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Pre-school Teacher's Pedagogical Content Knowledge for Teaching Informal Geometry: A Case of In-Service University Students

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Abstract: ***Pre-school pedagogical content Knowledge is increasingly taking a centre stage in teaching children in Early Childhood Education (ECE) centres. As such, this research examined pre-school teachers pedagogical content knowledge for teaching informal geometry in a quest to find out what early childhood informal geometry education was, design a degree course in early childhood hood education, where informal geometry is in early childhood education, construct and operationalise of the centre of excellence where informal geometry is concerned, come up with a syllabus and activities that teachers could be using to teach informal geometry. The purpose of this study was to establish the kind of geometry the students had come with before embarking on teaching. This was done so, so as to align the researchers teaching philosophy of teaching students in a similar way they are expected to teach the children. In the early childhood curriculum, there should be a connection between informal and formal geometry. The research was anchored on the theory of mathematical knowledge for teaching and Play based pedagogy because there is a shift globally to empower teachers with both instructional, educator –led approach based on training programmes and play-based approach. This research involved 84 university in-service students, 10 serving teachers, 01 early childhood specialist and 01 officer working for an NGO. Data was collected through participant observations, interviews, focus group discussions, questionnaire, audio-visual, presentations methods in different settings and during the residential. Best practices in mathematical knowledge for teaching and play based pedagogy were enhanced on designing a degree course for early childhood education, module writing, construction and operationalisation of the centre of excellence, coming up with a course outline and activities in numeracy and development with a bias towards informal geometry. This research faced challenges in mobility, appropriate equipment, time, management of new personnel and financial constraints. The researcher learnt on how to interpret and review teachers and children's activities in line with learning through play and use of Early Learning Development Standards (ELDS). The researcher further learnt how to interpret mathematical knowledge for teaching as it relates to teacher understanding of informal geometry and it's the teaching. It became clear that for teachers to understand informal geometry and its teaching, they needed to understand the theory of mathematics knowledge for teaching and play based pedagogy. The researcher found that university students had about 50% knowledge of content of students and teaching and no knowledge of the curriculum. The reason was that no curriculum and trained trainers existed at the time the study was being conducted. This study recommends that the nature of informal geometrical tasks provided be explored at all levels of education, psychology of informal geometry be introduced, a curriculum be designed, learning trajectories for informal geometry be spelt out from pre-school to university level, a robust mentorship and consultancy program be implemented, both transmissive and facilitative methods of teaching be employed when teaching informal geometry with a focus on lesson study.***

Keywords: Pre-school pedagogical content knowledge, Mathematical knowledge for teaching, Informal geometry, early childhood education, learning through pedagogy, Early Learning and Development standards.

1. Introduction

Pedagogical content knowledge is a teacher's level of capacity to develop and implement instruction on a particular content in particular ways in order to lead to enhanced student understanding (Schulman, 1989). It is an idea rooted in the belief that teaching requires considerably more than delivering subject content knowledge to students, and that student learning is considerably more than absorbing information for later accurate spitting out. PCK is the knowledge that teachers develop over time, and through experience, about how to teach particular content in particular ways in order to lead to enhanced student understanding. It is not a single entity that is the same for all teachers of a given subject area; it is a particular expertise with individual features and important differences that are influenced by (at least) the teaching context, content, and experience. It may be the same (or similar) for some teachers and different for others, but it is, nevertheless, a corner stone of teachers' professional knowledge and expertise. This PCK is put to test when teaching a subject where one is not an expert. The teacher has to understand content beyond basic facts. The teacher has to understand some of the difficulties and points of confusion that students experience with the content that may well be important in the selection of the term.

In the current study the researcher examined in-service university pre-school teachers PCK on basic informal geometry methodology and content, for example, on how they introduce geometry to the children, explanation of an activity they can use when introducing geometry, ways of starting a more systematic study of geometry, concepts to emphasize on when teaching informal geometry, ways of presenting a unit on quadrilaterals using the first Van Hiele levels, ways in which children learn geometric shapes, ways of teaching at level three of van Hiele, construction of a pyramid among others. Teachers PCK were compared to the curriculum on informal geometry (Copley, 2010), Zambia Teacher Education Syllabus (2016) and Zambia Pre-school syllabus (2013) so as to find out whether teachers PCK can be attributed to the documents that have been produced.

The researcher examined university students PCK as way of recognising early informal geometry education as a distinctive subject in University students mathematics learning (Zambia Education Curriculum framework (ZECF), 2013; UNICEF, 2018; Government of West Australia, 2019;

BUPL-ZNUT (2019); Flemmish Government (2018); Project Zero, (2015) Chimfwembe Gondwe 2020; The room 241 Team; 2018; Kathryn Hirsh- Pasek, 2014;…), that should be characterised by a curriculum that focuses on teacher understanding of instructional educator- led approach based on training programmes (Franziska Vogt, Bernhard Hauser, Rita Stebler, Karin Rechsteiner and Christa Urech (2018) to a play based learning/ teaching in early childhood geometry (Douglas H Clements, Candace Joswick, Julie Sarama, 2018) and Mathematical knowledge for teaching (Ball et al 2008 & 2011). This is on one hand as a way of responding to the questions that teachers ask when considering resources for shape and space said Jenni Way (2011) and on the other hand to help teachers deploy a lot of effective and innovative approaches to pedagogy, from instructional, educator-led approach based on training programmes to a play-based approach. This was envisaged as a way of enabling students to explore, experiment, discover and solve problems in line with sustainable geometry learning. Way said that, most teachers wander the benefits of using shape puzzles, tangrams and other mosaic-type resources. They do not know what children learn from mosaic-type resources and yet they like playing with them and do not know how activities could be structured to really teach some geometric ideas (Way, 2011). Engel et al. (2016) linked the time spent on mathematics, as reported by the educators (for example, Vogt F et al (2018), with children's mathematics achievement and found no correlation. They concluded that educators focus on curricular content, which is not sufficiently challenging for most children, for example, Counting and shapes. Despite focusing on curricular content which is not sufficiently challenging, learner performance has remained below 40% (SACMEQ111, 2015, NNF, 2020).

2. Purpose of the Study

The purpose of the study was to contribute in a small way in the researchers practice as a teacher trainer/educator, something that may help in improving learner performance in mathematics that has been below 40% at all levels (SACMEQ111, 2015; ECZ, 2015).This was done by examining pre-school teachers pedagogical content knowledge for teaching informal geometry for a sole purpose of establishing the background knowledge students had concerning informal geometry before embarking on teaching them (Kelly, 1955 cited in Mirriam, 2020). This was done so, so as to align the researchers teaching philosophy of teaching students in a similar way they are expected to teach the children. Although a teachers own criteria determines his or her own professional activities (ibid). The study stated the problem, objectives and questions. It reviewed necessary literature. It examined theory and linked it to research objectives, methodology findings and discussions of the study.

3. Literature Review

3.1 Approach to tasks by university students

Ball, Thames & Phelps (2008) explored Shulman's PCK and came up with six types of knowledge. The first three of these six knowledges is referred to as (PCK). PCK comprises of the following different types of knowledge: knowledge of content and students, teaching and curriculum. Ball, et al (2008) explained Knowledge of content and students as knowledge of cognitive steps that a learner is involved in during the mathematical thinking process that supports the development of mathematics concepts and skills. This cognitive steps involves mainly four processes that support learning for example, students must orchestrate processes essential to learning such as attention, emotion regulation, and inhibition of incorrect or inappropriate responses and have good memory (NAP, 2018). They should co-ordinate these varied capacities both consciously and unconsciously as they are needed to meeting learning challenges. Students should monitor and regulate cognitive processes and consciously regulate behavior including affective behavior. They should over ally regulate thinking, behaviour and the higher order processes that enable students to plan, sequence, initiate and sustain their behavior towards some goal, incorporating feedback and making adjustments. Furthermore, NAP says that self-regulation is learning that is focused by means of metacognition, strategic action, and motivation to learn. It is seen as involving management of cognitive, affective, motivational and behavioural components that allow the individual to adjust actions and goals to achieve desired results. And for students to do this, they should be able to hold information in mind, inhibit incorrect or pre-mature responses and sustain or switch attention to meet a goal. All learners need to choose among competing interests and then sustain attention to the chosen ones long enough to make progress, hold in mind multiple pieces of information, manipulate them productively and monitor their own progress. Students need to connect informal geometry they learnt outside the university to formal geometry at the University (Scott Alan Pattisan and Tracey Wright 2017) if meaningful learning has to take place although the two are distinct. However, as they are connecting this two types of geometry, the lecturer should be careful so as not to cognitively overload students. In line with cognitive overload, Heather Fry, Steve Ketteridge, Stephanie Marshall (2009) said that, it is unfortunate, but true, that some academics teach students without having much formal knowledge of how students learn. Many lecturers know how they learnt/learn best, but do not necessarily consider how their students learn, for example, students can be rationalists or realists, associates and constructivists. Furthermore, these lecturers do not know at all if the way they teach is predicated on enabling learning to happen. Nor do they necessarily have the concepts to understand, explain and articulate the process they sense is happening in their students. Actually, with the growing recognition of the importance of informal STEM education, there is need for academic practitioners and policy makers to pay more attention to how students learn and how these experiences might support mathematical thinking and learning and contribute to the broader goal of ensuring healthy, sustainable, economically, vibrant communities in this increasingly STEM rich world (Scott Alan Pattisan and Tracey Wright, 2017).

Students need to rely on intuitive understandings of informal geometry, draw on contextual cues from the situation, use tools and manipulatives to scaffold reasoning and avoid abstract notation, use empirical approaches to develop understanding of relationships, refer to other concepts, and be explicit in their verbal reasoning. However students at different levels approach certain mathematical tasks differently (Heather et al, 2009). For students to approach this type of task they have to be fully engaged and involved (Piaget, 1954 and Vygosky, 1978). The situated and flexible nature of everyday mathematics, as well as the possibility of using "social and empirical rules… alongside logical relationships, " often makes this more accurate and foolproof than school-based mathematics (Swanson & Williams, 2014). For example, Fisch and colleagues (2009) observed that third and fourth grade students playing an online game shifted approaches and used increasingly sophisticated mathematics strategies to solve game challenges when previous, simpler strategies were not effective. Drawing from Gee's theoretical work on learning through electronic games (Gee, 2007), they speculated that the informal nature of the game affords these changes by allowing for risk-taking without consequences and by creating new game scenarios and challenges that force players to "undo their routinized strategy to adapt to the new or changed conditions" (Fisch et al., 2009).

Research has indicated that the way students are taught has a strong influence and impact on their ideas about the subject and how the subject should be taught (Kember, David and Kam-por Kwan (2000).This thinking is supported by Kember, 1997 who says that, an academic's conceptions of teaching influences the approach to teaching. And that, it seems reasonable to assume that this will also be influenced by curriculum design and departmental and institutional pressures and also by the nature of the students. The extent to which these other factors modify the impact of beliefs upon the teaching approach is likely to vary with the nature of the institution, course and students. There is then evidence that the study approaches adopted by students are a function of the student's predisposition, the form of the teaching and the nature of the teaching and learning environment, or the curriculum in the broadest sense. The approach to study adopted by the student in turn affects the quality of the learning outcomes. In 1908 at the International Congress of Mathematicians in Rome the great mathematician Felix Klein (1849 - 1925) talked about the paradox of double forgetting. The essence of the paradox is the life experience of young teachers who have to forget about a lot of university training and its scientific thinking to successfully teach mathematics. Kitti Vidermanova & Dusan Vallo (2015) said that the paradox of double forgetting currently persists at Slovak primary and secondary schools. The problem is especially striking in teaching geometry. The duo said that many Slovak and Czech teachers agree that geometry has many applications in everyday life, but there is not enough "real-life" everyday problems in Slovak mathematics textbooks which are structured according to the deductive approach to teaching and learning. Therefore, there is need for a breadth of mathematics topics that can be productively be explored in designed informal learning environments and an explanation of how both experiences and professional development can be designed to enhance learning outcomes and promote positive mathematics attitude especially when it comes to tertiary mathematics education which has been researched less Kember, David and Kampor Kwan (2000)

In a bid to know content and students, students should know the difficulties that children encounter as they are learning informal geometry. There are findings related to difficulties with what in the NCTM Principles and Standards (NCTM, 2000) are denoted processes, e.g. non-routine problem-solving, proof and proving, reasoning, representing and modelling. Two of the more central, and recurrent, findings in research on problem-solving are: (i) students' focus on the rote learning of routine procedures, which is often not complemented by the development of other task-solving approaches; and (ii) students' extensive difficulties in solving non-routine problems (Schoenfeld, 1985; Lester, 1994; Selden et al., 1994). This unbalance seems to align poorly with most mathematics curricula goals. Although this has been well known for quite a while, this unbalance seems persistent at all educational levels (Hiebert, 2003).There are many studies on different aspects of learning, understanding and implementing proof (Hanna and Jahnke, 1996; Yackel and Hanna, 2003). Students have difficulties in differing proofs from other less rigorous types of argumentation (Chazan, 1993; Hoyles, 1997), understanding proof statements (Selden and Selden, 1995), making the transition from informal to formal reasoning (Tall, 1999) and constructing proofs. Even among university students, empirical sources of conviction (for example evidence from one or a few examples) dominate over more stable formal deductive reasoning (Balacheff, 1988; Harel and Sowder, 2007). There are also difficulties related to less formal but still central forms of reasoning. In line with this, Dickson Evbuomwan (2013) also analyzing performance in general and transformation geometry of rotation said that the performance of students was poor. Only a few students passed. Students performed poorly in naming transformation of rotation, finding the centre angle of rotation and locating the exact image of a rotated figure after rotation. They had difficulties at the level of abstraction and deduction of Van Hieles Model. On the same Makhubele Y (2011) analysed errors displayed by learners in learning of grade 11 and noted misapplied rules, weak conceptual knowledge, and very weak problem solving skills, inability to solve proof problems as some of the errors displayed by students. Makhubele's conclusion was that, there is need to establish the source of such errors and why learners do them. In order to do that teachers need knowledge of how students learn

A large part of the research results dealing with the reasons behind the difficulties discussed above can be characterized as an unwarranted and far too extensive reduction of the complexity of Informal geometrical concepts, processes and other ideas. This seems to be done in different situations by teachers, textbook writers and/or students in order to cope with curricula goals that are (too) hard to reach. Students are inclined to answer questions with a suspension of sense-making, and often use short-cut strategies (Schoenfeld, 1991; Heather, et al, 2009). There is pressure from students to reduce ambiguity and risk, and to improve classroom order, by reducing the academic demands in tasks (Doyle, 1988). As if this is not enough, teachers tend to take the reduction of mathematical complexity too far into inefficient rote learning (Lithner, 2011). In a historical perspective, McGinty et al. (1986) analysed grade 5 arithmetic textbooks from 1924, 1944 and 1984 and found that the number of word problems had decreased, the number of drill problems had increased, and that word problems had also become shorter and less rich. A brief comparison between some older calculus textbooks, for example (Courant and John, 1965), (de La Vallée Poussin, 1954) and some newer textbooks (Edwards and Penney, 2002), (Adams, 2006) indicates that the proportion of exercises that have more or less complete solution methods provided (for example. worked examples that are very similar to the exercises) has risen considerably. Cannon (2017) analyzing 14 textbooks on typical images anchors that despite extensive research on benefits and challenges of visualization there is little research into what types of figures students are exposed to through their textbooks. Vinner (1997) suggests a theoretical framework where two of the main notions are "pseudo-conceptual" and "pseudo-analytical". They are defined as thought processes that are not conceptual and analytical, respectively, but might give the impression of being so and could even produce correct solutions. Students' difficulties may often be better understood if they are interpreted within this "non-cognitive" framework than if they are seen as misconceptions within the domain of meaningful contexts: What may be a true learning and problem-solving situation for the teacher may not be so for the student. Because of the didactic contract (Brousseau, 1997) students may, consciously or not, try to please the education system with behaviour that, perhaps only superficially, is considered acceptable by the system. Leron and Hazzan (1997) emphasise additional non-cognitive means of trying to cope: attempts to guess and to find familiar surface clues for action, and the need to meet the expectations of the teacher or researcher.

Tall (1996) says that students mostly reduce their learning to rote learning by focusing on algorithmic procedures that can be carried out in order to solve advanced tasks without the need for conceptual understanding or constructive reasoning. The reasons behind students' focus on learning and applying routine procedures are that, when faced with conceptual difficulties, the student must learn to cope. In previous elementary mathematics, this coping involves learning computational and manipulative skills to pass exams. If the fundamental concepts of informal geometry prove difficult to master, one solution is to focus on the symbolic routines. At least this resonates with earlier experiences in which sequences of manipulations are performed to get an answer. The problem is that such routines become just that – routine – so that students begin to find it difficult to answer questions that are conceptually challenging. The teacher compensates by setting questions on examinations that students can answer and the vicious circle of procedural teaching and learning is set in motion. This scenario is not different in higher education where Heather (2007) says that, the students approach to a task maybe based on three approaches to learning thus, deep, surface and strategic.

The deep approach to learning is typified by an intention to understand and seek meaning, leading students to attempt to relate concepts to existing understanding and to each other, to distinguish between new ideas and existing knowledge, and to critically evaluate and determine key themes and concepts. In short, such an approach results from the students' intention to gain maximum meaning from their studying, which they achieve through high levels of cognitive are processing throughout learning. Facts are learnt in the context of meaning. There is some evidence that lecturers who take a student focused approach to teaching and learning will encourage students towards a deep approach to study (Prosser and Trigwell, 1999). The surface approach to learning is typified by an intention to complete the task, memorise information, and make no distinction between new ideas and existing knowledge; and to treat the task as externally imposed. Rote learning is the typical surface approach. Such an approach results from students' intention to offer the impression that maximum learning has taken place, which they achieve through superficial levels of cognitive processing. 'Facts' are learnt without a meaningful framework.

Biggs and Ramsden turned learning theory on its head in that rather than drawing on the work of philosophers or cognitive psychologists, they looked to students themselves for a distinctive perspective. Ramsden (1988) suggested that approach to learning was not implicit in the make-up of the student, but something between the student and the task and thus was both personal and situational. An approach to learning should, therefore, be seen not as a pure individual characteristic but rather as a response to the teaching environment in which the student is expected to learn. Biggs (1987) identified a third approach to study – the strategic or achieving approach, associated with assessment. Here the emphasis is on organising learning specifically to obtain a high examination grade. With this intention, a learner who often uses a deep approach may adopt some of the techniques of a surface approach to meet the requirements of a specific activity such as a test. A learner with a repertoire of approaches can select – or be guided towards – which one to use. Approaches need not be fixed and unchanging characteristics of the way a person learns. A misconception on the part of many students entering higher education is their belief that a subject consists only of large amounts of factual knowledge or a mastery of steps or rules, and, to become the expert, all one need do is add knowledge to one's existing store. It is the responsibility of the lecturer to challenge and change such limited conceptions and to ensure that their teaching, curricula they design, and assessments they set, take students into more stretching areas such as critical thinking, creativity, synthesis and so on. Biggs (1999) is one of the foremost proponents of the view that approaches to learning can be modified by the teaching and learning context, and are themselves learnt. He has also popularised the term constructive alignment to describe congruence between what the teachers intends learners to be able to do, know or understand, how they teach, and what and how they assess

A serious problem for those of us arranging undergraduate courses in mathematics, and probably for many engaged in teaching mathematics at any level and in any place in the world, is that we are unable to sufficiently help many students reach a desired level of mathematical competence (Lithner, 2011).

3.2 Perceptions on children's learning of informal geometry according to University students

The second type of knowledge is knowledge of content and teaching. This is the knowledge of the ways to support learner's cognitive development through progressively more sophisticated levels of paths. If children are to develop geometrical proficiency teachers must have a clear vision of classroom norms that support development of geometry proficiency. Unfortunately such concepts or elementary geometry or informal geometry are not what prospective teachers study. Instead the study of university geometry involves the increasing difficult in teaching (NAP 2001). The geometry curriculum in grades K–8 should provide an opportunity to experience shapes in as many different forms as possible. These should include shapes built with blocks, sticks, or tiles; shapes drawn on paper or with a computer; and shapes observed in art, nature, and architecture. Hands-on, reflective, and interactive experiences are at the heart of good geometry activities at the elementary and middle school levels. The geometry curriculum should aim at the development of geometric reasoning and spatial sense. Shapes, both two- and three-dimensional, exist in great variety. There are many different ways to see and describe similarities and differences among shapes. The more ways that one can classify and discriminate shapes, the better one understands them. Shapes have properties that can be used when describing and analyzing them. Awareness of these properties helps us appreciate shapes in our world. Properties can be explored and analyzed in a variety of ways. An analysis of geometric properties leads to deductive reasoning in a geometric environment.

Based on research outside of school, it is clear that children and adults regularly engage with geometry in their everyday lives and that the nature of this engagement is distinct from classroom practices. Independent of school, geometry is a central aspect of how children and adults solve challenges and complete tasks in their everyday and professional lives (for example Goldman & Booker, 2009; Nunes & Bryant, 2010; Roth, 2011). Furthermore, researchers have argued that these informal experiences represent critical resources and supports for mathematics learning in formal education settings. For example, Martin and colleagues highlighted the importance of explicitly connecting in-school and out-of-school mathematics by believing that when the mathematics of school and that of everyday life are seen as incommensurable, it impoverishes both contexts, separating the symbolic precision and power of school mathematics from the flexibility and creative sense-making of everyday life" (Martin & Gourley-Delaney, 2014).

Researchers have documented mathematics and mathematics learning in a range of everyday settings, including candy selling, carpet laying, video games, entertainment and play, sports, budgeting and money management, fishing, construction work, shopping and purchasing, farming, sewing, professional work in a variety of industries, and everyday family activities (Civil, 2002; Eloff, Maree, & Miller, 2006; Esmonde et al., 2013; Goldman & Booker, 2009; Hoyles, Noss, & Pozzi, 2001; Kliman, 2006; Martin, Goldman, & Jiménez, 2009; Martin & Gourley-Delaney, 2014; Masingila, Davidenko, & Prus-Wisniowska, 1996; Nasir, 2000; Nunes & Bryant, 2010; Nunes et al., 1993; Roth, 2011; Saxe, 1991; Taylor, 2009) For example, Nunes, Schliemann, and Carraher (1993) found that adult construction workers and fishermen who had no formal school mathematics training were able to solve proportional reasoning problems quite successfully, even compared to students who had studied proportions in school (Nunes & Bryant, 2010). Similarly, Nasir (2000) documented how high school basketball players were adept at solving basketball mathematics problems, especially when they were allowed to use informal estimation strategies.

Mathematical reasoning and learning have also been documented as a frequent part of family experiences and parent-child interactions (Benigno, 2012; Ginsburg, 2008; Hojnoski, Columba, & Polignano, 2014; Ramani, Rowe, Eason, & Leech, 2015), including cooking, meals, chores, shopping, and play activities, and the quantity and quality of mathematics related experiences between parents and preschool children have been found to be important predictors of children's developing mathematics skills and knowledge (Ramani et al., 2015). Studying the everyday mathematical experiences of four-year-old African-American children and their families through naturalistic observation, Benigno (2012) found substantial evidence of spontaneous mathematical experiences and practices that "reflected their unique family lives, individual predispositions, and knowledge development" including numbers and counting, geometric thinking and spatial reasoning, and discussions of difference and similarity. The process of parents helping their children with homework, although connected with formal schooling, can also create opportunities for rich, collaborative learning for both children and adults (Ginsburg, 2008).

Despite the unique and often sophisticated ways that people use mathematics in their daily lives, research indicates that children and adults often have a relatively narrow perspective on what counts as mathematics and may not connect concepts or skills learned in school with their everyday mathematical reasoning (Civil & Andrade, 2002; Ginsburg, Manly, & Schmitt, 2006; Goldman & Booker, 2009; Hoyles et al., 2001; Kliman, 2006; Kliman, Jaumot-Pascual, & Martin, 2013; Masingila et al., 1996).

Kliman and colleagues (2013) noted that, "even as awareness of science as a cultural and social activity is growing, adults of all backgrounds often view mathematics as a context-free topic consisting of facts and algorithms." Prior research in schools suggests that students tend to view mathematics as largely computational and involving problems that can be solved quickly. Students also often have difficulty finding applications for mathematics outside of school and bringing real-world knowledge to their mathematical problem-solving in the classroom (Martin & Gourley-Delaney, 2014). Outside of school, children seem to primarily associate mathematics with money, counting, and measuring, even though researchers have documented a diversity of examples of mathematical concepts and skills embedded in daily activities (Goldman & Booker, 2009; Hyatt, 2013; Jay & Xolocotzin, 2014), such as daily economics, trading and spending, counting, measuring and estimating distance and weight, exploring patterns and probability, and more. Some research suggests that even individuals in very technical fields, such as a fish culturist or field biologist, may not see themselves as doing mathematics (Roth, 2011).

A few researchers have explored and speculated about factors influencing how adults and children perceive mathematics outside of school. One study suggested that students are sensitive to the status of an activity when determining whether or not it is mathematical (Abreu & Cline, 2003). For example, a white-collar job, such as managing an office, might be more likely to be viewed as mathematical compared to a blue-collar job, such as taxi driving. Martin and colleagues (2014) found several factors that affected whether or not sixth grade students classified images of everyday and in-school activities as mathematical, including surface features, such as numbers, symbols, and money, and the possibility or necessity of mathematical action in the situation. The researchers also found that "consistent with common sense expectation, activities like dancing, playing music, and fishing were generally not seen as mathematical, while worksheets, school math presentations, and paying bills were" Students were also more likely to rate activities as mathematical if they had personal experience with them.

More broadly, Swanson and Williams (2014) have argued that the structure of everyday contexts, such as work environments, and the tools that we use in these situations can obscure the underlying mathematics of tasks and problems. Drawing from Vygotsky's work (Vygotsky, 1978), the researchers noted that mathematics can become "fossilized" in tools and procedures: "This fossilization (Vygotsky, 1997, p. 71) of the mathematics—often in physical artefacts, or in procedures, or fused in situated concepts—means that the acting subject is generally barely aware of the mathematics embedded there. It is concrete but not theoretical for them" (Swanson & Williams, 2014, p. 195). For example, in their research, professional and amateur dart players used "outs tables" to guide end-game strategies, based on the probabilities of achieving different combinations of points to win the game.

Although these strategies are highly mathematical, "much of this know-how has been crystallised in the outs table that players can download from the internet and carry in their pockets" (Swanson & Williams, 2014, p. 198). Swanson and Williams also argued that the hierarchy and division of labor in workplaces often produces knowledge barriers that relegate the mathematical aspects of work to certain individuals and obscure or routinize the math for many other workers. This hidden nature of mathematics can break down, however, in certain situations, such as intrinsic or vocational motivation or transitions to highly competitive situations, in which individuals or groups are motivated to explore and understand the mathematics at a deeper level.

It is also worth noting that there are ongoing debates even among educators and mathematicians about the nature of mathematics and what counts as math in different settings (Martin & Gourley-Delaney, 2014; Wright & Parkes, 2015). Given this, it may not be surprising that those who do not study mathematics or math education are also confused. One helpful framework for defining mathematics in out-of-school environments has emerged from researchers studying adult education and learning, who have coined the term "numeracy" to distinguish between more formal conceptions of mathematics and those math-related topics, skills, and dispositions "woven into the context of work, community, and personal life" (Ginsburg et al., 2006).

Students can also be supported through social mediation. Studies have also found that social mediation is frequently a central aspect of everyday mathematics. In the context of families, parents and caregivers often play an important role in facilitating their children's engagement with mathematics using a variety of strategies, including modeling, prompting and encouraging, engaging in distributed problem solving, asking questions, explaining and directing, or playing (Civil & Bernier, 2006; Civil, Díez-Palomar, Menéndez, & Acosta-Iriqui, 2008; Eloff et al., 2006; Goldman & Booker, 2009; Mokros, 2006). Some studies suggest that parents' cultural backgrounds and prior experiences with mathematics and school can be important influences on their approach to mathematics learning and discourse within the family (Civil & Bernier, 2006; Guberman, 2004; Rogoff, Paradise, Arauz, Correa-Chávez, & Angelillo, 2003). Parents and caregivers often report not feeling confident in their knowledge and abilities related to helping their children learn mathematics (Lopez & Donovan, 2009; Mokros, 2006), although this may be more true in the context of mathematics homework and school learning.

One way that parents engage children in math is through authentic involvement in everyday, mathematical activities. In studying four-year-old African-American children, Benigno (2012) documented a range of child driven, child-and-other-driven, and adult-driven mathematical experiences in the children's everyday lives and found that parents and other adults often played an important role by involving children meaningfully in everyday family practices through which mathematics naturally emerged, supporting mathematical understanding and exploration initiated by children, or purposely introducing and instructing children on specific mathematical skills and concepts.

The study highlighted how the young children and their families engaged in spontaneous mathematical events in the course of their daily activities and demonstrated distinct mathematical understandings that reflected the child's unique family life and individual predispositions and knowledge development.

Family mathematics can also arise in more pedagogical contexts. For example, a small but growing body of research suggests that parent-child shared book reading experiences are important contexts of early childhood mathematics learning. Hojnoski et al., (2014) says that children's literature can be used to support early mathematics development. Specifically, storybook text and illustrations contextualize mathematical concepts (for example, numbers and operations, measurement, shapes), storybook reading elicits mathematical behavior (for example, reasoning, problem solving), and the social nature inherent in shared reading mediates engagement in mathematical discourse (for example, the parent explains or elaborates upon mathematical ideas presented by his or her child).

Outside the family context, Taylor (2009) studied the mathematics of children's purchasing practices in convenience stores and found that store clerks often provided support to help children select items and make payments, especially during more complex transactions. Similarly, Nasir (2000) documented how social interactions with other players and coaches were important factors influencing the mathematical practices of middle school and high school basketball players. In the context of work settings, apprenticeship can be a common model through which adults learn and engage with mathematics (Masingila et al., 1996).

Students can also be supported through engaging them in mathematics in Designed Informal Learning Environments. Designed informal learning environments (National Research Council, 2009), such as mathematics-themed exhibits in museums, are another setting in which rich mathematical thinking and reasoning outside the classroom can occur. Unlike schools, these settings offer individuals and groups the opportunity to more freely choose how, what, where, and with whom they learn (Falk & Dierking, 2000, 2013). However, unlike everyday settings, designed informal learning environments are often created with explicit pedagogical goals, including supporting mathematical reasoning and learning (National Research Council, 2009). Because of this, designed informal learning environments may offer rich mathematics learning opportunities for families and children that are not widely available in formal classroom settings, including kinesthetic and social mathematics experiences (Cooper, 2011; Wright & Parkes, 2015).

An important example of these settings is the growing number of mathematics-focused exhibitions in museums and science centers (Cooper, 2011). Mathematics is a topic of growing interest in the informal science education field (Mokros, 2006) and there are an increasing number of museum and science center exhibitions focused on the topic, such as Design Zone, Mathematics Moves, and Geometry Playground (Danctep, Gutwill, & Sindorf, 2015), as well as a new museum focused entirely on mathematics.

Although they have been the focus of less research attention, libraries can also be spaces for facilitated and unfacilitated mathematics learning experiences (for example, Kliman et al., 2013). Similarly, online games are another opportunity for rich, informal mathematics learning. For example, studying third and fourth graders using an online mathematics game developed to complement the Cyberchase television series, Fisch and colleagues (2009) observed and tracked children using a range of sophisticated mathematical strategies that often became more advanced as they played the game and encountered new scenarios and challenges.

Although the literature is small (Anderson & Thompson, 2001; Cooper, 2011), there is a growing body of research and evaluation studies providing evidence of the mathematical thinking and learning that is possible in these settings. Investigators in science centers, for example, have documented evidence of algebraic and proportional reasoning (Garibay Group, 2013a; Pattison, 2011; Pattison, Ewing, & Frey, 2012; Rubin, Garibay, & Pattison, 2016; Selinda Research Associates, 2016); spatial reasoning (Danctep et al., 2015); qualitative, intuitive understandings of slope (Nemirovsky & Gyllenhaal, 2006; Wright & Parkes, 2015); connections with the mathematics in the experiences to school and everyday lives (Garibay Group, 2013a); and more general math-related discourse, such as description, counting and numbers, patterns, size estimation, problem-solving, comparison, spatial orientation, precision, shape identification, and fractions (Randi Korn & Associates, 2001; Vandermaas-Peeler, Massey, & Kendall, 2015). However, other studies have documented lost opportunities. For example, in observations of visitors at a zoo, a children's museum, and a history museum, Cooper (2011) found abundant opportunities for mathematical learning but limited evidence of mathematical-related conversations within families. In one of the few projects that took advantage of mathematical possibilities in institutions with live animal collections, the Math in Zoos and Aquariums project (Garibay, Martin, Rubin, & Wright, 2012) used animal behavior and animal characteristics as the basis for several family-oriented mathematics activities. One challenge for the field is that the majority of evaluation studies have focused on assessing project-specific goals and outcomes (e.g., Garibay Group, 2013a, 2013b, Randi Korn & Associates, 1999, 2001), providing few details on the nature of visitor mathematical reasoning, behaviors, or conversations.

A consistent finding from studies, also aligned with research from everyday settings, is that visitors are often not aware that they are engaging with mathematics or have relatively narrow conceptions of mathematics (Garibay Group, 2008; Gyllenhaal, 2006; Randi Korn & Associates, 1999). For example, in the front-end evaluation for the Design Zone project, Garibay Group (2008) noted that "both children and adults most commonly associated mathematics with numbers and operations" (p. 4) and that even older children and adults had a limited notion of algebra beyond solving for an unknown. In the evaluation of the Handling Calculus exhibition (Gyllenhaal, 2006), most visitors without formal calculus experience associated the exhibit activities with math in general, rather than the specific topic of calculus. However, for those who had taken calculus courses, the experience was often connected with both positive and negative school memories. Gyllenhall (2006) also reported that some individuals can become anxious when they learn that an experience involves math, potentially because of negative previous experiences with the topic.

Given these potential negative associations, some educators and developers working in informal learning environments have attempted to address these challenges and promote awareness of the mathematics without undermining other experience and learning outcomes. For example, exhibit developers often come face to face with the need to balance these two goals when they name an exhibition. While the developers of Geometry Playground purposely used a mathematics term in the title, the developers of Design Zone consciously avoided this association. Nonetheless, in the summative evaluation of the Design Zone exhibition (Garibay Group, 2013a), the majority of visitors felt that the exhibit experiences were connected to mathematics in school or in their everyday lives. Furthermore, 95% of respondents enjoyed their experience in the exhibition and 94% of the children in the target age range (10 to 14) who remembered using mathematics in the exhibition indicated that they felt comfortable with that aspect of the experience.

Also similar to mathematics in everyday settings, evaluation and research studies have repeatedly highlighted the importance of social mediation when visitors engage with mathematics in designed informal learning environments. In several evaluation studies of mathematics exhibitions at science centers and children's museums, Randi Korn & Associates (1999, 2001) found that parents and caregivers played an important role in facilitating mathematical reasoning and engagement and that the level and nature of that facilitation appeared to differ across activities. In one study, parent facilitation strategies included asking questions, making suggestions, pointing out details, instructing children, and engaging in dramatic play (Randi Korn & Associates, 1999). Similarly, in the evaluation of the Handling Calculus exhibition, Gyllenhaal (2006) found that adults and parents often facilitated learning for visitor groups, even when they knew little about the math content. In the summative evaluation of the Design Zone exhibition (Garibay Group, 2013a), evaluators found that parents and other adults played an important role in facilitating math learning and increased the likelihood that family members engaged in more sophisticated algebraic reasoning, such as conversations about the relationships between different variables in the exhibits. Aligned with the flexible nature of mathematics outside of school, one way adults might play an important role in these interactions is by helping their groups to adopt different mathematical strategies appropriate to the level of understanding within the group and to the problem or challenge relevant at a given moment (Rubin et al., 2016).

Only a few studies have explored the design characteristics of these settings that might support, or hinder, mathematical engagement and learning. One strand of this work has focused on the influence of exhibit size and scale, and in particular differences between immersive and tabletop exhibits. For example, Dancstep and colleagues (Danctep et al., 2015) used an experimental design to compare visitor experiences and outcomes at tabletop- and immersive-versions of exhibits as part of a larger exhibition designed to support spatial reasoning. At both versions of the exhibits, adults and children used spatial language and reasoning during the interactions, including static, dynamic, and causal language. Counter to their expectations, however, the visitors at the tabletop versions exhibited higher levels of spatial reasoning language compared to the visitors at the immersive versions, on average. In contrast, building on the notion of embodied cognition in mathematics (Abrahamson & Lindgren, 2014; Eisenberg, 2009; Hall & Nemirovsky, 2012), Nemirovsky and colleagues conducted several studies demonstrating the potential of interactive and immersive exhibits for supporting visitors in the development of more intuitive understandings of mathematical relationships and concepts (Nemirovsky & Gyllenhaal, 2006; Nemirovsky, Kelton, & Rhodehamel, 2013; Wright & Parkes, 2015). Similarly, a summative evaluation of the Math Moves! Exhibition indicated that visitor engagement "demonstrating increasing qualitative and kinesthetic fluency" (Selinda Research Associates, 2016, p. 69) was particularly noticeable at whole-body exhibits, although engagement times were longer at some smaller tabletop activities.

Another strand of research in this area has focused on supporting the role of parents or adult family members during interactions at mathematics exhibits. Vandermaas-Peeler and colleagues (2015) found that providing parents and family groups with additional orientation and guidance by a staff member before entering a math exhibition was associated with family groups asking a greater variety of guiding questions and talking more about measurements and size comparisons. Similarly, in research and evaluation studies of the Design Zone exhibition, investigators found evidence that carefully designed "parent panels" with supporting information for adult family members were important for encouraging algebraic reasoning (Garibay Group, 2013a; Rubin et al., 2016). Research on Design Zone also highlighted the promise of clear and explicit challenges posed in exhibit labels for enhancing math exploration (Garibay Group, 2013a), as well as the potential trade-offs of using technology, such as exhibit-embedded computer guides, to prompt challenges and structure the visitor experience (Pattison et al., 2012). Emerging evidence also suggests that museum educators can enhance visitor satisfaction and mathematical reasoning at interactive exhibits when staffs are supported by research-based professional development (Pattison et al., 2016, 2017).

These few studies provide early indications that, like classrooms and everyday settings, designed informal learning environments can offer rich opportunities for supporting mathematical reasoning and learning. However, with characteristics that are similar to and different from both everyday settings and classrooms, designed informal learning environments may also offer unique constraints and affordances (Rubin et al., 2016). For example, while educators and designers can provide rich mathematical representations for learners in these environments, the use of these tools may be dependent on the goals and social context of the experience. Similarly, although science centers and other informal learning environments create excellent opportunities for socially mediated learning (Astor-Jack, Whaley, Dierking, Perry, & Garibay, 2007; National Research Council, 2009; Pattison & Dierking, 2013), those in the position to support learning during these experiences, such as parents or staff facilitators, may have limited understanding of mathematical reasoning or the strategies for fostering mathematics learning. And in contrast to classrooms, where argumentation and proof may be explicit goals (National Research Council, 2005; Yackel & Cobb, 1996), in museums and science centers such modes of discourse may be at odds with social expectations (National Research Council, 2009). Given the importance of mathematical reasoning for success in school and life (National Research Council, 2005), there is a critical need to explore these tradeoffs and investigate the potential of designed informal learning environments for supporting mathematical learning

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3.3 Choosing and adapting a Curriculum

The third type is knowledge of content and curriculum. This type of knowledge is knowledge of how to choose and adapt curricula that are typically built on mathematics disciplinary perspective. In the first place choosing and adapting curricula that is typically built on informal geometry might be a difficult task as research explains (Duangduen Onnuam, Robert G. Underhill (1986). A curriculum can be interpreted from a set of personal constructs based on past experiences (Kelly, 1955) bearing in mind that, the presentation of geometry concepts in elementary school mathematics curricula vary widely. Even though Trafton and Le Blanc (1973) found that approximately 15 percent of pages in elementary textbooks were devoted to geometry a 1975 exploratory survey by the National advisory committee on mathematics education revealed that 78 percent of the teachers surveyed reported spending few than 15 class periods each year on topics of geometry. The situation may be explained in part by evidence collected by Brailey (1969). He reported that 70 percent of 183 prospective elementary teachers scored 70 percent or less on a test of geometry content based on elementary school textbooks.

There has been no generally accepted geometry scope and sequence. Trafton and Le Blanc (1973) concluded that the diffuse nature of geometry allows for greater flexibility and variability of organisation and content.

However, in 1957 P Van Hiele and D Van Hiele- Geldof completed theses at the university of Utrecht in which they presented 5 levels of understanding of geometry concepts which have been reported by Maryberry (1983). In the levels of understanding, level one is considered to be the basic. At the basic level, learners do not perceive properties of figures; they recognise figures solely by appearance, sort of a gestalt of undifferentiated particulars.

Although the debate on the diffuse nature of geometry allowing for greater flexibility and variability has continued, in 2007, in Ontario a curriculum was spelt out to guide the type of geometry to be included in a mathematics curriculum.

The mathematics curriculum should be future oriented that is it should provide a major opportunity to lead improved teaching and learning. This future orientation includes the consideration that society will be complex, with workers competing in a global market, needing to know how to learn, adapt, create, communicate, interpret and use information critically. It should be based on the following fundamental principles: Curriculum expectations must be coherent, focused, and well-articulated across the grades. Learning mathematics involves the meaningful acquisition of concepts, skills, and processes and the active involvement of students in building new knowledge from prior knowledge and experience. Learning tools such as manipulatives and technologies are important supports for teaching and learning mathematics. Effective teaching of mathematics requires that the teacher understand the mathematical concepts, procedures, and processes that students need to learn, and use a variety of instructional strategies to support meaningful learning. Assessment and evaluation must support learning, recognizing that students learn and demonstrate learning in various ways. Equity of opportunity for student success in mathematics involves meeting the diverse learning needs of students and promoting excellence for all students. Equity is achieved when curriculum expectations are grade- and destination-appropriate, when teaching and learning strategies meet a broad range of student needs, and when a variety of pathways through the mathematics curriculum are made available to students.

It must engage all students in mathematics and equip them to thrive in a society where mathematics is increasingly relevant in the workplace. It must engage and motivate as broad a group of students as possible, because early abandonment of the study of mathematics cuts students off from many career paths and postsecondary options. The unprecedented changes that are taking place in today’s world will profoundly affect the future of today’s students. To meet the demands of the world in which they live, students will need to adapt to changing conditions and to learn independently. They will require the ability to use technology effectively and the skills for processing large amounts of quantitative information. Today’s mathematics curriculum must prepare students for their future roles in society. It must equip them with an understanding of important mathematical ideas; essential mathematical knowledge and skills; skills of reasoning, problem solving, and communication; and, most importantly, the ability and the incentive to continue learning on their own. This curriculum provides a framework for accomplishing these goals. The development of mathematical knowledge is a gradual process. A coherent and continuous program is necessary to help students see the “big pictures”, or underlying principles, of mathematics. The fundamentals of important skills, concepts, processes, and attitudes are initiated in the early grades and fostered throughout university. The links between Grade 8 and Grade 9 and the transition from elementary school mathematics to secondary school mathematics and university mathematics are very important in developing the student’s confidence and competence. The university courses are based on principles that are consistent with those that under-pin early years, elementary, university program, facilitating the transition from secondary school. These courses reflect the belief that students learn mathematics effectively when they are given opportunities to investigate new ideas and concepts, make connections between new learning and prior knowledge, and develop an understanding of the abstract mathematics involved. Skill acquisition is an important part of the learning; skills are embedded in the contexts offered by various topics in the mathematics program and should be introduced as they are needed. The mathematics courses in this curriculum recognize the importance of not only focusing on content, but also of developing the thinking processes that underlie mathematics. By studying mathematics, students learn how to reason logically, think critically, and solve problems – key skills for success in today’s workplaces. Mathematical knowledge becomes meaningful and powerful in application. The curriculum should embed the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from related disciplines, such as computer science, business, recreation, tourism, biology, physics, and technology, as well as from subjects historically thought of as distant from mathematics, such as geography and art. It is important that these links between disciplines be carefully explored, analyzed, and discussed to emphasize for students the pervasiveness of mathematical concepts and mathematical thinking in all subject areas. The choice of specific concepts and skills to be taught must take into consideration new applications and new ways of doing mathematics. The development of sophisticated yet easy-to-use calculators and computers is changing the role of procedure and technique in mathematics. Operations that were an essential part of a procedures-focused curriculum for decades can now be accomplished quickly and effectively using technology, so that students can now solve problems that were previously too time-consuming to attempt, and can focus on underlying concepts. “In an effective mathematics program, students learn in the presence of technology. Technology should influence the mathematics content taught and how it is taught. Powerful assistive and enabling computer and hand held technologies should be used seamlessly in teaching, learning, and assessment. “The curriculum should integrate appropriate technologies into the learning and doing of mathematics, while recognizing the continuing importance of students’ mastering essential numeric and algebraic skills (The Ontario Curriculum, 2007).

In line with this The National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC) affirm that high-quality, challenging, and accessible mathematics education for 3- to 6-year-old children is a vital foundation for future mathematics learning. In every early childhood setting, children should experience effective, research-based curriculum and teaching practices. Such high-quality classroom practice requires policies, organizational supports, and adequate resources that enable teachers to do this challenging and important work.

Throughout the early years of life, children notice and explore mathematical dimensions of their world. They compare quantities, find patterns, navigate in space, and grapple with real problems such as balancing a tall block building or sharing a bowl of crackers fairly with a playmate. Mathematics helps children make sense of their world outside of school and helps them construct a solid foundation for success in school. In elementary and middle school, children need mathematical understanding and skills not only in mathematics courses but also in science, social studies, and other subjects. In high school, students need mathematical proficiency to succeed in course work that provides a gateway to technological literacy and higher education. Once out of school, all adults need a broad range of basic mathematical understanding to make informed decisions in their jobs, households, communities, and civic lives. Therefore, if children need a coherent curriculum, teachers must adapt a particular model for choosing and adapting an informal geometry curriculum.

Joan Moss, Zachary Hawes, Sara Naqvi, Beverly Caswell (2015) gives the Japan’s model of lesson study as one way of adapting and choosing a curriculum. Moss et al., (2015) says that, educators need to develop, test and revise conjectures regarding new approaches to the teaching and learning of geometry and spatial reasoning. The project involves them in engineering their participants’ development through new forms of practice, while, at the same time, systematically studying the effectiveness of those practices from the perspectives of both teacher change and student learning (Cobb et al. 2015).

Educators should re-conceptualize what it means for teachers to learn and teach geometry in the early years. Rather than approaching geometry as a subject that is largely static in nature and one mainly concerned with labeling and classifying shapes (Clements 2004), educators should introduce teachers and administrators to the idea of geometry as dynamic, spatial, and imaginative in nature. They should collaboratively plan, teach and reflect on classroom lessons. They should generally follow the following four steps: (1) goal setting/investigation; (2) planning; (3) implementation and research Lesson; and (4) debriefing/reflection (for example, Lewis et al. 2006).

Also follow the new adaptations which may include: (1) teachers engaging in informal geometry, (2) teachers designing and conducting task-based clinical interviews, (3) teachers and researchers co-designing and carrying out exploratory lessons and activities, and (4) the creation of resources for other educators.

The inclusion of these four adaptations would strengthen the lesson study process and provide optimal support for teacher growth in both their content knowledge and their attitudes towards geometry and spatial reasoning.

Educators should also come up with Professional Learning Team (PLT) consisting of k-2 teachers, early childhood educators, school administrators, district and provincial mathematics facilitators, university mathematics educators and researchers. PLT should participate in a range of geometry and spatial reasoning activities as learners. This approach departs from the traditional lesson study process, whereby teachers generally begin by identifying mathematics topics that their students find challenging to serve as the main focus for their inquiry. In Moss et al (2015) process, the researcher invited the PLT’s to work on mathematics challenges not typically addressed in elementary geometry curricula such as mental transformations, spatial visualization, and the composition/decomposition of 3D shapes. The researchers anticipated that teachers would become intrigued with these new types of mathematics problems and that through participating in the various activities, the students would see the benefit of trying similar activities in their own classrooms. They wondered if trying these unfamiliar tasks might begin to shift the PLT’s attention away from the notion of geometry as static (for example, naming and classifying shapes; teacher practices familiar to team members) towards a more dynamic and spatial view of geometry. The quartet did this because of their concern in improving the teaching of geometry and spatial sense in early childhood which according to them has not been receiving the attention it deserved globally. They also wanted to share their experiences from Japan which has

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been performing better where children performance in mathematics generally is concerned. In interpreting the curriculum in Japan, books start by training children on proofs. And facts are derived from proofs (Taro Fujita and Keith Jones, 2016).

4. Theoretical Framework

This study used a number of theories such as Jean Piaget's (1958), Pierre and Dina van Hiele (1957), Schulman (1986), Ball Deborah Loewenberg etal (2011) and UNICEF (2018). The focus was on examining how students at university level approach tasks in informal geometry, how they think is the way children they teach learn informal geometry and how teachers adapt an early childhood education geometry curriculum that may be relevant to societal aspirations and needs. The first objective focused on approaches that students at university use to approach tasks in informal geometry. While the second objective focused on what students think is the way children they teach learn informal geometry. In other words it focused on child informal geometry development and learning and the last objective focused on how to adapt an early childhood education geometry curriculum that may be relevant to societal and national aspirations where informal geometry teaching and learning was concerned. In the art of teaching, the teachers focus is on what to teach, how to represent it, how to question students about it and how to deal with problems of misunderstanding. Therefore, the four theories of geometrical thinking, pedagogical content knowledge, mathematical knowledge for teaching and learning through play were used as there is no single theory that can explain the art of teaching as it relates to content knowledge effectively (keith Jones and Taro Fujita, 2003).

Carol Bratton, Una Crossey, Dawn Crosby & Wendy Mc Keown (2005) revisited Piaget's theory of learning through play by saying that in addition to play, children also learn through movement, talk and sensory experience. Piaget considered play to be an important part of childhood as a path to the learning process. Learning through play for Piaget was defined as a movement through practice play, imaginative play, and continuing on to play with a set of rules. In play the teacher combines both instructional educator-led approaches and play based pedagogy. Meaning for learning and teaching of geometry to be effective both surface and deep learning should come into play. This should be combined with a strategic approach. A teacher should use both facilitative and Transmissive approaches in teaching.

The teacher further should have an understanding of how children develop geometric understanding. The van Hiele theory describes how young people learn and develop geometric understanding. It gives an analysis of teaching and learning. It postulates five levels of geometric thinking which are labelled visualization, analysis, abstraction, formal deduction and rigor. Each level uses its own language and symbols. Students or pupils pass through the levels step by step. It further, stresses the importance of the properties of the levels and teaching-learning act. Students' progress from one level to the next as the result of purposeful instruction organized into five phases of sequenced activities that emphasize exploration, discussion, and integration. The van Hiele model postulates that these five phases of instruction are necessary to enable students in each learning period to develop a higher level of geometric thinking.

This hierarchical order helps children to achieve better understanding and result. Van Hiele (1956) also explained that, at visual level the child may begin with nonverbal thinking. Shapes are judged by their appearance and generally viewed as a whole, rather than by distinguishing parts. Although children begin using basic shape names, they usually offer no explanation or associate the shapes with familiar objects. For example, a child might say, it's a square because it looks like one, or I know it's a rectangle because it looks like a box. However, there is a transfer of meanings as a child in imaginary play can think of a stick as a horse as he or she mentally designates the object or property as the word.

Van Hiele, said that, children must go through levels in order and that what was intrinsic in the preceding level becomes extrinsic in the current level. Each level has its own linguistic symbols and its own network of relationships connecting those symbols. What may be correct at one level may not be correct at another level. Two persons at different levels cannot understand each other. Learning process leads to complete understanding at the next level and this learning process is strictly not sequential. However despite Van Hiele explaining all these levels one wonders whether teachers have the competencies to observe, understand and interpret all the levels in the children that they teach. This may be might have prompted researchers like Schulman (1986) to start questioning teacher knowledge for teaching.

5. Conceptual Framework

The widespread acceptance among scholars for the need to revisit the conceptualization of pedagogical content knowledge and its interweaving with other knowledge bases has prompted other researchers to develop their own conceptual frameworks of teacher knowledge. One such conceptual framework is in-service university students' pre-school pedagogical content knowledge for teaching informal geometry (Gondwe-Chimfwembe (2020) which might be an extension of Pedagogical content Knowledge propounded by Ball et al (2008). In-service university students' pedagogical content knowledge for teaching informal geometry was theorized out of the proposition to shift the emphasis of this area of research from understanding how teachers' knowledge develop (Shulman, 1986), how this knowledge is used in and for teaching (Ball, Thames, & Phelps, 2008) to examining in-service university students pre-school teachers pedagogical content knowledge for teaching informal geometry – a concern that is captured from asking the question, what do teachers need to know and be able to do in order to teach effectively to how do students at university level approach tasks in informal geometry; what do students at University level think is the way in which children they teach at ECE level learn informal geometry and how do teachers choose, interpret and adapt an ECE informal geometry curriculum that is based on national needs and aspirations? (Chimfwembe, 2020).

In their work leading to the development of PCK, Ball, Thames, and Phelps (2008) raise three points central to their argument: (1) that a lot of the work teachers are expected to carry out require mathematical knowledge; (2) that this mathematical knowledge is often left out in discourses about what mathematics teachers need; and (3) that there is substantial evidence from their data to suspect that their insights can be extended to include the knowledge teachers need in other subjects. While Chimfwembe (2020) in this study raises three points central to the argument: (1) the teacher needs to understand the concepts and terms used in national documents (2) the teacher needs to understand that the way the child develops concepts is not separate from the way the child learns concepts, thus child development and early learning are not separate (3) since child development and early learning are not separate, a teacher should adapt a curricula that meets national needs and societal aspirations that are enshrined in national documents. While Ball et al (2011) interpretation of the data reaped from their research led to the development of the six domains that form mathematical knowledge for teaching, the current research has led to understanding of early childhood informal geometry education: its practices, challenges and solutions; how a bachelor's degree course outline in informal geometry is adapted: Practices, challenges and solutions; finding where informal geometry is in early childhood education, designing a syllabus and activities that teachers could be using to teach informal geometry and constructing and operationalising the centre of excellence where informal geometry should be taught.

6. Methodology

In examining pedagogical content knowledge for teaching informal geometry among university students mixed approaches were used. The sampling procedure used was exploratory in nature with a realist paradigm touch. The research was cross-sectional as the teachers observed were not the same teachers on which the questions for the class activities were administered.

A questionnaire, observation instruments, document analysis and voice records were used to find out how teachers understood how children learn, how they taught and how they adapted the curriculum. Thematic, narrative and statistical ways of analysing data were used. A further analysis was done to test students on how they understood the content, misconceptions and errors that children are likely to make**.** The researcher further analysed their knowledge of teaching geometry using the properties of levels according to van Hiele and observed departure (separation) points where they could not differentiate between SMK and PCK then the researcher went to the teaching and learning act by initiating a teaching program in geometry using a syllabus, modules, child assessment tool, demonstration school and the activities that the researcher designed.

7. Findings of the Study

7.1 Approach to tasks in informal geometry

Eleven questions were given to 84 in-service University students to examine their approach to tasks in informal geometry. About 50% of train teachers were not able to explain how they introduce geometry to the young ones; designing and explaining an activity they could use; other things the teacher should be aware of when introducing geometry; starting a systematic study of geometry; concepts that teachers should emphasise on in informal geometry and the most popular two dimension figures.

7.2 Way in which children learn informal geometry

About 50% of train teachers were not able to explain ways in which children learn geometric shapes; how children learn according to van Hiele; presentation of the four basic shapes using van hiele; teaching at the third level according to van Hiele; verification of the principle that the sum in degrees of angle measures of any triangle is always the same and formation of a pyramid.

Most of the ways in which children learn informal geometry such as through observation and interacting with objects daily, playing with toys, singing, daily routines- paper folding, sorting, classifying, sand play , puzzles, rhymes and riddles, games and other activities that were mentioned by the 84 students during the focus group were not seen in the 10 pre-school teacher's classes and this reflected even in the performance of 84 in-service train ECE teachers' individual activities during lecture hours. Most of the 84 in-service train ECE teachers during the class activity just mentioned one or two aspects as ways in which children learn informal geometry.

This study went on to analyze how preschool teachers link the early geometry content they teach. The researcher asked in-service pre-school teachers questions on completing the table and finding the number of diagonals for a polygon with N-sides, Verifying the principle that the sum in degrees of the angle measure of any triangle is always the same, explaining what the sum of three angle measures appear to be and finding the angle of the convex quadrilateral, using inductive reasoning or drawings to complete a table. The table below shows how train preschool teachers faired in these question:

7.3 Ways in which in-service preschool teachers are linking content they teach

Table 1: Explanation of activity to use when introducing geometry

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	5	13.5	13.5	13.5
	1	7	18.9	18.9	32.4
	18	1	2.7	2.7	35.1
	2	9	24.3	24.3	59.5
	3	5	13.5	13.5	73.0
	4	8	21.6	21.6	94.6
	5	2	5.4	5.4	100.0
	Total	37	100.0	100.0	

Most of the in-service preschool teachers got less than 50% in this particular question. One group could not even explain an activity to use when introducing geometry.

Table 2: Finding the number of diagonals for a polygon with N sides

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	3	8.1	8.1	8.1
	3	4	10.8	10.8	18.9
	6	29	78.4	78.4	97.3
	7	1	2.7	2.7	100.0
	Total	37	100.0	100.0	

A good number found the number of diagonals for a polygon with N sides but could not come up with the generalization thus $\frac{N(N-3)}{2}$

Table 3: Time of the day when minute hand is at 75 degrees

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	29	78.4	78.4	78.4
	1	5	13.5	13.5	91.9
	18	1	2.7	2.7	94.6
	2	1	2.7	2.7	97.3
	3	1	2.7	2.7	100.0
	Total	37	100.0	100.0	

29 groups of in-service preschool teachers got zero. They failed to show the time of the day when the minute hand is at 75^{o}. Only one group managed to arrive at an answer.

Table 4: Verification of the principle thus the sum in degrees of the angle measures of any triangle is always the same

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	21	56.8	56.8	56.8
	1	14	37.8	37.8	94.6
	18	1	2.7	2.7	97.3
	2	1	2.7	2.7	100.0
	Total	37	100.0	100.0	

21 groups got zero.one group could not write anything. 14 groups got one mark. One group left the question an attended to.

Table 5: Completion of the table by induction

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	3	8.1	8.1	8.1
	18	2	5.4	5.4	13.5
	2	1	2.7	2.7	16.2
	3	7	18.9	18.9	35.1
	6	23	62.2	62.2	97.3
	7	1	2.7	2.7	100.0
	Total	37	100.0	100.0	

A good number got more than 60%. Two groups could not do this activity where they were expected to use inductive reasoning or drawing to complete the table

7.4 Adaptation of the ECE informal geometry curriculum

The following train pre-school teachers were observed 20 pre-school teachers were observed. 8 from school L, 5 teachers from selected pre-schools were on the planned activities for the Non-Government Organisation Z and were not students at University C. 7 pre-school teachers were train Pre-school teachers at University C. Both categories had not received instruction in informal geometry at the University. But had basic ideas on plane shapes. These 20 pre-school teachers were observed using a teaching instrument designed by the ministry of general education and later after observing them the following follow up questions were asked. The following dialogue brings out what some of them said:

Interviewer: *Do you have access to the school syllabus*

In-service Pre-school teacher: *Yes, it was just given to me this term and I have been instructed to follow the syllabus and repeat on concept for the whole week.*

Interviewer: *What do you teach the whole week*?

In-service preschool teacher: *What is in the syllabus? That is what I was told to do during our teacher group meeting by our co-ordinator and during the workshop*

Interview: *What else do you teach*?

In-service pre-school teacher: *Madam I am a law abiding teacher, our books are just on lines and shapes? I do not want to teach things outside the guidelines. Learners draw lines, identify shapes, trace and colour basic shapes*

The interview and class observation from the 20 pre-school teachers enabled me to administer a min question to the 84 in service teachers who were admitted to the university in the academic year of 2015/2016.This cohort responded to the question on what the outcome in the pre-school syllabus *identify shapes* meant (pre-school syllabus, 2013)

In-service preschool teachers were able to define it as follows:

Identify means to *see, recognise and name the shape*

From this definition, it seems that there is consistence in the interpretation of the syllabus for teachers teaching at this particular level thus level 5-6 although the content in the syllabus was shallow as compared to Copley, J 2010 recommendations. This finding indicated that teachers interpreted the outcomes well as they could define the action words and able to present guided activities well in class. In short teachers had knowledge of content and syllabus but not a curriculum.

Further, I went on to investigate from the 84 in-service teachers on ways in which children learn informal geometry. They gave the following as the ways in which children learn informal geometry:

Through plane figures lines and points (Piaget, 1956, 1960);Through daily objects (roof, door, ball, wheel, nose);Through the shapes they can make with their body parts; Through the toys they play with (building blocks, monkey bars); manipulation of objects; Through observing their environments closely after being given classification of shapes; Through Songs e.g. this is a square looking like a window, this is a rectangle looking like a door and this is a triangle looking like a roof; Through day to day experiences but what they do not know are names of shapes; Through class activities for example making of different shapes using puzzles rhymes, riddles, paper folding; sorting, classifying, tracing, colouring, making shapes and modelling; Through class activities for example; Asking learners to mention things that have the same shape they were learning on in class ; They can be asked to look around; They can mention things like a door, desk or table, rhymes and riddles can be used; By identifying shapes using songs; Recognition of shapes; Tracing shapes; Drawing and colouring; Manipulating different objects; Children engaging themselves in different activities that require them to be creative; Comparing different shapes from one to the other; Give learners opportunities to explore geometry; Letting children physically and mentally change the position of objects around the environment; Describe shapes using the correct name and description; Use of songs and games; Through a game, shape dancing while moving in different ways; Through making comparisons in terms of height, length and size; Through daily routines, classification, songs and demonstrations-for example sand play

This finding also indicated teachers' knowledge of content, children and not the curriculum. However, this finding was from the focus group discussion by the 84 in-service train ECE teachers which differed with what I observed from the 20 pre-school teachers. In the 20 classes I went to most of them were bare apart from those that were on a non-governmental programme. Most of the aspects mentioned during the focus group were not seen in the 20 pre-school teacher's classes and this reflected even in the performance of 84 in-service train ECE teachers' individual activities during lecture hours. Most of the 84 in-service train ECE teachers during the class activity just mentioned one or two aspects as ways in which children learn informal geometry. My conclusion was that, despite the performance on the focus group discussion, teachers had a limited know how of interactions of knowledge of informal geometry and children' informal geometrical conceptions.

During free play, in most classes blocks were given to the learners but when teachers were asked the purpose or the rationale for giving the learners the blocks, most teachers could not explain the geometrical rationale behind that. The next interview justifies the above sentiments:

Interview: *I have just heard you telling learners to change their activities and go to other corners. My interest is on the block corner. What are children going to learn there?*

In-service Pre-school teacher: *They are going to play. Madam we tell learners to go there as they are waiting for parents or else they will be bored and start crying or fighting. These children are difficult to handle.*

Further, in some classes I went to teachers could give blocks during block play to children but could not make use of this time to teach geometry. One teacher when asked, what learners were doing in the block Centre, the answer she gave was "*I don't know ''Are they not playing?* "The teacher could not tell me exactly the geometric rationale, concepts, skills and values behind her telling children to go to the block corner.

The rationale behind block corners and other geometry corners is to develop spatial visualisation and orientation or to develop such concepts as decomposition and composition that are needed in later life to understand the four operations, transformations, and co-ordinate geometry just to mention a few. The other focus of informal geometry is to develop visual, verbal, drawing, logical and applied skills but it seems teachers were not able to spell out all these skills during a follow up interview.

In one particular class where I found a mirror, a teacher was asked the purpose of that mirror the answer I got was "*I don't know*." And yet that mirror was for the purpose of teaching reflection and discussing the relative position of objects with vocabulary such as above, below and next to.

My conclusion from this finding was that, teachers had minimal knowledge in content and teaching.

Teachers are further not making use of teachable moments as shown by the few highlighted episodes.

In some classes I went to they were literary no teaching aids and teachers could not make use of the classroom environment as geometry is found everywhere. The following episode illustrates the case:

Interview: *I have not seen any geometry corner in this class*

In-service pre-school teacher: *Yes madam. This class is used as a church. So all my teaching aids were stolen last week and others were torn.*

In some classrooms one could not differentiate between a pre-school and a grade one classroom. In these classes most teaching was in one direction where the teacher was a director. I could not see were learners were directing and initiating their own learning.

There was no integration of geometry across the curriculum. This study has also found that in-service pre-school teachers are not integrating geometry across the curricula. When asked as to why the proposed teaching approach seems to be failing, the expert on ECE from non-governmental organization said that, *the idea of integration was not received well. Teachers and some teacher trainers misunderstood the whole idea.* They prefer to teach using traditional methods that is according to subjects for instance geometry in mathematics alone and not in other learning areas.

The findings on lesson observations both guided and unguided play methods are being applied by the teachers but most geometry lessons were still on identification and recognition of shapes that's the teacher could say, "*what is this*?", then children could say, "*this is a square*". Or the teacher could ask the children to point at a named shape.

This study found no consistence in the interpretation of guided, unguided play and making use of teachable moments. The only aspect that is taught during guided play is identification of shapes that's to say learners are either taught to point to the mobiles, trace shapes and touch shapes.

8. Discussions, Conclusions and Recommendations

Objectives assessed teacher's level of capacity to develop and implement instruction on a particular content in order to lead to enhanced student understanding. This finding is consistent with Marton, 1975. In the 1970s, Marton (1975) conducted empirical work that has subsequently gained much credibility and currency in higher education. Considerable further work has taken place, including in and across a range of disciplinary contexts (For example, Lizzio et al., 2002). Marton's research, investigating the interaction between a student and a set learning task, led to the conclusion that students' approaches to the task (their intention) determined the extent to which they engaged with their subject and this affected the quality of outcomes. Exactly students in this research had a task of explaining how they introduce geometry to the young ones; designing and explaining an activity they could use; other things the teacher should be aware of when introducing geometry; starting a systematic study of geometry; concepts that teachers should emphasise on in informal geometry and the most popular two dimension figures. And based on this finding where they scored about 50% reviewed the level of background knowledge concerning informal geometry that they took into the university to enable them learn formal geometry. This percentage showed a 50-50 understanding in informal geometry. Among the reasons that may have caused this average performance might have been due to the fact that there might be gaps in their background knowledge, previous lesson delivery and implementation, a variety of books on the market, a curriculum that was not standard and insufficient informal geometry that they might have been exposed to before joining the university. Furthermore, most of the students had not entered the university with a pass as this was not an entry criteria in the department of early childhood education.

Having a variety of books on the market on informal geometry has its own advantages and disadvantages. For instance, some books may/may not have a short description of the history of the subject matter, a discussion on many of the traditional approaches to the subject, present methods for distributing teaching to different levels of formal education and to various types of non-formal education and informal education, describe different arrangements for raising informal geometry education and controlling its use, examine different methods for fore casting informal geometry educational resources and describe the index of informal geometry, discuss the concept of efficiency, develop a general approach for improving efficiency through intervention in the education system.

The 50 percentage created a link between the knowledge they had concerning geometry earlier and the so many activities given to them by the researcher that they were taught to discuss and come and present in class. The researchers conclusion was that, despite the performance on the focus group discussion, teachers had a limited know-how of how children learn geometry. They had limited knowledge of children's interactions of knowledge of informal geometry and children' informal geometrical conceptions. To help our student improve in their knowledge of content and students, this study recommends an introduction of a course or continuous professional development in the psychology of informal geometry whose focus should be on how children learn informal geometry, delivery of play based lessons, interpretation of an informal geometry curriculum, further research on how various stakeholders are implementing play activities. This will enable teachers to see the evolution of paradigms, questions, methodologies and most relevant research results during the last 30 years. Having a knowledge of paradigms, questions, methodologies and most relevant research results will enable teachers have an understanding of cognitively oriented research on learning and teaching specific content areas, transversal areas that are based on various environments especially technology rich environments. Furthermore, the knowledge of psychology will enables teachers to have an understanding of research on social, affective, cultural and cognitive aspects of informal geometry education and teacher training and professional life of informal geometry teachers (Angel Gutierrez and Paolo Boero, 2006).

Further objectives examined what University students think is the way children learn informal geometry. Under this objective 50% of the students could not tell how children learn informal geometry. Even those who mentioned the ways during the focus group discussions had no evidence of such in the classrooms that they teach. This research has shown that, in order to have any degree of success in a university geometry class a level III understanding of Van Hiele level of understanding is needed on the part of the students. Actually how children learn geometry greatly depends on the teachers and how they make instructional decisions at critical moments in the classroom. But how they make this critical instruction may be hampered by their teachers who were not successfully engaging them in appropriate geometry instruction (Unal H, Darryl L Corey & Elizabeth Jakubowski, 2009, ECE specialist, 2020-cell phone call). At times potential learning opportunities may abound for children but teachers whose geometric knowledge and/or spatial ability is limited may not have the capacity to make adjustments to curriculum to address the needs of students with varying needs. When the teacher has not achieved this level or level IV, then the instructional decisions and questions asked by students may lack the depth needed to ensure some mathematical richness of the problems by this teacher. This study shows evidence that knowing how children learn is important in the geometric understanding of in-service teachers. Therefore, further work should explore the nature of the mathematical tasks provided to teachers at secondary and tertiary education before they start university education so that appropriate tasks that would assist in teacher understanding of geometry and its teaching are taught.

Another area that is important to examine is the use of geometric reasoning by individuals (university students) with differing geometrical abilities (thus dynamism, perception of relationships (as of objects) in space and imagination). As well as the types of understanding represented by levels III and IV of the van Hiele model since necessary conditions, relationships, class inclusions and proof rely on an ability to reason through statements of relationships or properties.

Research objective number 3 focused on exploring how teachers adapt an ECE informal geometry curriculum. There is consistence in the interpretation of the syllabus but not the curriculum for teachers teaching at this particular level thus level 5-6 although the content in the syllabus was shallow as compared to Copley, J 2010 recommendations. This finding indicated that teachers interpreted the outcomes well in the syllabus as they could define the action words and able to present guided activities well in class but this knowledge was not sufficient. Because the syllabus they referred to was not a curriculum as there was no curriculum to refer to at the time of research. Therefore, this study recommends that, there is need to come up with an ECE informal geometry curriculum because what existed at the time of research was a topic on informal geometry in a module and course outline called Early Childhood Numeracy and Development. The curriculum will help students greatly because they are typically meeting informal geometry in transition to formal geometry for the first time. Under university geometry it is typically the first time that a student encounters formal proofs, this can obviously present some difficulties. It can also lead students to think that two-column proof is the only kind of proof there is – yet that form of proof is almost never used by practicing mathematicians. It would be easier if students had seen informal proofs earlier and were required to justify their statements and reasoning in elementary and middle school mathematics. This of course would not be done on the same formal level as at university level. Therefore, this research recommends that informal geometry learning trajectories be established from pre-school informal geometry to University to easy the dilemma that students have been facing of not really seeing the link between informal geometry and formal geometry and the link between what they learn at university and what they teach.

Teachers had problems with integration of geometry across the curriculum. Jones Keith (2014) says that, the inability to integrate geometry across the curriculum is because teachers fail to understand the nature of informal geometry, not knowing the importance of geometry in the curriculum at pre-school level and beyond, inability to know the type of informal geometry to include at school level, not knowing the aims of teaching geometry, inability to know how geometry can be taught and learnt, lack of understanding on how to use information and communication technology in geometry education. Teaching geometry well involves knowing how to recognise interesting geometrical problems and theorems, appreciating the history and cultural context of geometry, and understanding the many and varied uses to which geometry is put. It means appreciating what a full and rich geometry education can offer to learners when the mathematics curriculum is often dominated by other considerations (the demands of numeracy and algebra in particular). It means being able to put over all these things to learners in a way that is stimulating and engaging, and leads to understanding, and success in mathematics assessments. Teachers should understand that spatial thinking and visualisation are vital areas of education. Therefore, this study recommends that the university should conduct a lot of Continuous Development focused on the Japanese Model of lesson study. Both transmissive and facilitative methods should be used in teaching to ensure surface, deep and strategic learning among students.

This study has shown that most students at University entered the university with not enough extensive stock of informal knowledge about informal geometry. Many are also familiar with various patterns and some geometric shapes. This knowledge serves as a basis for developing mathematical proficiency in the early years. The level of children's knowledge, however, varies greatly across socioeconomic and ethnic groups. Some children have not had the experiences necessary to build the informal knowledge they need before they enter school.

A number of interventions have demonstrated that any immaturity of mathematical development can be overcome with targeted instructional activities. Lecturers and other caregivers, through games, puzzles, and other activities in the home, can also help students develop their informal knowledge and can augment the school's efforts. Support from home and school can have a catalytic effect on children's mathematical development, and the sooner that support is provided, the better: Lastly this study recommends that, school and preschool programs should provide rich activities with informal geometry and spatial sense from the very beginning, especially for children who enter without these experiences. Efforts should be made to educate parents and other caregivers as to why they should, and how they can, help their children develop a sense of informal geometry and spatial sense.

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