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Hybrid Hopscotch Method for Solving Two Dimensional System of Burgers' Equation

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Abstract: ***Hybrid finite schemes for solving Burgers equations are developed. Hybrid Hopscotch-Crank-Nicholson-Du Fort and Frankel-Lax Fredrich Scheme (HP-CN-DF-LF) proved to be the most accurate when compared with Hybrid Hopscotch-Crank Nicholson-Du Fort and Frankel (HP-CN-DF) and Hybrid Hopscotch-Crank Nicholson-Lax Fredrich (HP-CN-LF). The developed methods produced results that proved to be stable, consistent and thus convergent.***

Keywords: Hybrid Hopscotch, Burgers' equation

1. Introduction

We consider the 2-D system of Burgers equation given by:

$$\left.\begin{aligned} u_t &= -uu_x - vu_y + \frac{1}{Re}(u_{xx} + u_{yy}) \\ v_t &= -uv_x - vv_y + \frac{1}{Re}(v_{xx} + v_{yy}) \end{aligned}\right\} \quad (1.1)$$

Several approaches have been used to solve the 2D coupled Burgers equation (1.1) including Finite difference methods, operator splitting method, Adomian Decomposition Method among others with varied levels of accuracy. Hopscotch method is a type of finite-difference method for multi-variable partial differential equations in which implicit solutions are obtained using two sweeps in different directions. The Burgers' equation is considered as the fundamental partial differential equation in the field of applied mathematics such as fluid mechanics, nonlinear acoustics, gas dynamics, and traffic flow among others.

Hybrid Hopscotch-Crank-Nicholson-Lax-Friedrich's (HP-CN-LF) Scheme which is a scheme made by combining Hopscotch- Crank-Nicholson with Lax-Friedrich schemes to form a hybrid was used and discussed byMaritim*et. al.*(2018a)to solve system of 2D Burgers' equation. Similarly Maritim*et. al.* (2018b) developed Hybrid Hopscotch-Crank-Nicholson-Du-Fort and Frankel (HP-CN-DF) Scheme achieved acceptable level of accuracy.

Rotich *et. al*. (2016) developed the pure Crank-Nicholson (CN) Scheme and hybrid Crank-Nicholson-Lax-Fredrichs' (CN-LF) method by Operator Splitting. The developed schemes were then solved numerically using initially solved solution via Hopf-Cole transformation and separation of variables to generate the initial and boundary conditions.

Kweyu*et. al*. (2012) generated varied sets of exact initial and Dirichlet boundary conditions for the 2-D Burgers' equations from general analytical solution via Hopf-Cole transformation and separation of variables. The conditions were then used for the numerical solutions of the equations using finite difference methods.

Borah *et. al.* (2012)studied mathematical models for fluid flow which often involve systems of convection-diffusion equations as a main ingredient. According to the paper, in Operator splitting - one splits the time evolution into partial steps to separate the effects of convection and diffusion. The research showed that the temporal splitting error can be significant when there is a shock present in the solution, and is well-understood for scalar convection – diffusion equation. It is demonstrated numerically that operator splitting (OS) methods for systems of convection-diffusion equations in one-space dimensions, has a tendency to be too diffusive near viscous shock waves, the potential error is compensated for in the diffusion step (or in a separate correction step), in case of *scalar* case, the splitting error is closely related to the local linearization introduced implicitly in the convection steps due to the use of an entropy condition.

Espen (2011) discussed numerical quadratures in one and two dimensions, which is followed by a discussion regarding the differentiation of general operators in Banach spaces. In addition, the research discussed the standard and fractional Sobolev spaces Hs(R), and prove several properties of these spaces.The research showed that the operator splitting methods of the Godunov type and Strang type applied to the viscous Burgers' equation,$u_t = u_{xx} + uu_x$, and the Korteweg–de Vries (KdV) equation, $u_t = u_{xxx} + uu_x$, (and other equations), have the correct convergence rates in $H^s(\mathbb{R})$, for arbitrary integer $s \geq 1$. The research investigated the Godunov method and Strang method numerically for the viscous Burgers' equation and the KdV equation, and presented different numerical methods for the sub-equations from the splitting. It was discovered that the operator splitting methods work well numerically for the two equations. For the viscous Burgers' equation, it was found that several combination of numerical solvers for the sub-equations work well on the test problems, while for the KdV equation found only one combination of numerical solvers which works well on all test problems.

Holden *et. al.* (2011) proposed a new analytical approach to operator splitting for equations of the type $u_t = Au + uu_x$ where A is a linear differential operator such that the equation is well-posed. Particular examples include the viscous Burgers' equation, the Korteweg–de Vries (KdV) equation, the Benney–Lin equation, and the Kawahara equation. The research showed that the Strang splitting method converges with the expected rate if the initial data are sufficiently regular. In particular, for the KdV equation second-order convergence is obtained in H^r for initial data in H^{r+5} with arbitrary $r \geq 1$.

Geiser and Noack (2008) considered iterative operator-splitting methods for non-linear differential equations with respect to their eigenvalues. The main feature of the proposed idea is the fixed-point iterative scheme that linearizes their underlying equations. Based on the approximated eigenvalues of such linearized systems we choose the order of the the operators for our iterative splitting scheme. The convergence properties of such a mixed method are studied and demonstrated. The findings was confirmed with numerical applications of the effectiveness of the proposed scheme in comparison with the standard operator-splitting methods by providing improved results and convergence rates. The results were applied to deposition processes and proved to be more effective.

Holden *et. al.* (2000) presented an accurate numerical method for a large class of scalar, strongly degenerate convection–diffusion equations. According to the rerasearch, Important subclasses are hyperbolic conservation laws, porous medium type equations, two-phase reservoir flow equations, and strongly degenerate equations coming from the recent theory of sedimentation–consolidation processes. The method is based on splitting the convective and the diffusive terms. The nonlinear, convective part was solved using front tracking and dimensional splitting, while the nonlinear diffusion part is solved by an implicit–explicit finite difference scheme. A detailed convergence analysis of the operator splitting method was given in the research. The researcher presented numerical experiments with the method for examples modelling secondary oil recovery and sedimentation–consolidation processes. The research demonstrated that the splitting method resolves sharp gradients accurately, may use large time steps, has first order convergence, exhibits small grid orientation effects, has small mass balance errors, and is efficient.

Hongqing *et. al.* (2010) proposed the Adomian Decomposition Method (ADM) to numerically solve the two-dimensional Burger’s nonlinear difference equations. They attempted to solve the nonlinear problem, where the finite difference scheme is fully implicit sheme. With the help of symbolic computation software Maple 13, the proposed method was tested and compared with the exact solutions for various Reynolds numbers. Two test problems were considered to illustrate the accuracy of the proposed discrete decomposition method. They showed that the numerical results are in good agreement with the exact solutions for each problem. Thus the proposed discrete ADM is an efficient method for the solution of the two-dimensional Burgers’ equation.

Shukla *et. al.* (2014) discussed a numerical solution to the two dimensional nonlinear coupled viscous Burgers’ equation with the appropriate initial and boundary conditions using the modified cubic-Spline differential quadrature method (MCB-DQM). In the method, the weighting coefficients were computed using the modified cubic B-spline as a basis function in the differential quadrature method. Thus, the coupled Burger equation was reduced into a system of ordinary differential equations. An optimal five stage and fourth-order strong stability preserving Runge–Kutta scheme was applied for solving the resulting system of ordinary differential equations. The accuracy of the scheme was illustrated by taking two numerical examples. Computed results were compared with the exact solutions and other results available in literature. Obtained numerical result showed that the described method is efficient and reliable scheme for solving two dimensional coupled viscous Burgers’ equation.

Mittal and Jain (2012) proposed a numerical method to approximate the solution of the nonlinear Burgers’ equation. The method is based on collocation of modified cubic B-splines over finite elements so that continuity of the dependent variable and its first two derivatives throughout the solution range is achieved. The research applied modified cubic B-splines for spatial variable and derivatives which produced system of first order ordinary differential equations. The numerical approximate solutions of Burgers’ equation were computed without transforming the equation and without linearization. The method proved to be easy and economical to implement.

Kutluay and Yagmurlu (2012) proposed the modified bi-quintic B-spline base functions and successfully applied it to the two-dimensional unsteady Burgers' equation using the Galerkin method to obtain its numerical solutions. The accuracy of the numerical scheme was examined by the error norms L_2 and L_∞ .The obtained numerical results have been compared with the exact ones and were found to be in good agreement.

Jiang and Wang (2010) presented an improved numerical solution of the Burgers' equation based on the cubic B-spline quasi-interpolation and the compact finite difference method. At first the cubic B-spline quasi-interpolation and the compact finite difference method are introduced. Moreover, the numerical scheme is presented, by using the derivative of the quasi-interpolation to approximate the spatial derivative and a two-order compact scheme to approximate the time derivative. The accuracy of the developed scheme was demonstrated by two problems. The advantage of the resulting scheme is simple with better accuracy, so it is easy to implement.From the test examples, the research proved that the scheme is feasible and the accuracy is better than other quasi-interpolation methods.

Iltaf , Safyan and Arshed (2013) proposed a meshfree technique for the numerical solution of the 2D Burger’s equation. Collocation method using the Radial Basis Functions (RBFs) is coupled with first order accurate finite difference approximation. Different types of RBFs are used for this purpose. Performance of the proposed method is successfully tested in terms of various error norms. In the case of non-availability of exact solution, performance of the new method is compared with the results obtained from the existing numerical methods available in the literature. The elementary stability analysis is established theoretically and is also supported by numerical results.

Weinan (1992) presented a general framework for analyzing numerical methods for the evolutionary equations that admit semigroup formulations. This framework was then applied to spectral and pseudospectral methods for the Burgers' equation, using trigonometric, Chebyshev, and Legendre poly-nomials. Optimal order of convergence was obtained, which implies the spectral accuracy of these methods.

Shafiqul *et. al*. (2014) explained that there are many equations in mathematics which are used in our practical life and Burger's equation is one of them which is a good simplification of Navier-Stokes equation where the velocity is one spatial dimension and the external force is neglected in absence of pressure gradient. This equation is used to analyze traffic congestion and acoustics. It occurs in various areas of applied mathematics, such as modeling of various problems in fluid dynamics and traffic flow among others. Due to the complexity of the analytical solution, one needs to use numerical methods to solve this equation. For this the researchers investigated finite difference method for Burger's equation and presented an explicit central difference scheme. An implementation of the numerical solution by computer programming for artificial initial and boundary data and verify the qualitative behavior of the numerical solution of Burger's equation. The research considered Burger's equation as a fundamental partial differential equation from fluid mechanics. First the research showed derivation of Navier-Stokes equation, Burger's equation and numerical methods of Burger's equation. Finally the research showed that the numerical result based on the explicit central difference scheme agrees with basic qualitative behavior of viscous Burger's equation.

Burns *et. al.* (1998) considered Burgers' equation on the interval (0,1) with Neumann boundary conditions. The work showed that even for moderate values of the viscosity and for certain initial conditions, numerical solutions approach nonconstant shock type stationary solutions. Also the researchers showed that the only possible actual stationary solutions are constants.

Han *et. al.* (2006) discussed the numerical solution of Burgers' equation on unbounded domains. Two artificial boundaries are introduced and boundary conditions are obtained on the artificial boundaries, which are in nonlinear forms. Then the original problem was reduced to an equivalent problem on a bounded domain. Finite difference method is applied to the reduced problem, and some numerical examples are given to show the effectiveness of the new approach. Using the Hopf-Cole transformation the researchers obtained the boundary conditions on the artificial boundaries. These boundary conditions are in nonlinear forms. With the artificial boundaries, the original unbounded problem was solved in a much smaller domain. The numerical examples showed that the new approach was very effective; the numerical solutions converge fast to the exact solutions.

Aminikhah and Moradia (2014) proposed a numerical method for solving the systems of variable-coefficient coupled Burgers' equation based on two-dimensional Legendre wavelets. Two-dimensional operational matrices of integration are introduced and then employed to find a solution to the systems of variable-coefficient coupled Burgers' equation. It is shown that the numerical results are in good agreement with the exact solutions for each problem.

In the above review, there is no mention of use of Hybrid Hopscotch Crank-Nicholson Du-Fort and Frankel- Lax-Fredrich method. This research builds up on the work done by Maritim *et. al.* (2018a and 2018b) and extends to form a hybrid of Hopscotch Crank-Nicholson-Du Fort and Frankel-Lax Fredrich (HP-CN-DF-LF) to solve 2D coupled Burgers equation (1.1).

2. Approximation at the Boundaries

The analytical solution of Burgers system of equations (1.1) at any point (x, y, t) is given by the following equations, according to Rotich *et. al*. (2016):

$$u(x,y,t)=\frac{-2y-2\pi e^{-\frac{2\pi^2 t}{Re}}((cos\pi x-\sin\pi x)\sin\pi y)}{Re(100+xy+e^{-\frac{2\pi^2 t}{Re}}((cos\pi x-\sin\pi x)\sin\pi y)} \quad (2.1a)$$

$$v(x,y,t)=\frac{-2x-2\pi e^{-\frac{2\pi^2 t}{Re}}((cos\pi x-\sin\pi x)\sin\pi y)}{Re(100+xy+e^{-\frac{2\pi^2 t}{Re}}((cos\pi x-\sin\pi x)\sin\pi y)} \quad (2.1b)$$

From Maritim*et. al.* (2018a), taking$\Delta x = \Delta y = h$ and $\frac{\Delta t}{h^2} = \alpha$ and $\frac{\Delta t}{h^4 Re} = \beta$to obtain the Hopscotch Crank-Nicholson scheme as shown below:

$3\beta U_{i-2,j}^{n+1} + U_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + U_{i,j}^{n+1}(1 - 2\alpha Ui-1,j-1n+4\beta+Ui+1,jn+1\alpha Ui-1,j-1n-\beta+3\beta Ui,j-2n+2+Ui,j-1n+2\alpha Vi-1,j-1n-6\beta-Ui,jn+22\alpha Vi-1,j-1n-4\beta+Ui,j+1n+2\alpha Vi-1,j-1n-\beta=Ui,jn$ (2.2a)

$3\beta V_{i-2,j}^{n+1} + V_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + V_{i,j}^{n+1}(1 - 2\alpha Ui-1,j-1n+4\beta+Vi+1,jn+1\alpha Ui-1,j-1n-\beta+3\beta Vi,j-2n+2+Vi,j-1n+2\alpha Vi-1,j-1n-6\beta-Vi,jn+22\alpha Vi-1,j-1n-4\beta+Vi,j+1n+2\alpha Vi-1,j-1n-\beta=Vi,jn$ (2.2b)

3. Development of Hybrid Hopscotch-Crank-Nicholson-Du-Fort and Frankel -Lax-Friedrich's (HP-CN-DF-LF) Scheme

The developed Hybrid Hopscotch-Crank-Nicholson-Lax-Friedrich's (HP-CN-LF) Scheme given as:

$$3\beta U_{i-2,j}^{n+1} + U_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + U_{i,j}^{n+1}(1 - 2\alpha Ui-1,j-1n+4\beta+Ui+1,jn+1\alpha Ui-1,j-1n-\beta+3\beta Ui,j-2n+2+Ui,j-1n+2\alpha Vi-1,j-1n-6\beta-Ui,jn+22\alpha Vi-1,j-1n-4\beta+Ui,j+1n+2\alpha Vi-1,j-1n-\beta=12Ui+1,j+1n+Ui-1,j+1n+12Ui+1,j-1n+Ui-1,j-1n \quad (3.1a)$$

$$3\beta V_{i-2,j}^{n+1} + V_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + V_{i,j}^{n+1}(1 - 2\alpha Ui-1,j-1n+4\beta+Vi+1,jn+1\alpha Ui-1,j-1n-\beta+3\beta Vi,j-2n+2+Vi,j-1n+2\alpha Vi-1,j-1n-6\beta-Vi,jn+22\alpha Vi-1,j-1n-4\beta+Vi,j+1n+2\alpha Vi-1,j-1n-\beta=12Vi+1,j+1n+Vi-1,j+1n+12Vi+1,j-1n+Vi-1,j-1n \quad (3.1b)$$

fromMaritim*et. al.* (2018a) and using and Hybrid Hopscotch-Crank-Nicholson-Du-Fort and Frankel (HP-CN-DF) Scheme given by:

$3\beta U_{i-2,j}^{n+1} + U_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + U_{i,j}^{n+1}\left(1 -\right.$
2αUi−1,j−1n+4β−12+Ui+1,jn+1αUi−1,j−1n−β+3βU i,j−2n+2+Ui,j−1n+2αVi−1,j−1n−6β−Ui,jn+22αVi−1 ,j−1n−4β+Ui,j+1n+2αVi−1,j−1n−β=12Ui,jn−1

$3\beta V_{i-2,j}^{n+1} + V_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + V_{i,j}^{n+1}\left(1 -\right.$
2αUi−1,j−1n+4β−12+Vi+1,jn+1αUi−1,j−1n−β+3βV i,j−2n+2+Vi,j−1n+2αVi−1,j−1n−6β−Vi,jn+22αVi−1, j−1n−4β+Vi,j+1n+2αVi−1,j−1n−β=12Vi,jn−1

fromMaritimet. al. (2018b).
In the scheme (3.2a) and (3.2b) replace $U_{i,j}^{n-1}$ and $V_{i,j}^{n-1}$ by:
$\frac{1}{2}\left(U_{i+1,j+1}^{n-1} + U_{i-1,j+1}^{n-1}\right) + \frac{1}{2}\left(U_{i+1,j-1}^{n-1} + U_{i-1,j-1}^{n-1}\right)and\frac{1}{2}\left(V_{i+1,j+1}^{n-1} + V_{i-1,j+1}^{n-1}\right) + \frac{1}{2}\left(V_{i+1,j-1}^{n-1} + V_{i-1,j-1}^{n-1}\right)$
respectively to get:
$3\beta U_{i-2,j}^{n+1} + U_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + U_{i,j}^{n+1}\left(1 -\right.$
2αUi−1,j−1n+4β−12+Ui+1,jn+1αUi−1,j−1n−β+3βU i,j−2n+2+Ui,j−1n+2αVi−1,j−1n−6β−Ui,jn+22αVi−1 ,j−1n−4β+Ui,j+1n+2αVi−1,j−1n−β=
$\frac{1}{4}\left(U_{i+1,j+1}^{n-1} + U_{i-1,j+1}^{n-1}\right) + \frac{1}{4}\left(U_{i+1,j-1}^{n-1} + U_{i-1,j-1}^{n-1}\right)$

$$\begin{aligned}3\beta V_{i-2,j}^{n+1} + V_{i-1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - 6\beta\right) + V_{i,j}^{n+1}\Big(1 - 2\alpha U_{i-1,j-1}^{n} \\ + 4\beta - \frac{1}{2}\Big) + V_{i+1,j}^{n+1}\left(\alpha U_{i-1,j-1}^{n} - \beta\right) \\ + 3\beta V_{i,j-2}^{n+2} + V_{i,j-1}^{n+2}\left(\alpha V_{i-1,j-1}^{n} - 6\beta\right) \\ - V_{i,j}^{n+2}\left(2\alpha V_{i-1,j-1}^{n} - 4\beta\right) \\ + V_{i,j+1}^{n+2}\left(\alpha V_{i-1,j-1}^{n} - \beta\right) \\ = \frac{1}{4}\left(V_{i+1,j+1}^{n-1} + V_{i-1,j+1}^{n-1}\right) \\ + \frac{1}{4}\left(V_{i+1,j-1}^{n-1} + V_{i-1,j-1}^{n-1}\right)\end{aligned}$$

4. Presentation of Results

We present results in form of line graphs, 3D graphs and tables of values. The following data is used: $k = 0.001, h = 0.1, l = 0.1$ and $Re = 4000$ to obtain the results. The computational domain is taken as a square domain $\Omega = \{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$.The initial and boundary conditions for $u(x,y,t)$and $v(x,y,t)$ are taken from the numerical solutions by Kweyu*et. al.* (2012).

4.1 Two Dimensional plots of Absolute Errors in Solutions of $u(x,y,t)$ and $v(x,y,t)$

Figures 4.1 and 4.2 show the absolute error in solutions of $u(x,y,t)$ and $v(x,y,t)$ respectively plotted as a function of position along the x-axis for the four hybrid schemes used, fixing *t=1.0*.

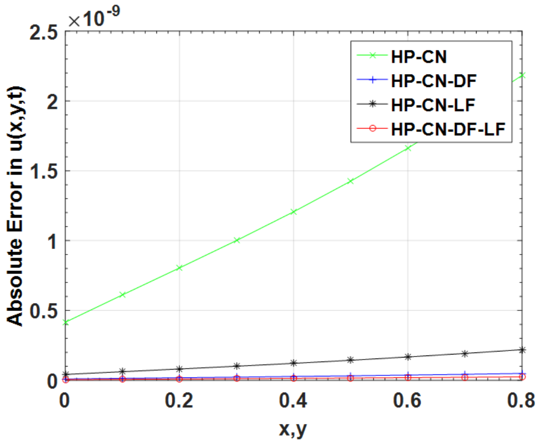

Figure 4.1: Absolute error in Solution of u for the 2-D Burgers' equation

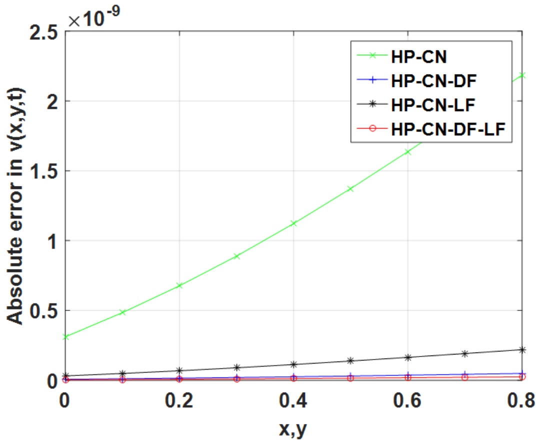

Figure 4.2: Absolute error in Solution of v for the 2-D Burgers' equation

The absolute errors are the variation of the numerical solutions of HP-CN, HP-CN-LF, HP-CN-DF and HP-CN-DF-LF when compared with values proposed by Kweyu*et. al.* (2012). The figures clearly shows that the hybrid HP-CN-DF-LF has the least absolute error then HP-CN-DF, HP-CN-LF and lastly HP-CN for both $u(x,y,t)$and $v(x,y,t)$. This shows that HP-CN-DF-LF is the most accurate and HP-CN is the least accurate. These figures show that the schemes developed are stable.

4.2 Three Dimensional Plots of Solutions of $u(x,y,t)$ and $v(x,y,t)$ for HP-CN-DF-LF

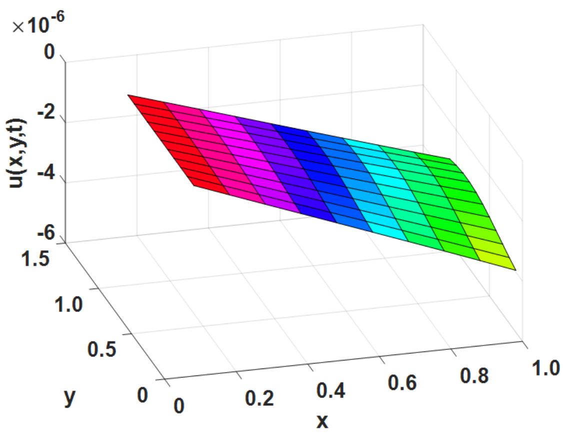

Figure 4.1: HP-CN-DF-LF Numerical Solution of u t=1.000

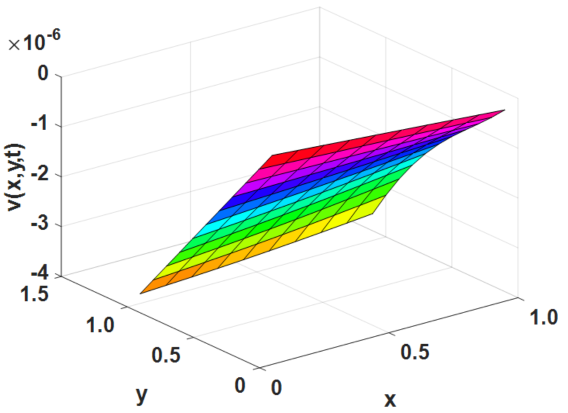

Figure 4.2: HP-CN-DF-LF Numerical Solution of v at t=1.000

The figures 4.1 and 4.2 are 3-D images of solutions $u(x,y,t)$ and $v(x,y,t)$ plotted against x and y respectively using MATLAB for the hybrid HP-CN-DF-LF scheme developed. The figures clearly show that the solutions are not changing suddenly for change in x and y hence the results for HP-CN-DF-LF Scheme developed are consistent.

Table 4.1: Percentage Absolute Errors in Solutions of u(x, y, t)

X	Y	HP-CN	HP-CN-LF	HP-CN-DF	HP-CN-DF-LF
0.1	0.1	**0.091912937**	0.002049272	0.009224257	0.001024662
0.2	0.2	0.067401849	0.001502792	0.006764432	0.000751415
0.3	0.3	0.059010038	0.001315697	0.005922273	0.000657865
0.4	0.4	0.054992840	0.001226131	0.005519119	0.000613081
0.5	0.5	0.053004852	0.001181804	0.00531959	0.000590917
0.6	0.6	0.052228321	0.001164483	0.00524162	0.000582256
0.7	0.7	0.052252640	0.001165014	0.005244004	**0.000582522**
0.8	0.8	0.052801786	0.001177243	0.005299043	0.000588636
0.9	0.9	0.053652403	0.001196192	0.005384327	0.000598111

The Table 4.1 shows percentage absolute errors in solutions of $u(x,y,t)$ when compared with the Kweyu *et. al.* (2012) values at different levels of $x$and y. It shows a maximum error of 0.091912937% and a minimum of 0.000582522%.

Table 4.2: Percentage Absolute Errors in Solutions of $v(x,y,t)$

X	Y	HP-CN	HP-CN-LF	HP-CN-DF	HP-CN-DF-LF
0.1	0.1	0.006254655	0.000627013	0.000139298	0.000069651
0.2	0.2	0.009823624	0.000984803	0.000218785	0.000109395
0.3	0.3	0.013828449	0.001386291	0.000307979	0.000153994
0.4	0.4	0.018334820	0.001838055	0.000408344	0.000204177
0.5	0.5	0.023367011	0.002342524	0.000520417	0.000260215
0.6	0.6	0.028905910	0.002897773	0.000643772	0.000321894
0.7	0.7	0.034892197	0.003497851	0.000777087	0.000388553
0.8	0.8	0.041233979	0.004133542	0.000918314	0.000459168
0.9	0.9	0.047817500	0.004793441	0.001064920	0.000532473

The Table 4.2 shows percentage absolute errors in solutions of $v(x,y,t)$ when compared with the Kweyu *et. al.* (2012) values at different levels of $x$and y. It shows a maximum error of 0.047817500% and a minimum of 0.000069651%

5. Conclusions

1) The hybrid scheme: Hopscotch-Crank-Nicholson Du-Fort and Frankel - Lax - Friedrichs' (HP-CN-DF-LF) were developed and used to solve 2D Burgers' equation.
2) The developed scheme proved to be stable because the errors did not 'blowup' (absolute errors less than 0.09% for $u(x,y,t)$ and 0.05% for $v(x,y,t)$ as shown on the table 4.1 and 4.2 of absolute errors. The schemes are also consistent because the results were not changing suddenly for small change in space hence convergent in line with Lax equivalence theorem.

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