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Particle Swarm Intelligence Based Reduction Method Applied to Power System Descriptor Models

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Abstract: ***To model and analyze a large complex dynamic system such as power systems is a very challenging task. Modelling of large real- time systems results in a large number of differential equations that lead to transfer function models that represents a higher order system. Higher order systems impose heavy computational burden, along with additional memory requirements. Therefore, it is necessary to reduce the power system model for simplifying the simulation and controller design. Pragmatic methods are preferred in power system model reduction for good performance, as they are simple to use along with their ability to maintain the physical structure of the model. A good algorithm to model the order reduction of power system applications should preserve the important characteristic and perform ability of the original system. The dimensions and density of typical accurate power system models give arise to difficulties which have been handled by several techniques. In this paper Particle Swarm Optimization (PSO) algorithm, evolutionary technique is employed to a two power system models. The first model considered is a single input single output (SISO) single machine connected to an infinite bus (SMIB) and the second model considered is a multiple input multiple output (MIMO) single machine connected to an infinite bus (SMIB). The PSO algorithm is based on the minimisation of the integral squared error between the transient responses of original higher order and reduced order model pertaining to a unit step input. The reduced models show the preservation of stability and other characteristic parameters of original system. The reduced order model so obtained shows minimum integral square error comparable with other reduction techniques.***

Keywords: Particle Swarm Optimization, Model Order Reduction, Single Machine Infinite Bus System, Single input single output, multi input multi output

1. Introduction

Real life systems and processes are quite complex and their mathematical modelling leads to high order differential equations. For simulation of their behavior, analysis and synthesis, the high order model of systems and processes needs to be reduced to lower order model whose behavior resembles that of original model as far as feasible. Various methods of model order reduction have been listed and described comprehensively and comparatively by GenesioRetal[1], Bosley and Lees[2], Fortuna et al[3] and many others. Shamash Y [4] showed that the Padé approximation method, Time Moments Method and Continued Fraction Expansion methods are equivalent to each other and these reduced models may turnout to be not stable even though the original model is stable. This instability problem has been addressed by Hutton M and others [5-9] by stability based reduction methods which make use of some stability criterion like Routh approximation or Mihailov stability criterion. Other methods which produce stable reduced models without using any stability criterion have been proposed by Chenand others [10-12].Lucas, Gutman [13, 14] proposed model order reduction (MOR) based on differentiation of the numerator and denominator polynomials but suffers with steady state error in the response of MOR as compared to original system. Many mixed techniques have also been developed by Shamash and others [7, 8, 12, 15, 16] in which the denominator of the reduced model is derived by using stability criterion and the numerator has been derived by some other method.

In order to obtain a better reduced model an optimisation is required and Luus [17] proposed numerical optimization to minimize the deviation between the frequency responses of the high and low order models. Howitt and Luus[18] considered the poles and zeros of the reduced models to be free parameters and are obtained by minimizing the integral square error in impulse or step responses and shows better results as compared with other methods available.

In the recent decade, bio-inspired evolutionary techniques such as Particle Swarm Optimization (PSO) Genetic Algorithm (GA), Differential evolution (DE),simulated annealing, Harmony Search (HS) algorithm, tabu search algorithm, cuckoo search algorithm have been applied for model reduction of high order systems. All of the GA, DE and PSO techniques are population based stochastic optimization versatile techniques, which utilize heuristics from nature and are capable of optimizing a solution in multi modal search spaces by minimization of an objective function which is often Integral Square Error (ISE) [19, 20]. PSO has been motivated by the behavior of organisms, such as fish schooling and bird flocking. Generally, PSO has few parameters, computationally efficient and has found applications in many areas. PSO has a flexible and well-balanced mechanism [21] to enhance the global and local exploration abilities as compared to other bio-inspired techniques such as Harmony Search (HA), Tabu Search algorithms. Both the PSO and GA methods are widely used for model order reduction. PSO utilizes the randomness of real numbers and the facilitation of global communication which occurs between swarm particles. The main advantage of the developed method in this paper is that it is applicable for all systems and not restricted to only stable or strictly proper systems.

Power systems are high order nonlinear large-scale systems with randomly changing operating conditions and are responsible for poor performance due to the occurrence of abrupt small load perturbations, parameter uncertainties [22]etc. A fairly complex and higher order model is obtained while modeling a large real time system from theoretical considerations. Accurate system modeling is an important step in power system engineering but the main problem is that the linearized models could be non-minimum phase, unstable or improper so an appropriate model reduction technique is a must. In this paper two linearized practical power systems, Single Machine Infinite Bus System (SMIB) Single input single output (SISO), and Single Machine Infinite Bus System (SMIB) Multi input multi output (MIMO) are employed to demonstrate the application of PSO algorithm for MOR. The worthiness and effectiveness of the method are investigated in terms of integral square error (ISE). PSO aids in finding the best values among the possible ones to match the requirements of the large-scale system. Therefore, it is most suitable for power systems. The proposed method is very effective as can be seen from simulation results. Despite having various optimization techniques for different problems, [23] the quest for a global optimization method which shall be one-size-fit-all for finding solution is underway.

Thepaper is divided into five sections including the introduction. Section II describes the statement of SISO and MIMO systems. Section III discusses the implementation of PSO algorithm. In section IV, two numerical examples from practical power system (SISO [24] and MIMO [3]) have been solved and their results are compared with the available techniques [3, 32] in literature. Section V concludes the paper.

2. Problem Formulation

A system can be generalized into time linear invariant linear differential algebraic equation

$$\sum : \begin{array}{l} \dot{x}(t) = Ax(t) + BU(t) \\ y(t) = Cx(t) + DU(t) \end{array} \Leftrightarrow \frac{|A|B|}{|C|D|} \tag{2.1}$$

where $A \in R^{nxn}$, $B \in R^{nxm}, C \in R^{pxn} D \in R^{pxm}$

If the state space description is easily available, directly model order reduction in time domain can be applied [25].

2.1 SISO system

If A, B, C and D are known than transfer function can be obtained using MatLab [26].

$$G(s) = \frac{\sum_{i=0}^{m-1} e_i s^i}{\sum_{j=0}^{m} d_j s^j} \tag{2.2}$$

$$R(s) = \frac{\sum_{i=0}^{s-1} b_i s^i}{\sum_{j=0}^{s} a_j s^j} \tag{2.3}$$

where a_j, b_i, d_j, e_i are coefficients of higher and lower order systems. The purpose is to find a reduced s^{th} order system model R(s) such that it maintains the important characteristics of G(s) for the same type of inputs.

2.2 MIMO system

The original system is described by transfer function G(s)

$$G(s) = \frac{1}{D(s)} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1p} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2p} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mp} \end{bmatrix} \tag{2.4}$$

where G(s) is a (mxp)transfer matrix

$$[G(s)] = [g_{ji}(s)], j = 1,2,\ldots\ldots m, i = 1,2\ldots\ldots p$$

The $g_{ji}(s)$ of [G(s)] is shown below

$$g_{ji}(s) = \frac{a_{ji}(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots\ldots\ldots a_{m-1} s^{m-1}}{b_0 + b_1 s + b_2 s^2 + \ldots\ldots b_{m-1} s^{m-1} + s^m} \tag{2.5}$$

The transfer matrix (nxq)reduced order system is given as

$$R(s) = \frac{1}{\bar{D}(s)} \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1q} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2q} \\ c_{31} & c_{32} & c_{33} & \cdots & c_{3q} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & c_{n3} & \cdots & c_{nq} \end{bmatrix} \tag{2.6}$$

where

$$[R(s)] = [r_{ji}(s)], j = 1,2,\ldots\ldots\ldots n, i = 1,2\ldots\ldots q$$

The $r_{ji}(s)$ of R(s) is given below

$$r_{ji}(s) = \frac{c_{ji}(s)}{\bar{D}(s)} = \frac{c_0 + c_1 s + c_2 s^2 + \ldots\ldots\ldots c_{n-1} s^{n-1}}{d_0 + d_1 s + d_2 s^2 + \ldots\ldots d_{n-1} s^{r-1} + s^n} \tag{2.7}$$

where a_{ji}, b_{ji}, $c_{ji}$and d_{ji} are scalar constants, and n is smaller than m.

3. Particle Swarm Optimisation (PSO)

Flocking of birds was an inspiration which led Eberhart and Kennedy [27] to formulate particle swarm optimization. In PSO, each particle or a ‘bird’ is treated as a solution in the search zone which modifies its flight according its own flying experience as well as the flying experience of other particles. The swarm consists of particles each of which has a memory. This memory enables it to remember its best position the search space ever visited by it.The main idea was to simulate the unpredictable choreography of a bird flock. Based on the observation of the evolution of the algorithm, it’s realized that the conceptual model is in fact an optimizer.

3.1 PSO Method

Imitation of bird flocks by Particle swarm optimization results in a desired outcome to the intricate non-linear optimization problem. The *m*-dimensional function *g* for PSO method is taken as:

$$g(x_1, x_2, \ldots\ldots x_m) = g(X) \qquad (3.1)$$

Where x_j signifies the set of independent variables of the given function. The objective is to find a valuex^* such that the function $g(x^*)$ is either a maximum or a minimum in the search space. The PSO algorithm is initialized by a population of random solutions, each having a randomized velocity. The particlesmake efforts to improve themselves by imitating the position of their own best fitness achieved so far and the best fitness achieved so far by any of their peers. The x_j^tin equation (3.2) denotes the position of particle which is updated at time step *t*.

$$x_j^{t+1} = x_j^t + v_j^{t+1} \quad \text{with} x_j^0 \sim U\,(x_{min}\,,\, x_{max}\,) \quad (3.2)$$

where, $v_i^t$is the velocity vector of particle responsible for the optimization process which is continuously being modified according to its flying experience.$U\ (x_{min}\ ,\ x_{max}\ )$ is the uniform distribution wherex_{min} ,x_{max} are its minimum and maximum values respectively. PSO is initialized by a population of random solutions and randomized velocity and evaluated to compute fitness together with finding the personal best (best value of each particle) and global best (best value of particle in the entire swarm). These two PSO algorithms, i.e.Global Best (*gbest*) and Local Best (*lbest*) differ in the size of their neighborhoods. In both algorithms, initially the particles' velocity is updated by the personal and global bests, and subsequently each particle's position is updated by the current velocity. AStopping criterion, predetermined in advance,ends the algorithm.

3.1.1 Global best PSO

The overall best position out of all the particles in the entire swarm is referred to as*gbest.* It uses a star social network topology since it leads to faster convergence than other topologies.In this method each individual particle, $j \in [1, \ldots, m]$ where m>1 , has a current position x_j in search space, a current velocity,v_j, and a personal best position in search space, $P_{best,j}$. In a minimization problem where individual *j* has the minimum value as obtained by the objective function g, the personal best position in the search space is denoted as $P_{best,j}$. Minimizing a function *f*, this is equivalent to maximizing a function *g*, where $f = -g$[28].$G_{best,j}$, called the global best position, is the smallest value in the entire swarm. The $P_{best,j}$and $G_{best,j}$ values are modified as shown in equations (3.3) and (3.4), respectively. Then $P_{best,j}$ at step, $t+1, t \in [0, \ldots.. m]$ is computed as shown in equation(3.3).

$$P_{best,j}^{t+1} = \begin{cases} P_{best,j}^t \ldots\ldots\ldots\ldots if \ldots. f\,(x_j^{t+1}) > P_{best,j}^t \\ x_j^{t+1} \ldots\ldots\ldots\ldots.. if \ldots. f\,(x_j^{t+1}) \le P_{best,j}^t \end{cases} \quad (3.3)$$

The global best position G_{best} at time step *t* is computed as

$$G_{best} = \min\{P_{best,j}^t\ \} \qquad (3.4)$$

where $j \in [1, \ldots, m]$ and m>1

It may be noted from above that the finest position visited by the individual bird since the first stage represents personal best P_{best} where as the finest position discovered by any of the bird in the entire flock is represented by the global best position $G_{best.}$

The velocity of particle *j* is computed by

$$v_{ji}^{t+1} = wv_{ji}^t + c_1 r_{1i}^t [P_{best,j}^t - x_{ji}^t] + c_2 r_{2i}^t [G_{best,j}^t - x_{ji}^t] \ (3.5)$$

Where v_{ji}^t is the velocity vector of particle *j* in dimension *i* at time *t* and

$$v_i^{min} \le v_{ji}^t \le v_i^{max}$$

x_{ji}^t is the position vector of particle *j* in dimension *i* at time *t* ; $P_{best,j}^t$ is the personal best position of particle*j* in dimension *I* found from initialization through time *t;*

$G_{best,j}$is the global best position of particle *j* in dimension *I* found from initialization through time *t* ; r_{ji}^1 and r_{ji}^2 are random numbers from uniform distribution *U (0, 1)* at time *t;*

c_1 and c_2 are constants which are used to uniform the involvement of the cognitive and social components respectively;

w is inertia weight factor that is used to control the effect of the previous velocities on the current velocity.

The flow chart of the *gbest* PSO algorithm for order reduction is shown in Fig. 1.

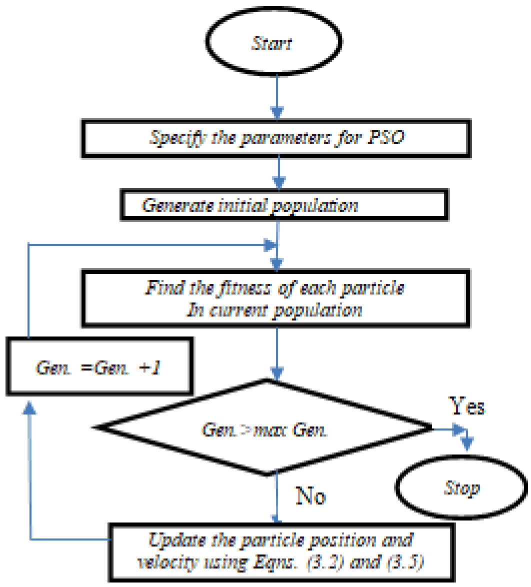


Figure 1: Flowchart of order reduction obtained by PSO

3.1.2 Local best PSO

The *lbest* PSO method, as the name suggests, restricts the influence of each particle to the best-fit particle selectedfrom its neighborhood. The velocity of particle is computed by:

$$v_{ji}^{t+1} = wv_{ji}^t + c_1 r_{1i}^t [P_{best,j}^t - x_{ji}^t] + c_2 r_{2i}^t [L_{best,j}^t - x_{ji}^t] \ (3.6)$$

The algorithm for '*lbest* ' PSOis same as that of '*gbest*' PSO except *gbest* is replaced by *lbest* and the velocity is given by above equation (3.6)

Basically, there are two dissimilarities between the '*gbest*' PSO and the '*lbest*' PSO: First is that because of the larger particle interconnectivity of the *gbest* PSO, sometimes the convergence is faster.Second it is less vulnerable to being stuck in local minima [29] due to the higher diversity of the *gbest* PSO, in contrast to the neighborhood information obtained in the *lbest* PSO algorithm. Therefore it can be inferred that every particle, under the *gbest* PSO algorithm, collects the information from the best particle in the entire swarm.

3.2 PSO algorithm parameters

The PSO algorithm is affected by some parameters. It becomes necessary to choose the optimum value of setting the parameter for the best performance of PSO for different types of applications. The swarm size or number of particles, number of iterations, velocity components, and acceleration coefficients are the basic PSO parameters.

The number of particles *n* contain in a swarm denotes the swarm size or population size. To ensure a bigger search space to be covered per iteration requires a big swarm. A big swarm may result in reducing the numbers of iterations required to acquire a noble optimization result [30]. In contrast, huge amounts of particles increase the computational complexity per iteration, and more time consuming. The number of iterations to attain a worthy result is problem-dependent. Too large iterations may unnecessarily add computational complexity and time needed whereas too low numbers may stop the search process prematurely. For updating particle's velocity, the velocity components are of utmost importance and consists of three parts; namely inertia, cognitive and social parts as shown in equations (3.5) and (3.6). The inertia component v_{ji}^t represents as a thrust which stops any drastic change in the path of the particles and biases it towards the present direction thus providing a memory of the previous flight direction which means movement in the immediate past. The term $c_1 r_{1j}^t [p_{best,j}^t - x_{ji}^t]$ is called cognitive component which measures the functioning of the particles comparative to past performances. It has a memory and stores best position for the particle and its effect corresponds to the particles tendency to return to the most satisfied past positions. The term $c_2 r_{2j}^t [G_{best} - x_{ij}^t]$ for *gbest* PSO or $c_2 r_{2j}^t [L_{best,i} - x_{ij}^t]$ for *lbest* PSO is called social component. As the term suggests, this measures the performance of the particles i relative to a group of particles or neighbors and its effect is to make individual particle fly near the best position obtained by the particle's neighborhood. The acceleration constantsc_1 and c_2,along with the random numbers $r_1$andr_2, maintain the stochastic impact of the cognitive and social components of the particle's velocity respectively. The constant c_1 and c_2 indicate the level of confidence a particle has in itself and in its neighbors respectively. Wrong initialization of c_1 and $c_2$may possibly result in deviating or cyclical performance [28]. From the different empirical researches, it has been proposed that the two acceleration constants should be $c_1 = c_2 = 2$[29].The inertia weight is a control parameter that determines how much a particle holds its current velocity in the next iteration. For efficient performance of PSO, theinertia weight (w) and the maximum allowable velocity is critical. Initially the inertia weight [31] was taken as constant, but subsequent experimental results proposed to start with a larger value initially (around 1.2) so as to mimic global exploration of the search space, and gradually decrease it towards zero in order to get more refined solutions[28].

4. Systems Under Consideration

A Single machine connected to infinite bus through transmission line is considered in both the examples as shown in fig.2. In this paper PSO algorithm is used to reduce the objective function, which is the integral square error (ISE) between the transient responses of higher and reduced order model system is:

$$ISE = \int_0^\infty (g(t) - y(t))^2 \qquad (4.1)$$

Where $g(t)$ and $y(t)$ are the higher order and lower order unit step responses respectively for SISO system. For MIMO system $g_{ji}(t)$ and $y_{ji}(t)$ can be taken in place of $g(t)$ and $y(t)$. The acceptability and reliability of reduced system is measured in terms of ISE. Lesser the ISE performance index, the closer is R(s) to G(S). Two numerical examples are undertaken in this study.

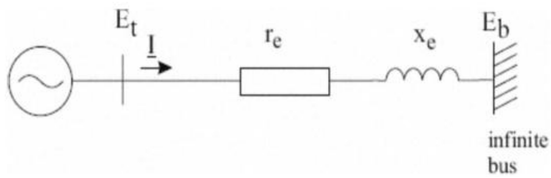


Figure 2: Single machine connected to infinite bus

*Case1:*A linearized SISO SMIB system has been taken from [24].The original seventh order transfer function is given below

$$G(s) = \frac{2s^6 + 420.4s^5 + 9435.01s^4 + 139000.01s^3 + 466300.01s^2 + 434200.03s + 187700}{s^7 + 23.28s^6 + 331.7s^5 + 2640.05s^4 + 17570.06s^3 + 51650.01s^2 + 35340.02s + 17290} \qquad (4.2)$$

Third order reduced model for SISO SIMB system using PSO algorithm is

$$R_3(s) = \frac{-15.04s^2 + 355.3429s + 1203.96774835}{0.5722029s^3 + 2.7132881s^2 + 38.83625s + 110.07886182} \qquad (4.3)$$

Simulation results along with performance index ISE are presented in Table I. From Table I It can be understood that the transient state responses of the PSO reduced order model is almost identical with that of the original model and shows better response as compared to previous work [32]. The systems mentioned in the Table II are found to be stable as indicated by the Eigen values calculated for both original and reduced models. It basically proves the "preservation of stability of a stable system" even with the reduced models. Fig 2 shows the step responses of original and reduced third order models. It can be easily seen from the Fig.2 that the GA [32] method possess some steady state error whereas proposed PSO does not.

Table I: Response Data, ISE of Original SMIB SISO and Reduced Power System Models

System	Rise Time	Settling Time	Peak Time	Peak	Overshoot	ISE
Original	0.1722	5.9332	0.4271	15.9952	47.34	---
GA [32]	0.1647	6.1503	0.4194	16.71	39.8342	6.9735
PSO(proposed)	0.1317	4.5092	0.4363	19.3024	76.4823	0.4999

Table II: Eigenvalue Analysis 0f SMIB SISO and Reduced Power System Models

System	Original	Third order GA [32]	Third order PSO (proposed)
Eigenvalue	-8.4107 ± 8.3263i, -0.3624 ± 0.5564i -0.9033 ± 8.3954i, -3.9273 + 0.0000i	-0.7153 ± 8.2085i, -1.8515 + 0.0000i	-0.8376 ±7.8761i, -3.0665 + 0.0000i

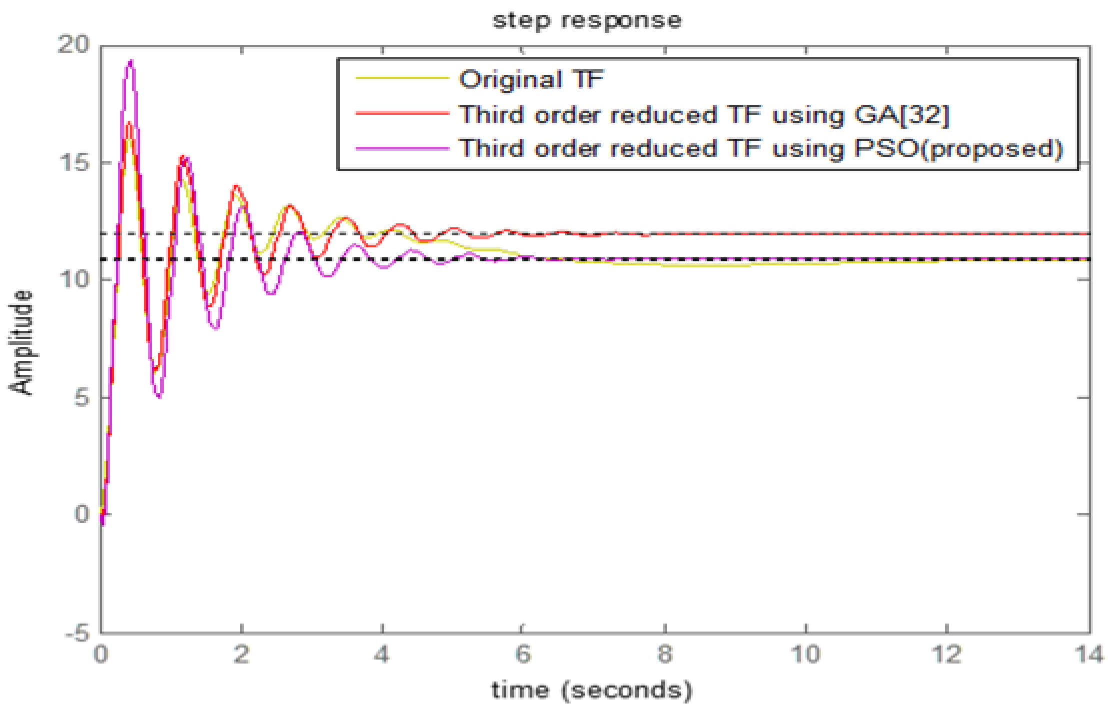


Figure 2: Step responses of original and reduced third order models for Case1.

Case 2: Amultiple input multiple output power system consisting of a salient-pole synchronous generator connected to an infinite bus-bar with two inputs and three outputs is taken from [3]. The elements of equation (2.4) of original seventh order transfer matrix is given below.

$$D(s) = s^7 + 258.64s^6 + 430959s^5 + 48281341s^4 + 1844291625s^3 + 2.50\times10s^2 + 5.4.6s + 1.18\times10 \quad (4.4)$$

The six numerators are given below

$$a_{11}(s) = -12.41377s^4 + 12124.7928s^3 - 2881410.04s^2 - 336887170.06s - 6506657015.653 \quad (4.5)$$

$$a_{12}(s) = 52.08s^5 + 10758.478s^4 + 21869382.1s^3 + 1373714674.95s^2 + 21862117031.65s + 16563848167.052 \quad (4.6)$$

$$a_{21}(s) = 52.08s^6 + 10758.478s^5 + 21869382.1s^4 - 2881410.04s^3 - 336887170.06s^2 - 6506657015.653 \quad (4.7)$$

$$a_{22}(s) = 52.08s^6 + 10758.478s^5 + 21869382.1s^4 + 1373714674.95s^3 + 21862117031.65s^2 + 16563848167.052s \quad (4.8)$$

$$a_{31}(s) = 0.2004556\, s^6 + 47.86274\, s^5 + 39267.396\, s^4 + 5153949.74\, s^3 + 231384193.179\, s^2 + 3482982404\, s + 5467885491 \quad (4.9)$$

$$a_{32}(s) = 7.453465\, s^5 + 27013.4259\, s^4 + 868109.387\, s^3 - 16222929.178\, s^2 - 632247763.006\, s - 8357195607.8251 \quad (4.10)$$

The third order reduced modelof each element of transfer matrix for multivariable system is obtained below using PSO algorithm.The following polynomials refer to elements of transfer matrix equation (2.7).

$$\bar{D}(s) = 0.92754064s^3 + 13.82695634s^2 + 34.16437298s + 7.49738098 \quad (4.11)$$

$$c_{11}(s) = -0.81957490s - 4.11417614 \quad (4.12)$$

$$c_{12}(s) = 11.5321s^2 + 13.43434s + 10.248 \quad (4.13)$$

$$c_{21}(s) = -0.13461163s^2 - 3.8387669s + 0.01739686 \quad (4.14)$$

$$c_{22}(s) = 13.51151505s^2 + 9.63296015s + 0.02369460 \quad (4.15)$$

$$c_{31}(s) = -0.203878s^2 + 2.123731s + 3.448266 \quad (4.16)$$

$$c_{32}(s) = 0.26789806s^2 - 0.44707174s - 5.29842535 \quad (4.17)$$

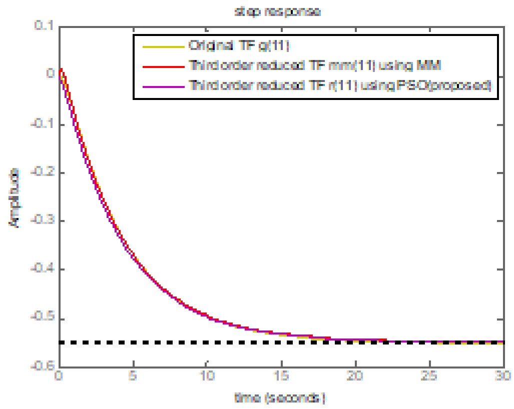
Table III and Table IV show time response parameters, performance index ISE and eigenvalue analysis of SMIB MIMO system respectively. The response parameters of reduced proposed systems are almost matching with the original one. Fig.4 shows comparison of time response of higher order system, proposed PSO reduced order model and previously proposed mm [3] method. The ISE is also small of almost all the PSO proposed reduced models as can be seen from Table III. The MATLAB (R2014A) software is used to obtain the results for SISO and MIMO systems.

Table III: Response Data, ISE of Original SMIB MIMO and Reduced Power System Models

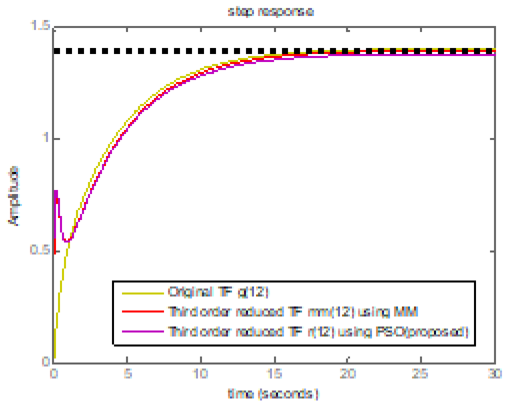
System	Rise Time	Settling Time	Peak Time	Peak	ISE
g_{11}	9.1347	16.6028	31.7375	0.5511	---
mm_{11} [3]	9.0885	16.4538	30.2314	0.5481	4.3681E-06
r_{11} **(proposed)**	9.0859	16.3569	30.1389	0.5484	7.1328E-04
g_{12}	8.2806	15.0933	29.4124	1.4028	---
mm_{12} [3]	8.7874	15.4456	25.7858	1.3940	0.0077
r_{12} **(proposed)**	8.7387	15.3754	24.5355	1.3787	0.0153
g_{21}	0	10.9215	0.1120	0.1301	---
mm_{21} [3]	0	17.6135	1.0602	0.1039	0.0040
r_{21} **(proposed)**	9.0439	17.4071	0.9192	0.0872	1.6446E-05
g_{22}	0	11.0817	0.0725	0.9011	---
mm_{22} [3]	0	11.5069	0.1820	0.7996	0.0022
r_{22} **(proposed)**	6.8035E-05	10.7407	0.1786	0.8050	1.8879E-05
g_{31}	8.9816	15.9761	52.7754	0.4634	---
mm_{31} [3]	8.8748	15.9160	26.1460	0.4690	5.5786E-06
r_{31} **(proposed)**	8.8325	15.8285	25.1568	0.4590	2.2642E-06
g_{32}	9.1348	16.5771	37.9377	0.7082	---
mm_{32} [3]	9.0856	16.4390	30.2314	0.7182	0.0015
r_{32} **(proposed)**	9.0681	16.4192	30.1389	0.7062	2.0385E-04

Table IV: Eigenvalue Analysis of SMIB MIMO and Reduced Order Power System

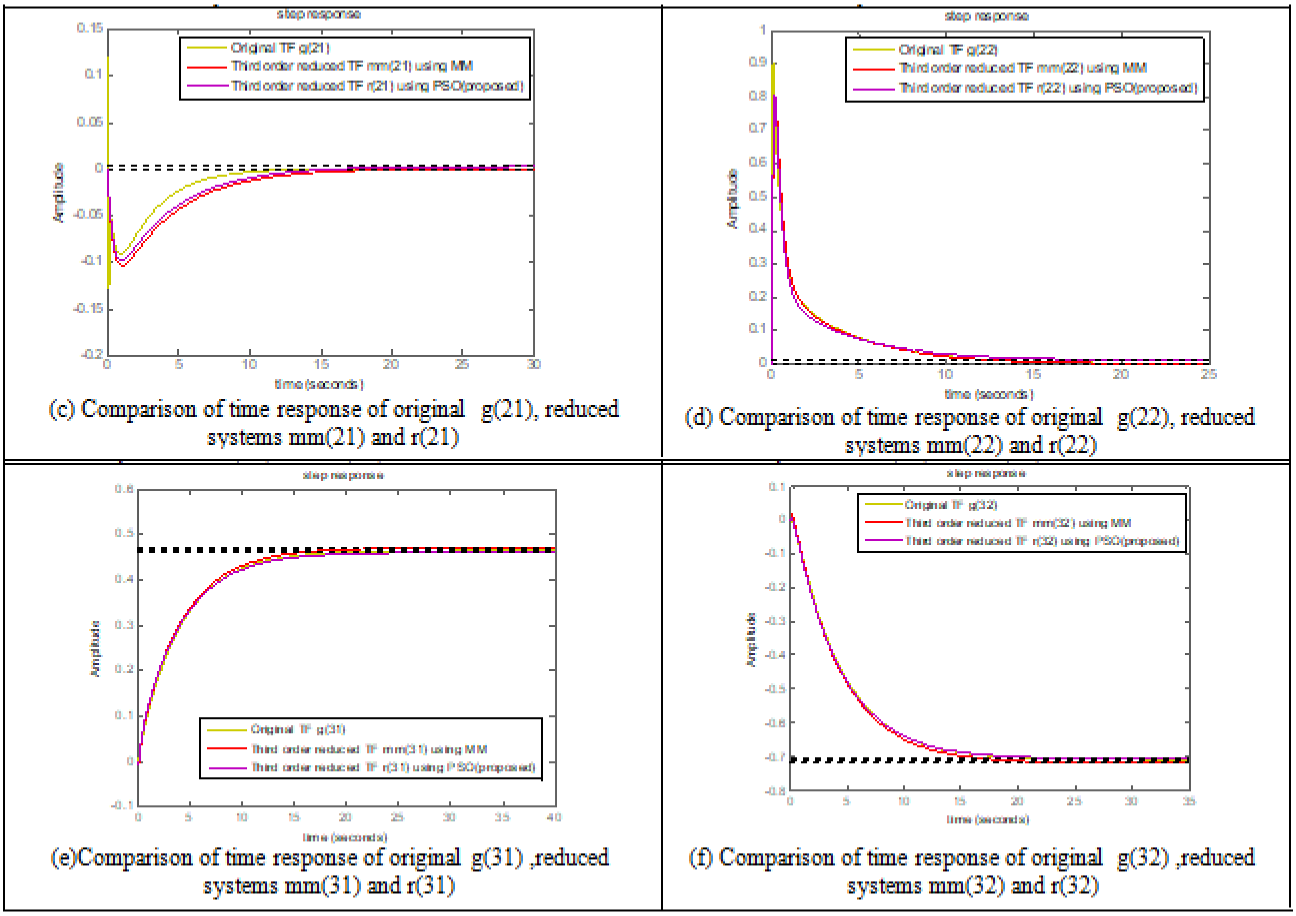
System	Original	Third order reduced (mm) [3]	Third order proposed (PSO)
Eigenvalue	-71.28 ± 636.28i,-0.4848, -2.38, -0.24,-27.42,-37.54	-11.6407 -2.6032 -0.2422	-11.8585 -2.8056 -0.2429



(a) Comparison of time response of original g(11),reduced systems mm(11) and r(11)



(b) Comparison of time response of original g(12), reduced systems mm(12) and r(12)



(c) Comparison of time response of original g(21), reduced systems mm(21) and r(21)

(d) Comparison of time response of original g(22), reduced systems mm(22) and r(22)

(e)Comparison of time response of original g(31) ,reduced systems mm(31) and r(31)

(f) Comparison of time response of original g(32) ,reduced systems mm(32) and r(32)

Figure 4: Step response of original and third order reduced SIMB MIMO Systems

5. Conclusion

The objective of model reduction is about getting a balance between complexity and misfit. Literature of MOR provides several techniques which can be used for approximation of different large scale practical systems. Every method is distinct and has a severe computational basic and aim to reduce the input-output behavior of the considered system. Empirical methods are favored as for approximations of power system model as they preserve the physical structure as well as performabilityof the model while being simple to use. The use of evolutionary algorithms such as PSO algorithm facilitates the process of solving complex model reduction problems. The coefficients of numerator and denominator polynomials are generated so as to minimize the error between the step responses of higher order and lower order models. Both the examples in the paper offer a significant contribution towards institution of a superior algorithm over other prevailing techniques based on the performance measure, step response and stability check.The results show a constructive improvement in the performance index ISE and transient response parameters. Eigenvalue analysis shows preservation of stability of reduced systems. This makes the proposed PSO algorithm for model order reduction for SMIB SISO power system and SMIB MIMO power system competitive to other methods available in the literature.

References

[1] R.Genesio, M.Mlianese., “A Note on Derivation and Use of Reducedorder Models”, Technical Notes and Correspondence, IEEE Transactions on Automatic Control, Vol. 21, pp. 118-122, February 1976.

[2] M.J. Bosley, F.P Lees., “A Survey of Simple Transfer function derivations from higher order State variable models”,*Automotica*, vol.8, pp.765-775, 1978.

[3] L. Fortuna, G. Nunnari, A.Gallo, “Model Order Reduction Techniques with applications in Electrical Engineering”, SpringerVerlag, 1992.

[4] Y.Shamash, “Stable Reduced-Order Models using Padé-typeapproximations”, Technical notes and Correspondence, IEEE Transactions on Automatic Control, pp. 615-616, October 1974.

[5] M.F.Hutton,B.Friedland, “Routh Approximations for reducing Orderof Linear Time Invariant System”, *IEEE Transactions onAutomatic Control*,vol.AC-20,No.3, pp. 329-336,June 1975.

[6] R. K.Appiah, "Linear model reduction using Hurwitz polynomial approximation," *International Journal of Control*, vol. 28, pp. 477-488, 1978/09/01 1978.

[7] Bai-Wu Wan, “Linear Model reduction using Mihailov Stability criterion and Padê approximation technique”, *International Journal of Control*, vol. 33, pp. 1073-1089, 1981.

[8] T. C. Chen, C. Y. Chang, and K. W. Han, "Reduction of Transfer Functions by the Stability-Equation Method," *Journal of the Franklin Institute*, vol. 308, pp. 389-404, 1979

[9] Y. Shamash, "Model reduction using the routh stability criterion and the Padê approximation technique," *International Journal of Control*, vol. 21, no. 3, pp. 475-484, 1975.
[10] V.Krishnamurthy and V.Seshadri, "Model reduction using the routh stability criterion," Automatic Control, IEEE Transactions on, vol. 23, no. 4, pp. 729-731, 1978.
[11] J.Pal, "Improved Padé Approximations using Stability Equation Method" Electronics Letters, vol.19, No. 11, pp. 426-427, May 1980.
[12] T.C. Chen, C.Y. Chang, K.W.Han, "Model reduction using the stability equation method and Padé approximation technique", *Journal of Franklin Institute*, vol. 309, pp. 473-490,1980
[13] T.N.Lucas, "Factor Division: A useful Algorithm in Model Reduction" *IEE Proceedings*, vol.130, Pt. D, No.6, pp. 362-364, November 1983.
[14] P.Gutman, C.Mannerfelt, and P.Molander,"Contributions to the model reduction problem," *Automatic Control*, *IEEE Transactions on*, vol. 27, pp. 454-455, 1982
[15] A. Sikander and R. Prasad, "Linear time-invariant system reduction using a mixed methods approach," *Applied Mathematical Modelling*, vol.39, pp. 4848 - 4858, 2015.
[16] C B.Vishwakarma, Prasad R., " Order Reduction using advantages of Differentiation method and Factor Division algorithm" *Indian Journal of Engineering & Material Sciences*, vol.15, pp. 447-451, December,2008.
[17] R. Luus. "Optimal reduction in linear system". *Journal of Franklin Institute*, vol. 336, issue3, pp.523-532,1999
[18] D.George,Hewitt, R.Luus, "Model reduction by minimisation of integral square error performance indices", *Journal of Franklin Institute*, vol.327, issue 3, pp. 343-357, 1990
[19] L. Shilpi, N. Deepak,"Evolutionary approach for model order reduction", *perspectives in sciences*, Vol.8, pp. 361-363, 2016
[20] A.Sikander and R.Prasad, "A novel order reduction method using cuckoo search algorithm," *IETE Journal of Research*, vol.61, no. 2, pp. 83-90, 2015
[21] J. Zhao, T. Li and JixinQian,"Applications of Particle Swarm Optimization Algorithm on Robust PID Controller Tuning ", *Springer-Verlag Berlin Heidelberg*, pp.948-957,(2005)
[22] D.Freitas,J.Rommes,N.Martins,"Grammian-Based Reduction Method Applied to Large Sparse Power System Descriptor Models"*IEEE Transactions on Power Systems*, vol. 23,No.3, pp. 1258-1270, 2008
[23] A. Narwal,R.Prasad,"A Novel order reduction approach for LTI systems using Cuckoo Search Optimization and Stability Equation",*IETE J.Res.*,Aug.2015
[24] M.Jamshidi,"Large Scale Systems: Modelling, Control and Fuzzy Logic". 2 edn., Printish Hall: North Holland, 1983
[25] D.P.Papadopoulos, Boglou A.K," Reduced order modelling of linear MIMO systems with the Padê approximation method", *International Journal of systems science*,vol.21, No.4,pp. 693-710,1990
[26] Ogata, Modern Control Engineering, 4rd edition, Prentice Hall, 2006.
[27] J.Kennedy and R.C.Eberhart, "Particle swarm optimization", *IEEE Int.Conf. on Neural Networks*, IV, 1942-1948, Piscataway, NJ, 1995.
[28] N.P.Padhy, S.P.Simon, Soft Computing with MATLAB Programming, Oxford university press, 2015.
[29] M.A. Abido " Optimal Design of Power System Stabilizers using Particle Swarm Optimization" *IEEE Transactions on Energy Conversion*, vol.17, No. 3, 2002, pp. 406-413.
[30] D. Bratton and J. Kennedy, "Defining a Standard for Particle Swarm Optimization," *IEEE Swarm Intelligence Symposium*, 2007, pp. 120-127.
[31] Y.Shi and R.C.Eberhart, "Parameter selection in particle swarm optimization " *Proc. 7th Ann. Conf. Evolutionary Program.*, Mar.1998, pp.591-600
[32] M. Bettayeb, R.Salim "Ga based h ∞ optimal model reduction:Application to power systems",*International Conference on Electric Power and Energy Conversion Systems* (EPECS '09), 2009 1–6