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On the Construction of Kumaraswamy-Epsilon Distribution with Applications

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Abstract: ***A new probability distribution function of the Kumaraswamy-G family is introduced in this study. It is constructed from a parent distribution called the epsilon distribution.This new distribution has more flexible probability density function shapes that suggest its possible use, for example, in modeling lifetime data generation processes. Its hazard rate functions and order statisticsdistributionswere derived. The distribution significantly provided good fit to two real life datasets with efficient parameter estimates within twice their standard errors. These attest to its practical applicability.***

Keywords: Kumaraswamy-G, epsilon distribution, hazard rate function, order statistic, deuterium excesses, monthly tax revenue.

1. Introduction

A beta-type distribution was introduced in 1980 for a double bounded random variable [16] andnamed after the founder, Kumaraswamy. A random variable, X, is said to be distributed according to the Kumaraswamy distribution with parameters a and b if its density and cumulative distribution functions are given, respectively, by

$$f_X(x;a,b) = abx^{a-1}(1-x^a)^{b-1} \quad (1)$$

and

$$F_X(x;a,b) = 1-(1-x^a)^b \quad (2)$$

where, in each case, $0 < x < 1$, and$a, b > 0$.The distribution has application in hydrology, reliability studies [17] and other areas.

Following the works of Jones [14] and Eugene et al [12], Cordeiro and de Castro [6] proposed the use of Kumaraswamy distribution as a generator of probability distributions of continuous random variables with parent distribution $G(x)$. These are referred to as the Kumaraswamy-G distribution. Its cumulative distribution function is given by

$$F_X(x;a,b) = 1-(1-G(x)^a)^b \quad (3)$$

and the probability density function given by

$$f_X(x;a,b) = abg(x)G(x)^{a-1}(1-G(x)^a)^{b-1} \quad (4)$$

where $g(x) = \frac{d}{dx}G(x)$. The parameters a and b in the equation (4) determine the skewness and tail weight, respectively.

In this study, a new distribution based on the Kumaraswamy generator is introduced.It is called the Kumaraswamy-epsilon distribution and, subsequently denoted by the K-epsilon distribution.

2. Literature Review

Of recent, interest in deriving new generalized probability distributions has grown tremendously. It has resulted in generalized classes of univariate distributions after introducing additional shape parameter to the baseline or parent distributions. The new generated families extend the well-known classical probability distributions and at the same time provide great flexibility in modeling data generating processes. For examples, beta generator [12]; gamma generator [19], the Weibull-G family [3], exponentiated family and generalized exponentiated family [8] of density functions have all been introduced. Many probability distributions have been constructed using these generators.

Many members of the Kumaraswamy generator have been constructed and applied. For instance, the Kumaraswamy Gumbel [7], Kumaraswamy Laplace [1], Kumaraswamy Pareto [4], Kumaraswamy generalized Rayleigh [13], the Kumaraswamy Lindley distribution [5], Kumaraswamy generalized gamma distribution [10] have been used in modeling bathtub shaped hazard rate functions. Also, they have the ability to model monotone and non-monotone failure rate functions, as such it can be useful in both lifetime data analysis and reliability studies. Oguntunde et al [18] introduced the Kumaraswamy inverse exponential distribution and studied some of its properties. The Kumaraswamy Weibull distribution was introduced in [9] and used in the study of failure data.

3. The K-epsilon distribution and its properties

The epsilon distribution was introduced in [11] and shown to be asymptotically equivalent to the exponential distribution. For a random variable X, distributed according to the epsilon distribution, its probability density and cumulative distribution functions are given, respectively, by

$$g_X(x;\lambda,\delta) = \lambda\left(\frac{\delta^2}{\delta^2-x^2}\right)\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta} \quad (5)$$

and

$$G_X(x;\lambda,\delta) = 1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta} \quad (6)$$

where $0 < x < \delta; \delta, \lambda > 0$.

Putting (6) into (3) and (4), the cumulative distribution and probability density functions of the K-epsilon distribution are given, respectively, by

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$$F_X(x) = 1-(1-Z^a)^b \tag{7}$$

and

$$f_X(x) = ab\lambda\Upsilon(1-Z)Z^{a-1}(1-Z^a)^{b-1} \tag{8}$$

where $\Upsilon = \left(\frac{\delta^2}{\delta^2-x^2}\right)$; $Z = 1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}$; $0 < x < \delta; a, b, \delta, \lambda > 0$.

The skewness and tail weight of the distribution are controlled by the parameters a and b. The plots of the K-epsilon density function (8) at various parameter values are given in Fig 1 below.

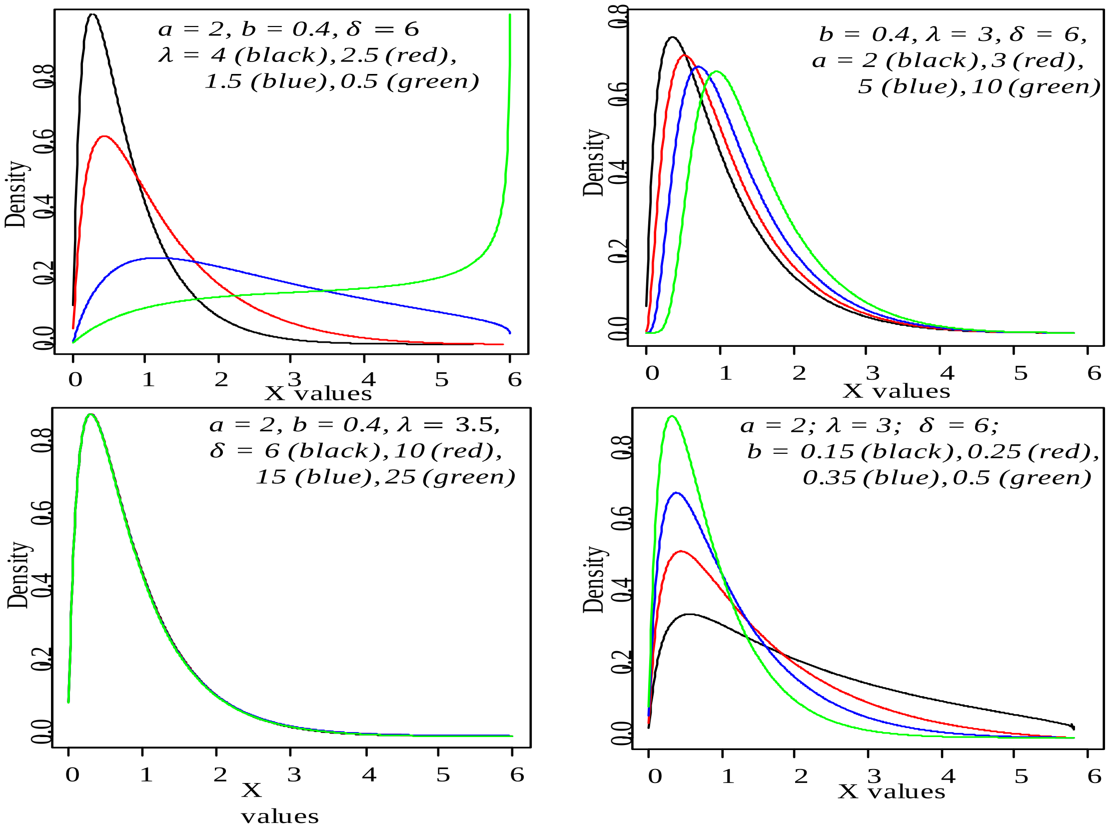

Figure 1: K-Epsilon probability density plots at various parameter values

From (7), the quantile function of the K-epsilon distribution is given by

$$x_q = \delta\frac{\left(1-\left(1-(1-p)^{\frac{1}{b}}\right)^{\frac{1}{a}}\right)^{-\frac{2}{\lambda\delta}}-1}{\left(1-\left(1-(1-p)^{\frac{1}{b}}\right)^{\frac{1}{a}}\right)^{-\frac{2}{\lambda\delta}}+1} \tag{9}$$

where $0 < p < 1$; $a, b, \delta, \lambda > 0$

Proposition 1

The K-epsilon distribution is a true probability density function with unimodal shape property. That is, we want to show that

(i) $f_X(x)$ is unimodal and

(ii) $\int_0^\delta f_X(x)\,dx = 1$

The proof of this proposition is given in the appendix.

3.1 Moments of the K-epsilon distribution

The $r^{th}, r = 1, 2, \ldots,$ moment of the K-epsilon can be derived from the expression

$$E(X^r) = \int_0^\delta x^r f(x)\,dx$$

$$= ab\lambda\int_0^\delta x^r\Upsilon(1-Z)Z^{a-1}(1-Z^a)^{b-1}\,dx$$

$$= \int_0^\delta x^{r-1}(1-Z^a)^b\,dx \tag{10}$$

where $\Upsilon = \left(\frac{\delta^2}{\delta^2-x^2}\right)$; $Z = 1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}$

From (10), it is obvious that the moments of the K-epsilon distribution cannot be obtained in explicit form. but can be obtained by numerical integration for the desired moment(s) when the parameter values are specified.

3.2 Survival and hazard rate functions

Survival and hazard rate functions form the focal points in survival analysis and reliability theory. The survival function, which is the complementary cumulative distribution or reliability function, is defined as the probability of surviving to the age of x or exceeding age x. On the other hand, the hazard rate function, also known as instantaneous failure rate or force of mortality, is interpreted as the amount of risk of failure associated with a unit of age x. The two functions are related; however, the hazard rate function is more informative about the underlying mechanism of failure.

The survival function of the K-epsilon distribution is given by

$$S_X(x) = (1 - Z^a)^b \quad (11)$$

and its hazard rate function is given by

$$h_X(x) = ab\lambda\Upsilon(1 - Z)\frac{Z^{a-1}}{1-Z^a} \quad (12)$$

where $\Upsilon = \left(\frac{\delta^2}{\delta^2 - x^2}\right)$, $Z = 1 - \left(\frac{x+\delta}{\delta - x}\right)^{-\frac{\lambda}{2}\delta}$

The plots of the hazard rate function of the K-epsilon distribution at various parameter values are presented in Fig. 2 below. The shapes suggest that the K-epsilon hazard rate function could be a good choice model for somefailure rate datasets.

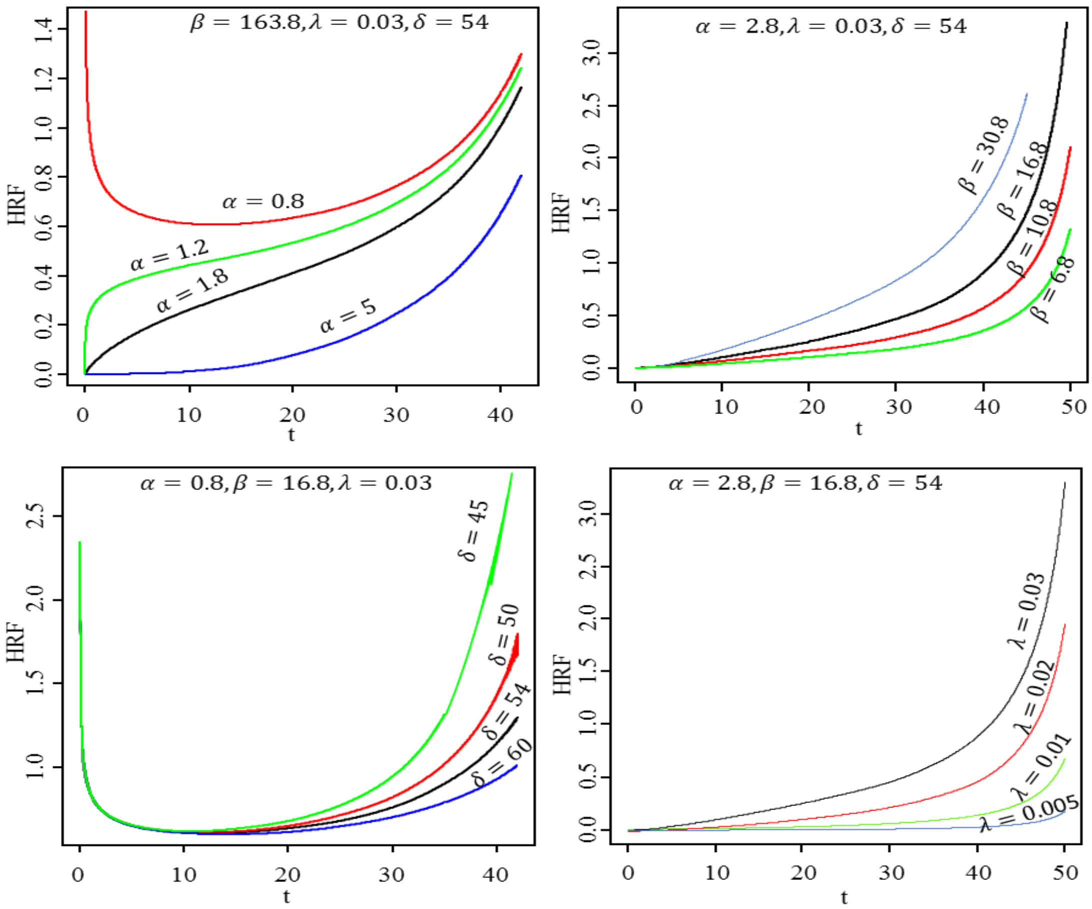

Figure 2: K-Epsilon distribution hazard rate function at various parameter values

3.3 Order statistics distribution

Order statistics play important role in life testing and reliability analysis. Given a random sample $x_1, x_2, \ldots, x_n$ of a random variable X having the K-epsilon distribution and density functions F_X and f_X, respectively. The distribution of the r^{th} order statistic, $X_{(r)}$, of the ordered sample, $x_{(1)}, \ldots, x_{(r)}, \ldots, x_{(n)}$, is given by

$$f_{X_{(r)}}(x) = ab\lambda\Lambda\Upsilon(1 - Z)Z^{a-1}\Psi^{b(n-r+1)-1}(1 - \Psi^b)^{r-1} \quad (13)$$

Where $\Lambda = \frac{n!}{(r-1)!(n-r)!}$; $\Upsilon = \left(\frac{\delta^2}{\delta^2 - x^2}\right)$;

$Z = 1 - \left(\frac{x+\delta}{\delta - x}\right)^{-\frac{\lambda}{2}\delta}$;and $\Psi = 1 - Z^a$. From (13), the K-epsilon distribution of the first order statistic, $X_{(1)}$, and the n^{th} order statistic, $X_{(n)}$, are given, respectively, by

$$f_1(x) = ab\lambda n\Upsilon(1 - Z)Z^{a-1}\Psi^{bn-1} \quad (14)$$

and

$$f_n(x) = ab\lambda n\Upsilon(1 - Z)Z^{a-1}\Psi^{b-1}(1 - \Psi^b)^{n-1} \quad (15)$$

4. Parameter Estimation

As noted earlier, the explicit expressions for the K-epsilon probability density, cumulative distribution and quantile functions has made the distribution a good candidate for the deployment of the **fitdistrplus** package in R. This has the advantage of producing parameter estimates, their standard errors, and many other important statistical properties.

5. Application

The K-epsilon distribution is applied to 2 datasets, namelydeuterium excesses and tax revenue datasets. These are represented in boxplots in Fig 3 below.

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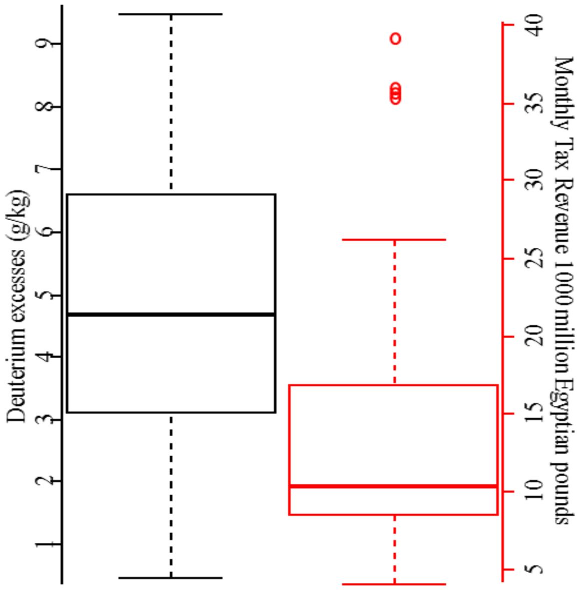

Figure 3: Boxplots of Deuterium excesses and Monthly Tax Revenue

The first dataset is hourly record of deuterium (heavy hydrogen – one of the two stable isotopes of hydrogen) excesses from 2015-11-11T00:00 to 2015-11-24T23:00 [2], given in the appendix.

Parameter estimates and Kolmogorov-Smirnov goodness-of-fit test results are given in Table 1 while the fitted K-epsilon densityplot is presented in Fig. 3.

Table 1: K-epsilon distribution fit results for deuterium excesses dataset

Distribution	$a\ (se)$	$b\ (se)$	$\lambda\ (se)$	$\delta\ (se)$	$-LL$	$KS(CV)$	Remark
K-epsilon	1.5866 (0.1061)	221.24 (95.9727)	0.0050 (0.0007)	9.7456 (0.2744)	676.68	0.0518 (0.0781)	Good fit

se = standard error, LL = log-likelihood, KS = Kolmogorov-Smirnov, CV = critical value

From Table 1, it could be observed that the K-epsilon distribution has fitted the deuterium excesses dataset.The results of the goodness-of-fit test shown in Table 1 above provide sufficient evidence that the K-epsilon distribution fits the dataset. Also, the standard errors of the parameter estimates obtained indicate they are of good precision.

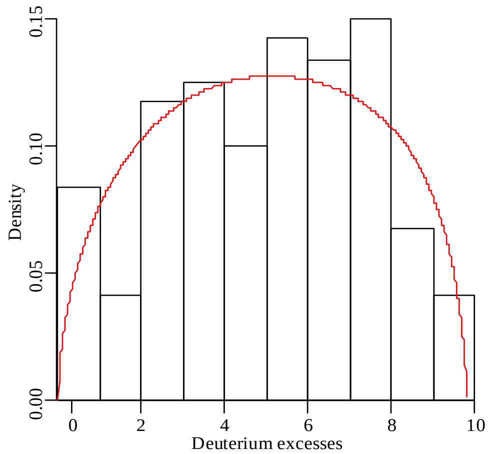

Figure 3: K-Epsilon distribution fit of hydrogen Isotope (Deuterium) excesses

The second dataset for application is the monthly records of tax revenue of Egypt between January 2006 and November 2010, in 1000 million Egyptian pounds, reported in [15]. It is presented in the appendix

.

The results for parameter estimate and goodness-of-fit test are presented in Table 2 while the fitted K-epsilon probability density plot is presented in Fig. 4.

Table 2: K-epsilon distribution fit results for Monthly Tax Revenue dataset

Distribution	$a\ (se)$	$b\ (se)$	$\lambda\ (se)$	$\delta\ (se)$	$-LL$	$KS(CV)$	Remark
K-epsilon	63.8712 (1.8779)	0.1877 (0.0378)	0.6924 (0.0637)	204.88 (0.2086)	187.973	0.0735 (0.1771)	Good fit

se = standard error, LL = log-likelihood, KS = Kolmogorov-Smirnov, CV = critical value

Table 2 also suggest that the K-epsilon distribution has successfully fitted the tax revenue dataset. The parameter estimates are of good precision.

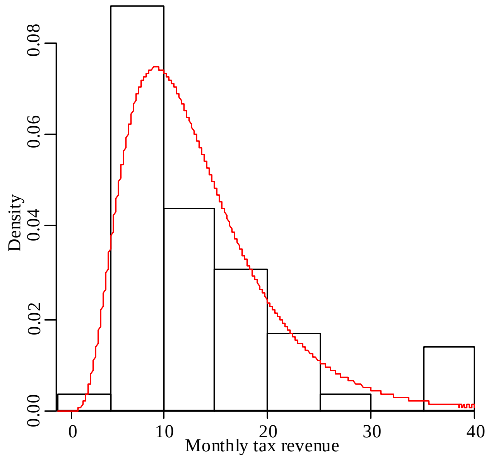

Figure 5: K-epsilon distribution fit of Monthly Tax Revenue

6. Conclusion

A new distribution, called the Kumaraswamy epsilon (K-epsilon) distribution, has been derived in this study. It is constructed from the Kumaraswamy generator (Kumaraswamy-G) of probability density functions. Here, the parent distribution is the epsilon distribution. The new distribution, unlike the beta distribution, has explicit expressions for the cumulative distribution and quantile functions. This makes parameter estimates much easier, particularly in R. Although the moments of the K-epsilon distribution could not be expressed in explicit form, other properties of the K-epsilon distribution such as the distribution of order statistics, survival and hazard rate functions have been derived. The shapes of the plots of the probability density and hazard rate functions at various parameter values suggest that the K-epsilon distribution holds the potential for wide applications within its parameter space.

The K-epsilon distribution was applied to two different lifetime datasets, namely, deuterium (hydrogen isotope) excesses and monthly tax revenue. The results of the fit of this distribution to these datasets are good, confirming its wide usefulness in practical applications.

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Appendix

Proof of Proposition 1

Proof

(i) We need to find the limiting values of $f_X(x)$. That is

$$\lim_{x\to 0} f_X(x) = \lim_{x\to 0} ab\lambda\left(\frac{\delta^2}{\delta^2-x^2}\right)\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a-1}\left(1-\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a}\right)^{b-1}$$

$$= ab\lambda \lim_{x\to 0}\left(\frac{\delta^2}{\delta^2-x^2}\right)\lim_{x\to 0}\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\lim_{x\to 0}\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a-1}\lim_{x\to 0}\left(1-\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a}\right)^{b-1}$$

$$= ab\lambda\cdot 1\cdot 1\cdot(1-1)^{a-1}\cdot(1-(1-1)^{a})^{b-1}$$

$$= 0$$

Also

$$\lim_{x\to \delta} f_X(x) = \lim_{x\to \delta} ab\lambda\left(\frac{\delta^2}{\delta^2-x^2}\right)\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a-1}\left(1-\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a}\right)^{b-1}$$

$$= ab\lambda \lim_{x\to \delta}\left(\frac{\delta^2}{\delta^2-x^2}\right)\lim_{x\to \delta}\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\lim_{x\to \delta}\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a-1}\lim_{x\to \delta}\left(1-\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a}\right)^{b-1}$$

$$= ab\lambda\cdot\infty\cdot 0\cdot(1-0)^{a-1}\cdot(1-(1-0)^{a})^{b-1}$$

$$= 0$$

Since $f_X(x)$ at both limits is zero (0), it is unimodal.

(ii) $\int_0^{\delta} f_X(x)\,dx = \lim_{x\to\delta}\int_0^{x} f_X(t)\,dt$

$$= \lim_{x\to\delta} F_X(x)$$

$$= \lim_{x\to\delta}\left(1-\left(1-\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a}\right)^{b}\right)$$

$$= 1-\lim_{x\to\delta}\left(1-\left(1-\left(\frac{x+\delta}{\delta-x}\right)^{-\frac{\lambda}{2}\delta}\right)^{a}\right)^{b}$$

$$= 1-\left(1-\left(1-\left(\frac{\delta+\delta}{\delta-\delta}\right)^{-\frac{\lambda}{2}\delta}\right)^{a}\right)^{b}$$

$$= 1-(1-(1-0)^{a})^{b}$$

$$= 1$$

Hourly record of deuterium excesses from 2015-11-11T00:00 to 2015-11-24T23:00

3.02, 2.57, 1.74, 2.25, 1.55, 0.66, 0.56, 0.92, 1.24, 1.20, 0.86, 1.18, 1.34, 1.94, 1.97, 1.79, 1.82, 1.94, 2.06, 1.84, 2.24, 2.01, 2.32, 3.15, 3.45, 3.38, 3.54, 3.89, 4.98, 3.43, 3.98, 4.19, 4.36, 4.97, 4.65, 4.15, 3.49, 3.19, 3.37, 3.58, 3.82, 4.00, 4.21, 4.69, 4.86, 5.06, 4.93, 4.77, 4.94, 4.95, 5.34, 5.26, 4.76, 5.35, 5.59, 5.54, 5.78, 5.92, 5.80, 5.69, 5.85, 5.74, 5.64, 5.82, 5.21, 5.48, 6.19, 7.92, 8.93, 8.45, 8.74, 8.35, 8.40, 8.00, 7.69, 6.22, 6.99, 6.85, 7.45, 7.88, 7.80, 8.05, 8.22, 8.40, 7.90, 6.65, 6.29, 6.74, 5.72, 4.75, 4.12, 4.20, 3.86, 3.52, 3.53, 3.27, 3.23, 3.13, 2.62, 3.94, 4.52, 5.12, 5.77, 5.94, 5.92, 5.86, 5.86, 5.79, 5.35, 5.57, 5.71, 5.81, 5.96, 6.04, 6.58, 6.69, 6.70, 6.97, 6.99, 7.23, 7.32, 6.61, 7.20, 7.99, 8.17, 8.50, 8.58, 8.51, 8.97, 8.88, 8.73, 8.62, 8.93, 9.45, 9.12, 9.09, 8.73, 8.98, 8.76, 8.91, 9.18, 9.30, 9.23, 9.07, 8.59, 9.08, 9.47, 9.12, 8.73, 8.47, 8.89, 8.15, 6.77, 6.72, 6.41, 6.17, 4.79, 4.70, 4.35, 4.32, 4.32, 4.11, 3.98, 4.25, 4.21, 3.97, 3.51, 2.70, 2.56, 2.67, 2.12, 2.58, 2.37, 1.99, 1.57, 1.20, 1.19, 0.70, 0.94, 1.07, 1.09, 1.31, 1.18, 0.86, 0.81, 0.47, 0.70, 0.83, 1.04, 1.69, 0.65, 1.33, 1.55, 2.13, 2.38, 2.67, 2.49, 2.19, 2.20, 1.69, 1.66, 1.87, 2.67, 2.43, 1.93, 2.63, 3.07, 3.61, 3.40, 3.14, 3.54, 2.93, 3.39, 2.45, 2.50, 3.41, 3.29, 3.57, 3.41, 3.67, 3.03, 3.37, 3.71, 3.74, 3.84, 3.52, 3.47, 3.75, 3.65, 3.48, 3.49, 4.40, 3.67, 4.41, 3.65, 3.49, 2.85, 3.79, 3.86, 4.00, 3.88, 4.69, 4.27, 3.80, 3.72, 3.89, 3.71, 2.93, 2.10, 1.79, 2.98, 3.91, 5.28, 5.51, 5.83, 5.54, 5.42, 4.93, 4.50, 3.12, 4.33, 4.60, 4.00, 4.30, 4.85, 5.01, 4.90, 5.50, 6.07, 6.36, 6.50, 6.83, 7.39, 7.56, 7.22, 7.28, 7.87, 7.63, 7.68, 7.00, 7.28, 7.19, 6.86, 6.91, 6.64, 6.72, 6.87, 7.14, 6.86, 6.80, 6.30, 6.23, 5.90, 6.02, 6.16, 5.85, 5.98, 5.55, 5.53, 5.45, 5.61, 5.31, 5.24, 4.92, 5.27, 2.24, 2.50, 2.57.

Monthly records of tax revenue of Egypt between January 2006 and November 2010 (1000 million Egyptian pounds)
5.9, 20.4, 14.9, 16.2, 17.2, 7.8, 6.1, 9.2, 10.2, 9.6, 13.3, 8.5, 21.6, 18.5, 5.1, 6.7, 17, 8.6, 9.7, 39.2, 35.7, 15.7, 9.7, 10,4.1, 36, 8.5, 8, 9.2, 26.2, 21.9, 16.7, 21.3, 35.4, 14.3, 8.5, 10.6, 19.1, 20.5, 7.1, 7.7, 18.1, 16.5, 11.9, 7, 8.6, 12.5,10.3, 11.2, 6.1, 8.4, 11, 11.6, 11.9, 5.2, 6.8, 8.9, 7.1,10.