraded if the pressure exerted on it exceeds its resistance and will act satisfactorily if $\{(x, y) | x > y\}$ occurs. Therefore, $R = P(X > Y)$ is a measure of component reliability. Therefore, in this study, we consider the estimation of $P(Y < X)$ based on the second type of sequential censored data and hybrid censorship for both variables. In the first chapter of this study we will examine the relevant definitions and the introduction of the research. In Chapter Two, we review the research on censorship. The third chapter deals with the methodology of the research and finally, the fourth chapter presents the results and discusses them, and the fifth chapter summarizes the research results.

Keywords: Increasing Type II Right Censorship, Hybrid Combined Censorship, Pressure-Life Strength

1. Introduction

In statistical inference and common classical methods for estimating the unknown parameter of society, the information obtained from the random sample is used. However, in many cases, in addition to information provided by a random sample of the unknown parameter of society,

There is other useful information (non-sample information) that can help us improve the estimation of the unknown parameter. This non-sample information is usually obtainable through prior knowledge or results of previous experiments. For example, a quality control engineer based on personal experience or previous test results can make an initial guess about the average lifetime of the test piece. According to Fisher (Saleh, p. 1), this non-sample information can be expressed as a preliminary hypothesis test and used to estimate the unknown parameter. The estimator obtained with this preliminary test is called the pre-test estimator. That is, if the classical estimator of the unknown parameter θ is represented by $\hat{\theta}_j$ and there is an initial guess $\theta = \theta_0$ about the unknown parameter θ , then the hypothesis test $H_0: \theta = \theta_0$ versus $H_1: \theta \neq \theta_0$ is first performed. Then, based on the rejection or acceptance of the assumption, H is the pre-test estimator (PTE) for θ as follows $(\hat{\theta}^{PT}) = \theta_0$ if H_0 is accepted and $(\hat{\theta}^{PT}) \neq \theta_0$ if H_0 is rejected.

The pre-test estimator was introduced by Bancroft and Han and Bancroft. Since then, many researchers have studied this estimator, including Jajj and Bok Akbariya and Saleh, Saleh and Kobriya, Banda, Chiao and Han., Han, Shanbogh and Jihali, Arashi et al., He Singh. Beckley [13] investigated the pre-test estimator in an exponential two-parameter distribution based on Type I censored data. Kebiria and Saleh [14] investigated the pre-test estimator for exponential and Paratoux distribution parameters based on doubly censored samples. Baklizi [15] and Zakar Zadeh and Karimi [16] also studied the pre-test estimator in exponential two-parameter distribution based on the record data. Recently, Mir Farah and Ahmadi [17] compared pre-test and classical

two-parameter exponential estimators based on Pittman's proximity criterion with the record data.

In longevity and reliability tests, there are many cases where prior trials are excluded or excluded from the test when they fail or fail. This deletion may occur unintentionally (stopping the lifetime test due to unforeseen conditions). Usually the removal of test units is premeditated and deliberate and is done by the examiner for reasons such as saving time and money. In this case it is said that censorship has occurred, meaning only part of the lifetime data is observed. In this case the observed sample is called the censored sample. Typical censors Type I and II are the most common type of censor. One of the drawbacks to conventional type I and II censors is that they do not allow the removal of test units at times other than the time of termination. But the increasing type II censorship has no drawback. This censorship is described as follows. n Expose the test unit at zero time to a lifetime test. Upon first failure, R_1 units of the remaining healthy units are excluded from the test process. By observing the second failure, R_2 units of the remaining healthy units are excluded from the experiment and this continues until all remaining remaining units have been mean

$$R_m = n - R_1 - R_2 - \dots - R_{m-1} - m$$

Exit lifetime test. In this type of censorship, the values of m and $(R = R_1, \dots, R_m)$ are predetermined. Note that if

$$R_1 = R_2 = \dots = R_{m-1} = 0$$

Then a typical type II censorship scheme is obtained. Also if we have:

$$R_1 = R_2 = \dots = R_m = 0$$

Then we will have an uncensored design (full sample).

Unified Hybrid Censorship is a combination of two generalized type I and II hybrid censors.

Consider a lifetime test with n units. Suppose the units have independent lifetime and even distribution with density

function $f(y, \theta)$ and distribution function $F(y, \theta)$ and $Y_-(1:n) < 0$. $< Y_-(n:n)$ Until they are destroyed or destroyed. Epstein (1954) first examined a scheme in a survival experiment in which the experiment was terminated at $T^* = \min(Y_r: n, T)$ and the values of T and r were predetermined. Childs et al (1) called this scheme Type I hybrid censorship. In this scheme, very few failures can occur until T . To solve this problem, Childs et al. Designed an experiment, in which the experiment terminates at $T = \max(X(Y_r: n, T))$. This scheme became known as the Type II hybrid censorship scheme. Not all units may fail before T time, but the time required for Chandrasekar et al. (2004) introduced two generalized type I and II hybrid censorship schemes such that the two previous schemes (Lack of minimum failure in Type I hybrid censorship design and extension of testing time in Type II hybrid censorship design) have improved somewhat.

Weibull distribution is one of the continuous probability distributions. Although the distribution was first recognized by French scientist Ferré in Year 1, and then by Resin and Ramler in Year 1, it was used to describe the particle size distribution, but its name derives from the name of Valdie Weibull. Described in detail in Year 2.

Probability density function:

$$f(x; k, \lambda) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$$

In statistics, Maximum Likelihood Estimation (MLE) is a method for estimating the parameters of a statistical model. When operating on a dataset, a statistical model is obtained, then maximum likelihood can provide an estimate of the model parameters. The maximum likelihood method is similar to many known statistical estimation methods. Suppose it is important for a person to be aware of the height of adult female giraffes in a population, and because of the cost or time constraints they cannot measure the height of each individual giraffe, the person only knows that these lengths follow the normal distribution but Does not know the mean and variance of the distribution Now, using the maximum likelihood method and with the information about a limited sample of population, can estimate the mean and variance of this distribution. MLE does this by assuming variance and mean unknown, and then assigns values that are most likely to be available with the available information. In general, the MLE method for a given set of data is to assign values to the model's parameters, thereby producing a distribution that is most likely to be attributed to the observed data (ie, the values of the parameter that maximize the likelihood function). MLE provides a specific estimation mechanism that works well for normal distribution and many other distributions. However, in some cases there are problems such as whether or not maximum likelihood estimators are in place.

In this study, we first estimate the Weibull scale and shape distribution parameters by maximum likelihood estimation based on Type II incremental right censorship and hybrid censorship, assuming that the parameters are independent and have a prior gamma distribution, with Bayesian estimation of the parameters with The sampling aid is

obtained from important points and compared in a simulation study of different designs.

2. Data Analysis Method

2.1 Weibull distribution based on type II incremental right censorship

The problem of estimating R is raised in mechanical assurance systems. Suppose X represents the resistance of a component with pressure Y , then R can be considered as a measure of the performance of this system. The system will be out of control if the system pressure exceeds its resistance. Since R represents a relationship between system pressure and resistance, it is generally considered as the system pressure-resistance parameter. Several authors and researchers have studied the problem of estimating R . Also MLE for $P(X < Y)$ when X and Y have exponential bivariate distributions has also been studied by Avad et al. Gupta and Gupta (1990) examined the estimation of $P(Y < X)$ when X and Y are distributed as normal and multivariate. Ahmad, Fakhri, and Jahan (1997) studied the estimation when X and Y are random variables of type X boron. Kondo and Gupta (2005 and 2006) obtained an estimate of R in which X and Y are generalized based on a complete sample of random Weibull variables and exponential variables. Bakab et al. (2008) provided an estimate of R in which X and Y are distributed as two general exponential random variables with three parameters and different shape and similar parameters for the same scale and position. Baklizi (2008) discussed the probability and estimation of R -bases on the basis of lower values than the general exponential distribution. Recently, Rezai et al. (2010) investigated the estimation of $P(Y < X)$ in which X and Y are two independent and general Pareto distributions with different parameters. Although many studies have been conducted on estimating R by distributing complete sample cases, none of the previous studies in this regard on the second type deleted sample have been advanced. This type of deletion is discussed below, after the start of the life test with N units. N units are included in the life test and only $n (< N)$ units are fully observed until failure. The deletion process is progressive and advanced in n steps. These n stages of failure times suggest n fully observed units. During the first failure (first stage) r_1 of the remaining $N-1$ units is randomly deducted from the experiment (deliberately omitted) During the second failure (second stage), r_2 is removed from the $N-2-R_1$ remaining unit And the process goes on like this. Finally, at time n (stage n), all $r_n = N-n-r_1-\dots-r_{n-1}$ remaining units are eliminated. We call this the advanced sequential type II deletion process with patterns (r_1, r_2, \dots, r_n) . It is clear that this pattern includes the second type of right-handed incremental censorship model that is $r_1 = r_2 = \dots = r_{n-1} = 0$ and $r_n = Nn$ and the complete pattern is $N = n$ and $r_1 = r_2 = \dots = r_{n-1} = 0$. The regular life expectancy data from this type of pattern is the increasing type II right censorship, called regular type II censorship statistics. For more information on this type of sequential deletion and related references, see the article by Balakrishnan and Agarwala (2000). Also read the articles by Konda and Girder on the censorship model called the second type of sequential hybrid censorship, a combination of the hybrid and second successive type of censorship. Hybrid censorship is also a

combination of the first and second types of censorship. The Weibull distribution with two parameters is represented by $W(\alpha \text{ and } \theta)$, which has the following probability density function:

$$f(x, \alpha, \theta) = \frac{\alpha}{\theta} x^{\alpha-1} e^{-\frac{x^\alpha}{\theta}}, \quad x > 0, \alpha, \theta >$$

And its cumulative distribution function is as follows:

$$F(x, \alpha, \theta) = 1 - e^{-\frac{x^\alpha}{\theta}}, \quad x > 0, \alpha, \theta >$$

Here, the alpha is the shape parameter and the theta is the scale parameter. The statistical inference for the distribution of Weibull under the second type of sequential censorship has also been examined by many authors. Estimates of the Weibull distribution are discussed in detail in our work in 1971, by Vivrose and Balakrishnan (1994), Balakrishnan and Agarwala (2000) and Wu (2002). The inference of the unknown parameters of the Weibull distribution in the presence of sequential censorship by Kondord 2008 has also been studied. Suppose that $X \sim W(\alpha, \theta)$, $Y \sim W(\beta, \theta)$ are random independent variables. In this study, we consider the estimation of $P(Y < X)$ based on the second type of sequential censored data for both variables.

2.2 Weibull Distribution Based on Hybrid (Censored) Removal

For some applications of R, see the article by Coates et al. Many researchers and authors have studied the pressure-resistance parameter R. Among them, Ahmed et al. In 1997, Avad et al. In 1981, Kondo and Gupta in 2005 and 2006, Adimari and Chiogena (2006), Rial Baklizi (2008), Bakab et al. (2008) and Rezai (2010).) Can be named. A combination of type I and type II censorship patterns is known as a hybrid pattern and can be described as follows. Suppose n similar units are in a test. The lifetime of the sample units are independent and similarly are randomly distributed variables. The test ends when the r number that is predetermined by n is broken by the unit or the predetermined time T is reached. It is also assumed that damaged items will not be replaced. Therefore, in the hybrid model, the experimental time and the number of failures will not exceed T and r. It is clear that the removal patterns of types I and II can be obtained as special cases of the combination censorship model with $r = n$ and $T = \infty$. Now, we will explain the existing data in the pattern we want. Note that according to this hybrid model, it is assumed that r and t are known. Therefore, in accordance with this hybrid pattern, we will have one of two types of observations:

Case I : $\{y_{1:n} < y_{2:n} < \dots < y_{r:n}\}$ if $y_{r:n} < T$

Case II : $\{y_{1:n} < y_{2:n} < \dots < y_{d:n}\}$ if $d < r$

Where $y_{1:n} < y_{2:n} < \dots$ represents the observed sequential times of failure of the experimental units. Further details on combinatorial removal and related references can be found in the research by Epstein (1954), Firebanks et al. (1982), Childs et al. (2003), Copta and Condo (1998), Condo (2007) and Ebrahimi (1990, 1992). Read. The Weibull distribution with two parameters is represented as $W(\alpha, \theta)$, which has the following probability density function:

$$f(x, \alpha, \theta) = \frac{\alpha}{\theta} x^{\alpha-1} e^{-\frac{x^\alpha}{\theta}}, \quad x > 0, \alpha, \theta > 0,$$

And its cumulative distribution function is as follows:

$$F(x, \alpha, \theta) = 1 - e^{-\frac{x^\alpha}{\theta}}, \quad x > 0, \alpha, \theta > 0,$$

Here α is the shape parameter and θ is the scale parameter. Based on the examples Y and X, Kondo and Kupta consider $R = P(X < Y)$ in 2006 when $X \sim W(\alpha, \theta)$ and $Y \sim W(\alpha, \theta)$ are two independent Weibull distributions. And the scale parameters are different and the shape parameters are similar. In this study, we generalize the results obtained when the samples are removed in combination, to the other. This article is organized as follows. In the second part we will obtain the maximum estimator of the probability R. It can be seen that this estimator cannot be obtained in closed form. We propose an AMLE for R in Section III, which can be obtained explicitly.

3. Results

3.1 Weibull distribution based on type II incremental right censorship

3.1.1 Numerical calculations

This section presents real data analysis and a Monte Carlo simulation to show all the proposed methods in the previous sections.

3.1.1.1 Data Analysis

In this section, we analyze the resistance data. These data were originally reported by Beard and Priest in 1982 and indicate the GPA measured power for a single carbon fiber and 1000 carbon fiber cords. Pressure unit fibers were tested in a 20 mm and 10 mm gauge gauge. These data have already been reported by Rakab and Kondo (2005), Kondo and Gupta (2006), Kondo and Rakab (2009). The data are reported in Tables 1 and 2.

Table 1: Datasheet 1 (gauge length 20 mm)

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958
1.966	1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179
2.224	2.240	2.253	2.270	2.272	2.274	2.301	2.301	2.359	2.382
2.382	2.426	2.434	2.435	2.478	2.490	2.511	2.514	2.535	2.554
2.566	2.570	2.586	2.629	2.633	2.642	2.648	2.684	2.697	2.726
2.770	2.773	2.800	2.809	2.818	2.821	2.848	2.880	2.954	3.012
3.067	3.084	3.090	3.096	3.128	3.233	3.433	3.585	3.585	

Table 2: Data Set 2 (Pile Length 10 mm)

1.901	2.132	2.203	2.228	2.257	2.350	2.361	2.396	2.397	2.445
2.454	2.474	2.518	2.522	2.525	2.532	2.575	2.614	2.616	2.618
2.624	2.659	2.675	2.738	2.740	2.856	2.917	2.928	2.937	2.937
2.977	2.996	3.030	3.125	3.139	3.145	3.220	3.223	3.235	3.243
3.264	3.272	3.294	3.332	3.346	3.377	3.408	3.435	3.493	3.501
3.537	3.554	3.562	3.628	3.852	3.871	3.886	3.971	4.024	4.027
4.225	4.395	5.020							

Table 3: Shape, Scale, X, and P Parameters for Weibull Fit Models in Data Set 1 and 2

Data Set	Shape Parameter	Scale Parameter	K-S	p value
1	5.505	214.131	0.0578	0.9658
2	5.049	424.574	0.0867	0.7197

We fitted the Weibull model to two separate datasets. We also report the shape and scale parameters, the Kolmogorov-Smirnoff travel between the fitted and empirically distributed functions, and the corresponding p values in Table 3. It can be seen from Table 3 that the Weibull model fits perfectly with the dataset. Since the two scale parameters are not equal, we converted the above dataset to the Weibull

model with similar scale parameters. We know that if the random variable X follows $W(\alpha, \theta)$, then the random variable $Q = W(\theta \wedge (1 / \alpha))$ will have the standard distribution $W(\alpha, \theta)$. Therefore, we will convert the above dataset by dividing them into $\theta \wedge (1 / \alpha)$ and $\theta \wedge (1 / \beta)$ into $W(\alpha, 1)$ and $W(\beta, 1)$ states. Then, we have the above dataset from Weibull distributions:

Table 4: Converted Datasets 1

0.495	0.496	0.558	0.585	0.641	0.680	0.702	0.704	0.733	0.739
0.742	0.753	0.757	0.762	0.765	0.775	0.778	0.791	0.807	0.822
0.839	0.845	0.850	0.856	0.857	0.858	0.868	0.868	0.890	0.899
0.899	0.915	0.918	0.919	0.935	0.939	0.947	0.948	0.956	0.963
0.968	0.970	0.976	0.992	0.993	0.997	0.999	1.013	1.017	1.028
1.045	1.046	1.056	1.060	1.063	1.064	1.074	1.086	1.114	1.136
1.157	1.163	1.166	1.168	1.180	1.220	1.295	1.352	1.352	

Table 5: Converted Datasets 2

0.573	0.643	0.665	0.672	0.681	0.709	0.712	0.723	0.723	0.738
0.740	0.746	0.760	0.761	0.762	0.764	0.777	0.789	0.789	0.790
0.792	0.802	0.807	0.826	0.827	0.862	0.880	0.883	0.886	0.886
0.898	0.904	0.914	0.943	0.947	0.949	0.971	0.972	0.976	0.978
0.985	0.987	0.994	1.005	1.009	1.019	1.028	1.036	1.054	1.056
1.067	1.072	1.074	1.094	1.162	1.168	1.172	1.198	1.214	1.215
1.274	1.326	1.514							

Table 6: Shape, Scale, K-S, and p Parameters for Weibull Fit Models in Data Set 1 and 2

Data Set	Shape Parameter	Scale Parameter	K-S	p-value
1	5.505	1	0.0578	0.9658
2	5.049	1	0.0867	0.7197

We fitted the Weibull model separately to the two transformed datasets, and the results in Table 4-6 show that the Weibull models with similar scales fit well with the converted datasets. In order to show the results, we

generated two sequential deletion samples using two sampling schemes from the above sets in Tables 4 and 5. The data and schemas generated corresponding to each are reported in Table 4-7.

Table-data and schemas generated delete corresponding to each data

i	1	2	3	4	5	6	7	8	9	10
r_i	1	0	1	2	0	0	3	0	1	50
x_i	1.312	1.479	1.552	1.803	1.944	1.958	1.966	2.027	2.055	2.098
r'_i	0	2	1	0	1	1	2	0	0	44
y_i	1.901	2.132	2.257	2.361	2.396	2.445	2.474	2.525	2.532	2.575

Based on Equation 2-2, 3-22, and 4-5, ML, AML and B-R estimates are 0.5500, 0.5172 and 0.5221, respectively. In order to calculate Bace estimates, we have no prior information assuming that $a_1 = a_2 = a_3 = b_1 = b_2 = b_3 = 0$. In this case, when this happens, prior information is invalid and only Jeffrey's PRIORS are considered. 95% confidence

intervals and ML, AML and corresponding bisections for each R for (0.555, 0.4488), (0.556, 0.4644) and (0.5831, 0.4652), respectively. are. The Boot-p and Boot-t confidence intervals were also obtained from 1-4 and 2-4 respectively (0.7758, 0.4758) and (0.558, 0.4726), respectively. After press research in 2001, we also used small non-negative

values of the large parameters in this case and used previous PRIORS. We tested the relationship $a_1 = a_2 = a_3 = b_1 = b_2 = b_3 = 0.0001$. In this case, the estimate of the base R is equal to 0.5169 and also the estimate of the base interval is (0.55750, 0.4563). We observed that the results were not significantly different from the corresponding results obtained from non-optimal PRIORS.

3.1.1.2 Simulation Study

In this section, we propose some results based on Monte Carlo simulations in order to compare the performance of different methods for different schemas and compare different values of parameters. We compare the performance of the estimation methods of Bice, AMLE and MLE in terms of errors, mean squared error with respect to the squared error function. We also compare the different confidence intervals called confidence intervals obtained by using the hypothetical distribution of MLE, AMLE and the self confidence interval and the HPD validity interval in terms of mean confidence length and coverage percentage. We use different values of parameters, different and large parameters and different schemas for sampling. Also from the three sets of parameter values respectively $(\alpha = 1.5, \beta = 2, \theta = 1)$, $(\alpha = 1, \beta =, \theta = 1)$, $(\alpha = 0.5, \beta = 0.5, \theta = 1)$ mainly We use the Bayes estimator to compare different MLEs. To calculate these valid HPD estimators and intervals, we assume the following three modes:

$$a_j = 0.0001, b_j = 0.0001, j = 1, 2, 3,$$

$$a_j = 1, b_j = 1, j = 1, 2, 3,$$

$$a_j = 3, b_j = 3, j = 1, 2, 3.$$

Note that state 3 contains more information than state two because the variance of state 2 is smaller than state two, and both are better than state 1. Also, three removal schemes are proposed in Table 4-8 below.

Table 8: Deleting Schemes

	(n, N)	C. S.
r ₁	(15, 30)	(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,15)
r ₂	(15, 30)	(15,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
r ₃	(15, 30)	(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)

For different parameters, different deletion schemes and different PRIORS, Table 9 shows the mean differences, MSE for MLE and AMLE, and B-R estimates over 500 iterations. In simulation experiments for ordinary methods, we calculated confidence intervals based on re-sampling with 250 sampling times. The bypass estimates and the validity intervals corresponding to each are based on 500 sampling, which is T = 500. All calculations were made using Visual Maple software version 12.

Table 9: MSE Differences for MLE, MLE, and B-R Estimates

(α, β, θ)	C.S.		MLE	AMLE	BS		
					prior 1	prior 2	prior 3
(1.5, 2, 1)	r ₁ , r ₁	Biase	0.052	0.059	0.091	0.082	0.076
		MSE	0.007	0.009	0.011	0.011	0.010
	r ₁ , r ₂	Biase	0.074	0.078	0.105	0.094	0.087
		MSE	0.006	0.008	0.010	0.009	0.010
	r ₁ , r ₃	Biase	0.076	0.084	0.107	0.097	0.094
		MSE	0.010	0.011	0.013	0.013	0.012
r ₂ , r ₂	Biase	0.026	0.032	0.044	0.036	0.034	
	MSE	0.004	0.006	0.008	0.007	0.007	
r ₂ , r ₃	Biase	0.050	0.061	0.080	0.079	0.069	
	MSE	0.007	0.009	0.012	0.011	0.011	
r ₃ , r ₃	Biase	0.067	0.084	0.111	0.099	0.093	
	MSE	0.008	0.011	0.014	0.012	0.009	
(1, 1, 1)	r ₁ , r ₁	Biase	0.139	0.149	0.177	0.161	0.157
		MSE	0.020	0.022	0.026	0.026	0.025
	r ₁ , r ₂	Biase	0.152	0.161	0.189	0.180	0.172
		MSE	0.023	0.027	0.031	0.028	0.029
	r ₁ , r ₃	Biase	0.184	0.185	0.203	0.201	0.196
		MSE	0.036	0.039	0.041	0.042	0.040
r ₂ , r ₂	Biase	0.093	0.104	0.123	0.121	0.116	
	MSE	0.013	0.016	0.020	0.021	0.018	
r ₂ , r ₃	Biase	0.136	0.141	0.165	0.161	0.154	
	MSE	0.024	0.025	0.033	0.031	0.030	
r ₃ , r ₃	Biase	0.151	0.162	0.203	0.193	0.186	
	MSE	0.026	0.029	0.034	0.033	0.031	
(0.5, 0.5, 1)	r ₁ , r ₁	Biase	0.230	0.231	0.251	0.245	0.245
		MSE	0.053	0.055	0.056	0.058	0.054
	r ₁ , r ₂	Biase	0.270	0.276	0.293	0.293	0.283
		MSE	0.074	0.077	0.078	0.079	0.075
	r ₁ , r ₃	Biase	0.311	0.321	0.350	0.344	0.339
		MSE	0.101	0.106	0.116	0.111	0.110
r ₂ , r ₂	Biase	0.225	0.235	0.260	0.252	0.244	
	MSE	0.043	0.050	0.053	0.054	0.055	
r ₂ , r ₃	Biase	0.261	0.263	0.287	0.274	0.270	
	MSE	0.065	0.065	0.072	0.074	0.075	
r ₃ , r ₃	Biase	0.273	0.279	0.295	0.290	0.285	
	MSE	0.073	0.077	0.083	0.083	0.081	

From Table 9, it can be seen that MLE and AMLE are well compared in terms of disparities and MSEs with bias estimates. It is also observed that the MSE and the MLE difference are also close to AMLE. Comparison of two bisect estimators based on modes 2 and 3, it is clearly shown that the bisect estimators based on the third case perform better than the previous two states and this performance is better in terms of difference and MSE. Bayes estimators performed better than the results obtained from state 1 based on the above mentioned states. From Table 9, also by comparing the different schemas we can see that the schemas (r2, r2) offer less deviation and MSEs for different parameter values. We also calculated 95% confidence intervals for R based on the assumed distributions of MLE and AMLE. We then calculated the Boot-p and Boot-t confidence intervals of the rat and examined the validity of the HPD. In Table 10, we show the average validity intervals and corresponding coverage percentages. The nominal level of confidence intervals or credit intervals in each case is

0.95. It can be seen from Table 10 that the HPD validity ranges are wider than the other confidence intervals. We also found that the ML method is a valid way of constructing confidence intervals for R and different schemas and works best for different parameter values. Different AMLE values for hypothetical confidence intervals are also the best confidence intervals. It was also observed that the Boot-p confidence interval performs significantly better than the Boot-t. It can be seen from Table 10 that the valid Boot-t and HPD intervals provide the most probability coverage. It can be seen from Tables 9 and 10 that the Bayes estimates and HPD confidence intervals are very sensitive to the hypothetical values of the previous parameters. Note that sequential deletion type II schemas include the usual type II deletion schemas and complete sampling schemas. Therefore, all the results obtained in this article can be generalized to the complete case and the second type omission

Table 10: Average reliability / validity interval length and percentage of estimator cover R

$\alpha = 1.5, \beta = 2, \theta = 1$							
C.S.	ML	AML	p-boot	t-boot	BS		
					prior 1	prior 2	prior 3
r1, r1	0.127(0.939)	0.134(0.942)	0.146(0.938)	0.156(0.943)	0.205(0.947)	0.193(0.949)	0.190(0.951)
r1, r2	0.139(0.935)	0.149(0.942)	0.157(0.943)	0.167(0.947)	0.206(0.949)	0.197(0.947)	0.195(0.946)
r1, r3	0.161(0.935)	0.170(0.937)	0.174(0.944)	0.181(0.947)	0.218(0.946)	0.213(0.946)	0.203(0.942)
r2, r2	0.132(0.942)	0.140(0.945)	0.147(0.946)	0.156(0.949)	0.201(0.944)	0.202(0.944)	0.202(0.947)
r2, r3	0.153(0.935)	0.157(0.937)	0.160(0.946)	0.175(0.947)	0.209(0.948)	0.201(0.949)	0.197(0.948)
r3, r3	0.173(0.934)	0.175(0.944)	0.182(0.951)	0.197(0.952)	0.217(0.952)	0.212(0.949)	0.202(0.951)
$\alpha = 1, \beta = 1, \theta = 1$							
C.S.	ML	AML	p-boot	t-boot	BS		
					prior 1	prior 2	prior 3
r1, r1	0.203(0.929)	0.211(0.937)	0.227(0.942)	0.220(0.947)	0.239(0.945)	0.235(0.948)	0.230(0.945)
r1, r2	0.212(0.941)	0.224(0.946)	0.231(0.949)	0.237(0.949)	0.261(0.949)	0.254(0.949)	0.246(0.952)
r1, r3	0.219(0.937)	0.229(0.941)	0.237(0.942)	0.245(0.940)	0.267(0.942)	0.261(0.946)	0.255(0.945)
r2, r2	0.215(0.944)	0.223(0.943)	0.229(0.942)	0.237(0.948)	0.250(0.949)	0.246(0.949)	0.241(0.949)
r2, r3	0.220(0.938)	0.224(0.943)	0.232(0.943)	0.239(0.944)	0.263(0.947)	0.256(0.949)	0.251(0.950)
r3, r3	0.237(0.942)	0.241(0.941)	0.247(0.945)	0.263(0.943)	0.277(0.947)	0.272(0.946)	0.268(0.947)
$\alpha = 0.5, \beta = 0.5, \theta = 1$							
C.S.	ML	AML	p-boot	t-boot	BS		
					prior 1	prior 2	prior 3
r1, r1	0.217(0.935)	0.224(0.942)	0.229(0.946)	0.234(0.948)	0.252(0.948)	0.247(0.948)	0.244(0.951)
r1, r2	0.228(0.943)	0.231(0.943)	0.238(0.941)	0.246(0.942)	0.269(0.945)	0.263(0.948)	0.255(0.945)
r1, r3	0.237(0.943)	0.243(0.944)	0.247(0.948)	0.255(0.944)	0.281(0.948)	0.272(0.947)	0.264(0.947)
r2, r2	0.218(0.937)	0.224(0.941)	0.233(0.944)	0.244(0.948)	0.258(0.943)	0.252(0.945)	0.247(0.951)
r2, r3	0.227(0.938)	0.232(0.941)	0.237(0.945)	0.245(0.943)	0.271(0.946)	0.265(0.946)	0.259(0.951)
r3, r3	0.236(0.942)	0.242(0.943)	0.250(0.945)	0.252(0.944)	0.279(0.948)	0.272(0.950)	0.264(0.949)

3.1.2 Final Review

Proof of Theorem 1: It is known that if $X_1: N < X_2: N < \dots < X_n: N$ is a successive elimination of a second type of sample $W(\alpha, \theta)$, then $Z_1: N < Z_2: N < \dots < Z_n: N$ where $Z_i: N = (X_{(i):N})^\alpha / \theta$ and $(i = 1, \dots, n)$ is an example of successive deletion of the second type of standard exponential distribution. therefore:

$$E[(X_{i:N})^\alpha] = \theta E(Z_{i:N}) = \theta \mu_i.$$

In order to calculate:

$$E[(X_{i:N})^\alpha \ln(X_{i:N})]$$

And

$$E[(X_{i:N})^\alpha (\ln X_{i:N})^2]$$

We need the following. Distribution $X_i: N$ equals:

$$f_{X_{i:N}}(x) = c_{i-1} \sum_{k=1}^i a_{i,k} f(x) [1 - F(x)]^{\gamma_k - 1} = c_{i-1} \sum_{k=1}^i a_{i,k} \frac{\alpha}{\theta} x^{\alpha-1} e^{-\gamma_k \frac{x^\alpha}{\theta}}$$

For $x > 0$ and $\theta > 0$, then:

$$\begin{aligned}
 E[(X_{i:N})^\alpha \ln(X_{i:N})] &= \int_0^\infty x_i^\alpha \ln x_i f_{X_{i:N}}(x_i) dx_i \\
 &= c_{i-1} \sum_{k=1}^i a_{i,k} \int_0^\infty x_i^\alpha \ln x_i \frac{\alpha}{\theta} x_i^{\alpha-1} e^{-\gamma_k \frac{x_i^\alpha}{\theta}} dx_i \\
 &= c_{i-1} \sum_{k=1}^i a_{i,k} \int_0^\infty \frac{\theta}{\gamma_k^2} t \left[\frac{1}{\alpha} (\ln t + \ln \frac{\theta}{\gamma_k}) \right] e^{-t} dt \\
 &= c_{i-1} \sum_{k=1}^i a_{i,k} \left[\frac{\theta}{\alpha \gamma_k^2} \int_0^\infty t \ln t e^{-t} dt + \frac{\theta \ln \frac{\theta}{\gamma_k}}{\alpha \gamma_k^2} \int_0^\infty t e^{-t} dt \right] \\
 &= \frac{\theta}{\alpha} c_{i-1} \sum_{k=1}^i \frac{a_{i,k}}{\gamma_k^2} \left[\Gamma'(2) + \ln \frac{\theta}{\gamma_k} \Gamma(2) \right].
 \end{aligned}$$

We have this:

$$\begin{aligned}
 E[(X_{i:N})^\alpha (\ln X_{i:N})^2] &= \int_0^\infty x_i^\alpha (\ln x_i)^2 f_{X_{i:N}}(x_i) dx_i \\
 &= c_{i-1} \sum_{k=1}^i a_{i,k} \int_0^\infty x_i^\alpha (\ln x_i)^2 \frac{\alpha}{\theta} x_i^{\alpha-1} e^{-\gamma_k \frac{x_i^\alpha}{\theta}} dx_i \\
 &= c_{i-1} \sum_{k=1}^i a_{i,k} \int_0^\infty \frac{\theta}{\gamma_k^2} t \left[\frac{1}{\alpha} (\ln t + \ln \frac{\theta}{\gamma_k}) \right]^2 e^{-t} dt \\
 &= c_{i-1} \sum_{k=1}^i a_{i,k} \left[\frac{\theta}{\alpha^2 \gamma_k^2} \int_0^\infty t (\ln t)^2 e^{-t} dt + 2 \frac{\theta \ln \frac{\theta}{\gamma_k}}{\alpha^2 \gamma_k^2} \int_0^\infty t \ln t e^{-t} dt \right. \\
 &\quad \left. + \frac{\theta (\ln \frac{\theta}{\gamma_k})^2}{\alpha^2 \gamma_k^2} \int_0^\infty t e^{-t} dt \right] \\
 &= \frac{\theta}{\alpha^2} c_{i-1} \sum_{k=1}^i \frac{a_{i,k}}{\gamma_k^2} \left[\Gamma''(2) + 2 \ln \frac{\theta}{\gamma_k} \Gamma'(2) + (\ln \frac{\theta}{\gamma_k})^2 \Gamma(2) \right].
 \end{aligned}$$

Proof of Theorem 2: Here $R^\wedge = g(\alpha^\wedge, \beta^\wedge, \theta^\wedge)$ where

$$g(\alpha, \beta, \theta) = 1 - \int_0^\infty \frac{\alpha}{\theta} x^{\alpha-1} e^{-\frac{1}{\theta}(x^\alpha + x^\beta)} dx.$$

Using theorem 1 and data methods, B can be obtained as follows:

$$B = \left(\frac{\partial g}{\partial \alpha}, \frac{\partial g}{\partial \beta}, \frac{\partial g}{\partial \theta} \right) A^{-1}(\alpha, \beta, \theta) \begin{pmatrix} \frac{\partial g}{\partial \alpha} \\ \frac{\partial g}{\partial \beta} \\ \frac{\partial g}{\partial \theta} \end{pmatrix}$$

Under regular conditions, we have:

$$\frac{\partial g}{\partial \alpha} = h_1(\alpha, \beta, \theta), \quad \frac{\partial g}{\partial \beta} = h_2(\alpha, \beta, \theta),$$

Therefore, the proof is obtained.

3.2 Weibull Distribution Based on Hybrid (Censored) Removal

3.2.1 Numerical calculations

In this section, analysis of real data sets and Monte Carlo simulations is suggested to show all the estimation methods explained in the previous sections.

3.2.1.1 Simulation Study

In this section, we will compare the performance of the MLE and AMLE methods of the bisection with respect to the squared error function in terms of the relative deviations and then examine the mean squared error. We will also compare different confidence intervals called confidence intervals obtained using the hypothetical MLE distribution, and the other two confidence intervals and the valid HPD interval in terms of the average confidence interval. To calculate the bisector estimator and the HPD confidence intervals, we assume the following two modes:

Prior 1: $a_j = 0, b_j = 0, j = 1, 2, 3,$

Prior 2: $a_j = 1, b_j = 2, j = 1, 2, 3.$

Prior1 mode is the non-information PRIOR gamma mode for scale and shape parameters and Prior2 is the information gamma PRIOR mode. For different deletion schemas and different PRIORS, we will report average estimates, MSE for MLE and AMLE, and B-R estimates at 1000 iterations. The results are reported in Table 11. In the simulation experiments along the methods we have calculated, we calculated confidence intervals based on 250 sampling times.

Table 11: Mean and MSE estimates for MLE, AMLE, and bypass estimates R ($m = n = 30$ and $(\alpha, \theta_1, \theta_2) = (1.5, 1, 1)$)

(r_1, T_1)	(r_2, T_2)		MLE	AMLE	BS	
					prior 1	prior 2
(20, 1)	(20, 1)	A.E	0.4929	0.4921	0.5074	0.5067
		MSE	0.0081	0.0093	0.0047	0.0031
	(25, 1)	A.E	0.4941	0.5040	0.5068	0.5037
		MSE	0.0072	0.0084	0.0035	0.0032
	(20, 2)	A.E	0.5049	0.4951	0.5043	0.5039
		MSE	0.0074	0.0090	0.0045	0.0039
(25, 2)	A.E	0.4957	0.4953	0.5049	0.5036	
	MSE	0.0068	0.0071	0.0049	0.0041	
(30, 2)	A.E	0.4986	0.4979	0.5033	0.5027	
	MSE	0.0065	0.0074	0.0053	0.0037	
(25, 1)	(20, 1)	A.E	0.5058	0.4947	0.5051	0.5047
		MSE	0.0071	0.0078	0.0066	0.0061
	(25, 1)	A.E	0.5045	0.5052	0.5038	0.5031
		MSE	0.0066	0.0067	0.0054	0.0056
	(20, 2)	A.E	0.5041	0.5046	0.5039	0.5022
		MSE	0.0067	0.0069	0.0061	0.0057
(25, 2)	A.E	0.5033	0.5039	0.5032	0.5023	
	MSE	0.0058	0.0061	0.0055	0.0046	
(30, 2)	A.E	0.4979	0.4983	0.5024	0.5022	
	MSE	0.0058	0.0049	0.0049	0.0038	
(20, 2)	(20, 1)	A.E	0.5035	0.5036	0.5033	0.5027
		MSE	0.0067	0.0074	0.0057	0.0043
	(25, 1)	A.E	0.4971	0.4956	0.5029	0.5024
		MSE	0.0054	0.0069	0.0046	0.0033
	(20, 2)	A.E	0.5029	0.5031	0.5026	0.5025
		MSE	0.0066	0.0069	0.0048	0.0037
(25, 2)	A.E	0.4979	0.4968	0.5022	0.5020	
	MSE	0.0065	0.0059	0.0043	0.0031	
(30, 2)	A.E	0.4986	0.4977	0.5018	0.5016	
	MSE	0.0056	0.0061	0.0039	0.0023	
(25, 2)	(20, 1)	A.E	0.4974	0.4959	0.5027	0.5019
		MSE	0.0062	0.0071	0.0045	0.0037
	(25, 1)	A.E	0.4979	0.4964	0.5020	0.5018
		MSE	0.0051	0.0061	0.0043	0.0023
	(20, 2)	A.E	0.5023	0.4968	0.5021	0.5020
		MSE	0.0057	0.0070	0.0049	0.0027
(25, 2)	A.E	0.5019	0.4982	0.5019	0.5017	
	MSE	0.0050	0.0054	0.0036	0.0021	
(30, 2)	A.E	0.4988	0.4983	0.5014	0.5010	
	MSE	0.0044	0.0041	0.0041	0.0013	
(30, 2)	(20, 1)	A.E	0.5047	0.4952	0.5042	0.5029
		MSE	0.0066	0.0076	0.0059	0.0043
	(25, 1)	A.E	0.4951	0.4959	0.5031	0.5016
		MSE	0.0059	0.0064	0.0048	0.0032
	(20, 2)	A.E	0.5068	0.4953	0.5043	0.5021
		MSE	0.0064	0.0068	0.0065	0.0029
(25, 2)	A.E	0.5034	0.5029	0.5016	0.5009	
	MSE	0.0050	0.0057	0.0047	0.0014	
(30, 2)	A.E	0.5005	0.5013	0.5006	0.5003	
	MSE	0.0043	0.0049	0.0051	0.0008	

Table 12: Reliable and valid length for R estimators

(r_1, T_1)	(r_2, T_2)	MLE	Boot-t	Boot-p	BS	
					prior 1	prior 2
(20, 1)	(20, 1)	0.3854	0.4071	0.3966	0.3717	0.3416
		0.3779	0.3988	0.3824	0.3714	0.3197
	(20, 2)	0.3815	0.3874	0.3902	0.3759	0.2989
		0.3426	0.3747	0.3592	0.3217	0.2996
	(30, 2)	0.3140	0.3706	0.3269	0.2975	0.2661
		0.3785	0.3884	0.3877	0.3419	0.3055
(25, 1)	(25, 1)	0.3766	0.3899	0.3813	0.3124	0.2892
	(20, 2)	0.3711	0.3914	0.3732	0.3053	0.2971
	(25, 2)	0.3463	0.3618	0.3557	0.2902	0.2546
	(30, 2)	0.2909	0.3314	0.3273	0.2613	0.2387
(20, 2)	(20, 1)	0.3461	0.3597	0.3444	0.3143	0.2819
	(25, 1)	0.3211	0.3424	0.3229	0.3078	0.2413
	(20, 2)	0.3357	0.3527	0.3289	0.3053	0.2673
	(25, 2)	0.3103	0.3315	0.3071	0.2642	0.2374
(25, 2)	(30, 2)	0.2846	0.3363	0.2978	0.2560	0.1956
	(20, 1)	0.3484	0.3677	0.3565	0.2835	0.2634
	(25, 1)	0.3192	0.3275	0.2905	0.2764	0.2312
	(20, 2)	0.3248	0.3350	0.3132	0.2714	0.2276
(30, 2)	(25, 2)	0.2925	0.2850	0.3013	0.2558	0.2170
	(30, 2)	0.2614	0.2831	0.2738	0.2203	0.2111
	(20, 1)	0.3417	0.3605	0.3452	0.2804	0.2079
	(25, 1)	0.3311	0.3225	0.3308	0.2244	0.1716
(30, 2)	(20, 2)	0.3255	0.3600	0.3396	0.2372	0.1948
	(25, 2)	0.2779	0.2813	0.2860	0.2052	0.1620
(30, 2)	0.2462	0.2804	0.2748	0.2034	0.0913	

3.1.2 Data Analysis

In this section, we analyze the resistance data obtained by Badar and Priest (1982). These data show the eliminated resistance in terms of mean values for single carbon fibers, 1000 fiber carbon fiber cords. Tensile unit fibers were tested in a 20 mm and 10 mm gage. These data have already been

used by Rakab and Kondo (2005), Kondo and Gupta (2006), Kondo and Rakab (2009) and Agerzadeh et al. (2011). The data are presented in Tables 3 and 4.

Table 13: First data set (20mm gauge gauge)

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958
1.966	1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179
2.224	2.240	2.253	2.270	2.272	2.274	2.301	2.301	2.359	2.382
2.382	2.426	2.434	2.435	2.478	2.490	2.511	2.514	2.535	2.554
2.566	2.570	2.586	2.629	2.633	2.642	2.648	2.684	2.697	2.726
2.770	2.773	2.800	2.809	2.818	2.821	2.848	2.880	2.954	3.012
3.067	3.084	3.090	3.096	3.128	3.233	3.433	3.585	3.585	

Table 14: First data set (10mm gauge gauge)

1.901	2.132	2.203	2.228	2.257	2.350	2.361	2.396	2.397	2.445
2.454	2.474	2.518	2.522	2.525	2.532	2.575	2.614	2.616	2.618
2.624	2.659	2.675	2.738	2.740	2.856	2.917	2.928	2.937	2.937
2.977	2.996	3.030	3.125	3.139	3.145	3.220	3.223	3.235	3.243
3.264	3.272	3.294	3.332	3.346	3.377	3.408	3.435	3.493	3.501
3.537	3.554	3.562	3.628	3.852	3.871	3.886	3.971	4.024	4.027
4.225	4.395	5.020							

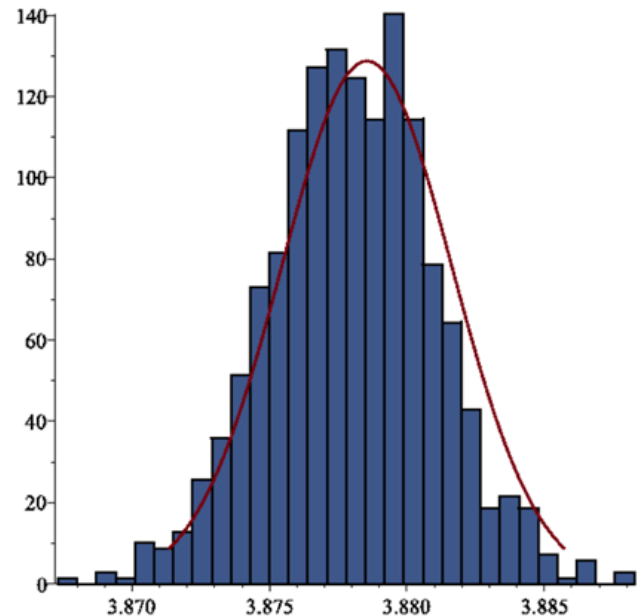


Figure 1: The alpha density function

Kondo and Gupta (2006) analyzed this dataset using a two-parameter Weibull distribution after deducting 0.75 from both datasets. After subtracting this value from all points in the proposed dataset, Kondo and Gupta (2006) observed that the Weibull distributions fit the same set of parameters with both datasets. Based on the complete data, we plot the histogram of alpha samples produced by MCMC along with the exact density function in Fig. 1. It can be seen from Fig. 1 that the true compaction function fits well with the simulated samples obtained by MCMC. In order to show these results, we generated two different combination deletions after subtracting 0.75 from two datasets:

$$r_1 = 45, T_1 = 2.5, r_2 = 40, T_2 = 2.5.$$

Therefore, MLE and AMLE for R were 0.3958 and 0.3872 and the corresponding confidence intervals were 95% (0.5193, 0.2723) and (0.4891, 0.22617), respectively. We also obtained Boot-t and Boot-p confidence intervals as (0.555, 0.3045) and (0.5526, 0.3106). To estimate the R

Bayesian, we used inaccurate PRIORS at $\theta_1, \theta_2, \alpha$ i.e. $a_1 = a_2 = a_3 = b_1 = b_2 = b_3 = 0$. On the basis of the above, we obtain 0.3933 as an estimate of the R byte under the squared error function. We also estimated 95% as the highest density confidence interval for R (0.4829, 0.2937).

$$r_1 = 35, T_1 = 1.7, r_2 = 25, T_2 = 2.2.$$

In this case, the MLE and AMLE and the RB estimates will be 0.4024, 0.4092, and 0.4238, respectively, and the corresponding 95% confidence interval will be equal to (0.55267, 0.2781), (5531/0, 2960/0) and (5371/0, 2870/0).

We also obtained 95% confidence intervals of Boot-p and Boot-t equal to (0.557, 0.3359) and (0.5964, 0.3484).

3.2.2 Final review

AMLE for R to:

When $u_1 = T_1$ and $u_2 = T_2$, we generalize $p(z_i)$ and $q(z^* i)$ to the tilose sets around the points $\mu_{(i)}$ and $\mu_{(r_1)}^*$. Then we generalize the terms $p_{(w_j)}$ and $q_{(u_2)}^*$ around the points $\mu_{(i)}$ and $\mu_{(r_1)}^*$. Similarly, only with respect to first-order derivatives and by ignoring higher-degree derivatives, the alpha, θ_1 and θ_2 AMLEs can be obtained as follows:

$$\tilde{\mu}_1 = A_1 + B_1\tilde{\sigma}, \quad \tilde{\mu}_2 = A_2 + B_2\tilde{\sigma}, \quad \text{and} \quad \tilde{\sigma} = \frac{-D + \sqrt{D^2 - 4(R_1 + R_2)E}}{2(R_1 + R_2)},$$

where in

$$A_1 = \frac{\sum_{i=1}^{r_1} \beta_i t_i + (n - r_1)\beta_{r_1} \ln(T_1)}{\sum_{i=1}^{r_1} \beta_i + (n - r_1)\beta_{r_1}}, \quad B_1 = \frac{\sum_{i=1}^{r_1} \alpha_i - (n - r_1)(1 - \alpha_{r_1})}{\sum_{i=1}^{r_1} \beta_i + (n - r_1)\beta_{r_1}},$$

$$A_2 = \frac{\sum_{j=1}^{r_2} \beta_j s_j + (m - r_2)\beta_{r_2} s_{r_2}}{\sum_{j=1}^{r_2} \beta_j + (m - r_2)\beta_{r_2}}, \quad B_2 = \frac{\sum_{j=1}^{r_2} \alpha_j - (m - r_2)(1 - \alpha_{r_2})}{\sum_{j=1}^{r_2} \beta_j + (m - r_2)\beta_{r_2}},$$

$$D = \sum_{i=1}^{r_1} \alpha_i (t_i - 3A_1) - (n - r_1)(1 - \alpha_{r_1})(\ln(T_1) - 3A_1) + 2A_1 B_1 \sum_{i=1}^{r_1} \beta_i + 2A_1 B_1 (n - r_1)\beta_{r_1} \\ + \sum_{j=1}^{r_2} \alpha_j (s_j - 3A_2) - (m - r_2)(1 - \alpha_{r_2})(\ln(T_2) - 3A_2) + 2A_2 B_2 \sum_{j=1}^{r_2} \beta_j + 2A_2 B_2 (m - r_2)\beta_{r_2},$$

$$E = \sum_{i=1}^{r_1} \beta_i t_i (t_i - A_1) + (n - r_1)\beta_{r_1} (\ln(T_1))^2 - (n - r_1)A_1 \beta_{r_1} \ln(T_1) + \sum_{j=1}^{r_2} \beta_j s_j (s_j - A_2) \\ + (m - r_2)\beta_{r_2} (\ln(T_2))^2 - (m - r_2)A_2 \beta_{r_2} \ln(T_2),$$

And therefore,

$$\alpha_{r_i} = 1 + \ln(q_{r_i})(1 - \ln(-\ln(q_{r_i}))), \quad \beta_{r_i} = \ln(q_{r_i}), \quad i = 1, 2.$$

Therefore, the approximate MLE for R is as follows:

$$\tilde{R} = \frac{\tilde{\theta}_1}{\tilde{\theta}_1 + \tilde{\theta}_2},$$

where in

$$\tilde{\theta}_1 = \exp\left(\frac{1}{\tilde{\sigma}}(A_1 + B_1\tilde{\sigma})\right),$$

And

$$\tilde{\theta}_2 = \exp\left(\frac{1}{\tilde{\sigma}}(A_2 + B_2\tilde{\sigma})\right)$$

Also, AMLE for R can be obtained for other cases (when $u_1 = T_1, u_2 = xR_2$ or $u_1 = xR_1$ and $u_2 = T_2$).

4. Conclusion

On the basis of the second-type omitted samples, we investigated the estimation of $R = P(X < Y)$ when X and Y are independent Weibull distributions with different shape parameters and scale-like parameters. The maximum likelihood estimator, approximate maximum likelihood estimator and the bisector R estimator are obtained. Based on the hypothetical distribution of R, the confidence interval R was also obtained. Spontaneous confidence intervals were also suggested. Data analysis is proposed to display modes. Monte Carlo simulations are also presented to compare the different methods proposed.

The hybrid model of a hybrid deletion scheme consists of a combination of deletion schemes type I and II. Based on the omitted composite samples, this study seeks to deduce $R = P(X > Y)$ in which X and Y are two independent Weibull distributions with different scale parameters having similar

shape parameters. The maximum estimator of probability and approximation R is obtained. The hypothetical maximum distribution of the estimator of probability R can be obtained. Based on the hypothetical distribution, the confidence interval R can also be obtained. Two spontaneous confidence intervals are also suggested. We consider the Bayes estimation of R and propose a valid and corresponding interval for R . Monte Carlo simulations are also used to calculate the various proposed methods. Real dataset analysis is also provided to study the proposed modes.

In this study, we propose $R = P(X > Y)$ when X and Y are independent and their Weibull distribution with similar shape and different scale parameters is discussed. It is assumed that the combined data on X and Y are omitted. We propose MLE for R and it is clear that MLE for R cannot be obtained explicitly. But it can be obtained by solving a one-dimensional nonlinear equation. We also obtain AMLE for R and we can say that it is obtained explicitly. Extensive Monte Carlo simulations show that the performance of MLE and AMLE are similar and, therefore, AMLE can be used for all practical purposes. We also consider the R-based inference based on generalized inverse gamma PRIORS for scale parameters and an independent case in the form of shape parameters. The Bayes estimator cannot be obtained explicitly, so we used the Gibbs sampling technique to calculate the Bayesian estimation and to calculate the valid interval. It is found that the estimator functions of the bisect are very satisfactory. And if prior information was also available on the unknown parameters, the Bayes estimator should be preferred to MLE and AMLE as expected.

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