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Dependency between Stock Movements Using the Clayton Copula Method (Ghana Stock Exchange)

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Abstract: ***This study examines the dependence structure of Ghana's financial market using copula methods and the correlation method. Modeling multivariate probability distributions can be difficult if the marginal probability density functions of the random variables of the components differ. Most microeconomic modeling situations have marginal distributions that cannot be easily combined into joint distributions. Since there are few or no joint parametric distributions based on the margins of different families, the copula method provides a simple and general approach to building joint distributions in these situations. Financial markets are concerned with whether prices of different assets exhibit dependence. For these reasons, copulas have become very important as a technique for modeling these non-constant correlations. This has been a great blessing for financial engineering because it is possible to flexibly model these nonlinear relationships. Copula is a suitable tool for modeling dependence between random variables with any marginal distributions. This is why the copula method will be used to study how the various selected stocks move together. How can the Copula method be used on a stock exchange market? This report introduces the idea of a copula, consisting of correlation and dependence, completes the basic mathematics behind its composition and the applications in financial engineering, in particular the structure of dependency in the Ghanaian financial market (promotions). This report examines the linear and non-linear dependency (structure) between the stocks selected on the Ghanaian stock market using the Joe Clayton Copula.***

Keywords: Dependence, Correlation, Concordance, Risk, Copulas, Asymmetry, Clayton Copula**.**

1. Introduction

We are exposed to some form of risk every day, whether eating, walking, driving, investing etc. Risk is defined as the deviation from the expected outcome. Many risks arise due to the uncertainty arising out of various factors that influence an investment or a situation. Most financial-economics setups are also prone to risk in their everyday businesses. In most financial and economic areas, modeling association/dependence is important in high-risk areas where uncertainty plays an important role. It is a critical element of decision making in case of uncertainty and risk analysis. Since its introduction, copula has received a lot of attention in various areas of applied mathematics, such as finance, insurance and reliability theory. It is currently a well-known tool on the market (financial, insurance and reliability markets) for credit models, risk aggregation, portfolio selection, etc. Copulas functions can be used with different variable distributions and these Distributions are called univariate edge distributions. If only two variables are taken into account, the second distribution is determined and depends on the first distribution after the copulation function has been defined and the first distribution has been defined.

The theory of Copulas was introduced by [1] and has long been recognized as a powerful tool for modeling the dependence between random variables. The use of Copulas theory for financial applications is a relatively new area [2] and is growing rapidly. From a practical point of view, the advantage of the relational modeling approach is that the appropriate marginal distributions for the components of a multivariable system can be freely selected and then coupled using an appropriate ratio.

Every multivariate distribution function can serve as a copula. This report looks at copula, how it extends from correlation and dependency and goes through the basic maths that underlies its composition and discuss some of its applications in financial engineering, particularly the dependence structure of the Ghana financial market (stocks). This report studies the linear and nonlinear dependency (structure) between the stocks selected on the Ghanaian stock exchange with the help of Joe Clayton's Copula.

1.1 Motivation for the Study

Insight into copulas will result in the increase of knowledge in its application, since copulas extend from dependency and correlation and having the notion that copulas play a significant role in the financial engineering and risk management, they are highly applicable in the field of risk management.

The results will be useful to the different users of the report (researchers, students, actuarists, investors, risk managers, financial engineers) as they help to uncover and understand the various errors related to correlation and dependencies.

For researchers / students, this research will help to discover and explore areas of copulas that are more suitable for financial engineering than many researchers have been unable to do so far. Thus, other copulas applications in financial engineering may be arrived at.

2. Related Literatures

2.1 Copulas

In the first financial literatures, researchers generally assumed that equity returns are normally distributed, and therefore the emphasis is placed on the linear relationship between equity returns. However, current financial developments show that equity returns are rarely normal [3] and therefore non-linear associations such as tail dependence play an important role in modeling equity returns [4]. In the light of these concerns, the correlation coefficient can lead to misleading conclusions and lead to investors missing important business opportunities due to the loss of information about dependence. The concept of the copulas was introduced by [1] and has long been recognized as a powerful tool for modeling the dependency between random variables. The use of Copulas theory in financial applications is a relatively new area [2] and is growing rapidly. Moreover, copulas facilitate a bottom-up approach to multivariate model building. This is particularly useful in risk management, where we very often have a much better idea about the marginal behavior of individual risk factors than we do about their dependence structure. This report will present the idea of a copula, how it extends from correlation and dependence, run through the basic math that underlies its composition and discuss in-depth some of its applications in financial engineering. In fact, it is very important to remember that correlation is just a special case of dependence

2.2 Dependence

In statistics, dependency refers to a relationship between two variables. Therefore, correlation refers to each of a large class of dependency-related statistical relationships. Correlation in its most natural form is a measure of linear dependence. Not all dependencies are linear. Correlation implies dependence, but dependency does not imply correlation. The dependence between two random variables (such as risk factors) means that there is a relationship between them, i.e., information about a random variable, providing information about the value of the other random Variables.

2.3 Correlation

2.3.1 Definition

The correlation can be defined as a measure of the linear relationship between two quantitative variables. This means that there is a kind of relationship between two variables. If the values of one variable increase while the values of the other variables increase (that is, if both variables increase together), there is a positive correlation. If the values of one variable decrease while the values of another variable increase to form an inverse relationship (ie when one value decreases and the other increases), there is a negative value correlation. When it is possible to predict the values of one variable with reasonable accuracy based on the value of the other variables (that is, when two sets of variables are closely related), one speaks of a strong correlation between two variables.

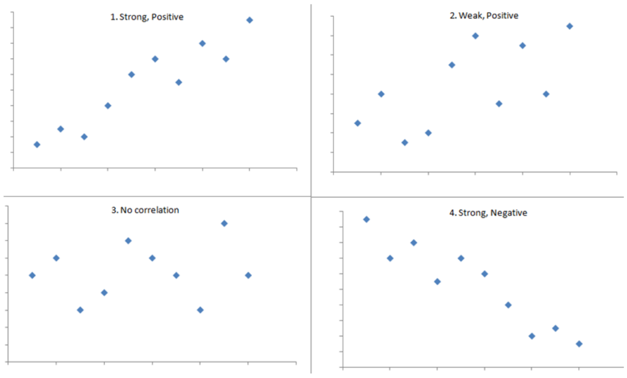

Figure 1: Scatter Diagram for Correlation

When on average, the values of one variable are related to the other, then there is a weak correlation. But there are some exceptions.

- Correlations are used to evaluate the strength and direction of a relationship between two or more variables.
- Possible correlation values depend on the marginal distribution of risks.
- Positive risks to workers do not necessarily have a correlation of 1;
- A correlation of zero (for example,
- Correlation is not invariable in monotonous transformations (a transformation through a strictly growing function
- Correlation is only defined if the risk variances have ended.

2.3.2 Types of Correlation

The following describes the types of correlation;

Pearson's Correlation Coefficient (Linear correlation) was originated by Karl Pearson in 1900s. It is the most widely used non-parametric correlation statistic to measure the degree of the relationship between linearly correlated variables. For Pearson's correlation, both sets of data should be normally distributed, linear and homoscedastic (equally distributed about the regression line). Pearson's Correlation is denoted by ρ_{XY} for population data and r_{XY} for sample data. The joint variation of X and Y is measured by the covariance of X and Y, denoted by $Cov(X,Y)$. It is defined as

$$Cov(X,Y) = E[X - E(X)][Y - E(Y)] \quad (1)$$

The $Cov(X,Y)$ may be positive, negative or zero. When $Cov(X,Y)$ is divided by the standard deviation of X and Y, σ_X and σ_Y, we get the correlation coefficient, $\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X\sigma_Y}$. If $\rho = \pm 1$, it is a perfect correlation. If $\rho = -1$, it is a perfect negative correlation. If there is no correlation between X and Y, then X and Y are independent and $\rho = 0$. Correlation coefficient lies between −1 and +1, i.e. $-1 \leq r_{XY} \leq +1$. Copulas and rank correlation methods are two ways to model and / or explain the dependence between two or more variables. The most commonly used methods of measuring association are the Tau des Kendall and the Rho des Spearman. Both measure a form of dependency called concordance [5]. The rank methods of correlation allow the modeler to compensate for the existence of outliers by correlating the rankings of a dataset rather than the values themselves, that is, the relative positioning of a dataset within the dataset is related. To overcome the shortcomings of Pearson's linear correlation, the correlations of the two ranks are briefly discussed, namely; the Spearman's rho and the Kendal's tau. Spearman's rank correlation is a non-parametric method to measure the degree of association between two sets of variables. In Spearman's method, we correlate the range pairs and extract the coefficient using Pearson's linear regression method. The Kendall's tau offers quite a distinct technique. That is, it considers the probability of concordance minus probability of discordance, given a sample (Xi, Yi), (Xj, Yj) of observations. Here, the strength of association between any two sets of variable is measured. [5]. It measures the strength of the dependency between two sets of variables.

2.4 Copula

2.4.1 Definition

Copula is a statistical measure that represents a uniform multivariate distribution that examines the association or dependency between many variables. A copula is a function that connects or "links" a multivariate distribution function with its one-dimensional marginal distribution function. [6]. The only thing we have to remember is that we have to evaluate the degree of association one way or another in order to build a copula.

Copulas are of interest for statisticians for two reasons:

- A way to study dependency measures without scale
- As a starting point to build families of multivariate bivariate distributions, sometimes with a view to simulation.

The main advantage of copula functions is that it clearly distinct between the structure of dependence and the univariate distribution of the variables. With this feature, copula functions can model dependencies for distribution functions of any type. The copula functions separate the dependency structure of the independent distribution functions. They are used when we have to model dependencies with non-normal distributions, although the Copulas method also works for normal distributions.

2.4.2 Advantages of Copulas

- Unlike correlation, copulas have a good ability to be invariant under strictly increasing transformations of random variables.
- Unlike correlation as a scalar dependency measure, we use copulas as a model, so the dependency structure reflects a more detailed understanding of the risk management problem we're dealing with.
- The use of copulas is consistent with a typical actuarial and financial risk modeling process, in which marginal risk distributions are first determined for each risk and then considered separately for the aggregation process.

2.4.3 Disadvantages of Copulas

- There is usually not enough data to perform a credible calibration of a copula, especially in the tail.
- Any economic capital model becomes more of a 'Black Box'. There is often a lack of transparency in the modelling process.
- Communication both internally and externally becomes more of an issue when dealing with non-technical people.

3. Methodology

3.1 Measures of Association/Dependence

Copulas are a general basis for defining multivariate distributions and modeling multivariate data. Correlation describes dependency between random variables, but recent studies have determined the advantage of copulas for modeling dependency because they offer much more flexibility than the correlation approach. We first focus on the types of correlation and then copulas and its classes, and which classes we shall use for our study.

3.1.1 Pearson's Correlation Coefficient

Also known as linear correlation, it was created in 1900 by Karl Pearson. The Pearson's, a non-parametric measure, is the commonly used measure of correlation that measures the degree/extent of association between variables that have a linear relation. Pearson correlation is not invariant under strictly increasing transformations of the underlying random variables. The main reason why correlation fails as an invariant measure of dependence is that Pearson's correlation coefficient depends not only on the relationship but also on the marginal distributions. Therefore, the measure is affected by the size changes in the boundary variables.

To correct Pearson's correlation error, the two widely used methods for measuring the strength of association are Kendall's tau and Rho of Spearman. Both measure a form of dependence called **concordance** [5].

The observations (X_1, Y_1), and (X_2, Y_2), are said to be concordant if $(X_1 < X_2)$, and $(Y_1 < Y_2)$, or if $(X_1 > X_2)$, and $(Y_1 > Y_2)$. This means that large (small) values of the random variable X are associated with large (small) values of the random variable Y. If the opposite is, there is discordance.

Because the Copula of a multivariate distribution describes the dependency structure, we can use dependency measurements based on the Copula. Concordance measures; Kendall's tau and Spearman's suit, as well as the tail dependency coefficient, can be expressed, unlike the correlation coefficient of the rank order, with regard to only the underlying Copula.

3.1.2 Spearman's Rank Correlation

This is a non-parametric method used to measure the degree of association between two sets of variables. In the Spearman method, we correlate the ranking pairs and extract

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the coefficient with the Pearson linear regression method. It can be calculated from [7]:

$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)} \quad (2)$$

d_i = differences between the ranks of corresponding variables
n = number of observations

In this sense, Spearman's correlation of scope can be seen as Pearson's linear correlation between ranked variables. Variables are classified by assigning the highest range to the highest value. The most interesting feature of Spearman's correlation is that it makes no assumptions about the frequency distribution of the two variables. Another interesting feature of the Spearman correlation is the ability to capture the nonlinear dependency between the two variables.

3.1.3 Kendall's tau

For investigating more deeply the dependence, we need a measure for gauging it. It is known as Kendall τ (Tau). It is a rank correlation measure; it has been invariant under strictly increasing transformations of underlying random variables. It is a slightly different approach that examines the likelihood of concordance minus the likelihood of discordance, (X_i, Y_i), (X_j, Y_j) of random observations from the samples [5]. It measures the strength of dependence between two sets of variables. When we consider two variables x and y, where each sample size is n, we know that the total number of pairings with x, y is $\frac{n(n-1)}{2}$. The following is the formula used to calculate the Kendall's rank correlation;

$$\tau = \frac{n_c - n_d}{\frac{1}{2}n(n-1)} \quad (3)$$

n_c = number of concordant
n_d = number of discordant

The most common approach to determining dependence is the Pearson correlation coefficient for determining the weight of the link, as it is a quantity that can be calculated quickly and easily. However, the Pearson coefficient measures the linear correlation between two time series, and this is a fairly serious limitation, since nonlinearity has proven to be an important feature of financial markets. However, the Kendall's measure takes into account monotonic nonlinearity, while other measures such as tail dependence quantify dependence extreme events.
If X and Y are variables with continuous marginal distributions and unique copula then Spearman's rho and Kendall's tau can be expressed as follows [8]:

$$\rho_s(X,Y) = 12 \iint_0^1 C(u,v)\, dudv@ - 3 \quad (4)$$

$$\rho_\tau(X,Y) = 4 \iint_0^1 C(u,v)\, dC(u,v) - 1 \quad (5)$$

Where $C(u,v)$ is the copula of the bivariate distribution function of X and Y. Both $\rho_s(X,Y)$ and $\rho_\tau(X,Y)$ may be considered as measures of the degree of monotonic dependence between X and Y, whereas linear correlation measures the degree of linear dependence only. According to [9] it is slightly better to use these measures than the linear correlation coefficient. In their opinion, however, one should choose a model for the dependence structure that reflects more detailed knowledge of the problem at hand instead of summarizing dependence with a single number like linear correlation or rank [10]

3.2 Copulas

3.2.1 Mathematical Definition of Copulas

Given a random vector $X_1, X_2, \dots, X_p$, its marginal cumulative distribution functions (CDFs) are

$$C(u_1, u_2, \dots, u_p) = P[U_1 \le u_1, U_2 \le u_2, \dots, U_p \le u_p] .$$

$$Fi(x) = P[Xi \le x] \quad (6)$$

By applying the probability integral transform to each component, the marginal distributions of $(U_1, U_2, \dots, U_p) = (F_1(X_1), F_2(X_2), \dots, F_p(X_p))$ are uniform (from Wikipedia). Then the copula of $X_1, X_2, \dots, X_p$ is defined as the joint cumulative distribution function of $U_1, U_2, \dots, U_p$, for which the marginal distribution of each variable U is uniform as $U(0,1)$
The Copulas function contains all the dependency characteristics of boundary distributions and best describes the linear and nonlinear relationship between variables using probability. They allow you to model boundary distributions independently of each other, and you do not need to assume the common behavior of marginalized groups.

3.2.2 Properties of Copulas

Definition: A d-dimensional copula is a function $C : [0,1]^d \Rightarrow [0,1]$ with the following properties;

i. For every $U \in [0,1]$
$$C(0,u) = C(u,0) = 0 \quad (7)$$

ii. For every $U \in [0,1]$
$$C(u,1) = u \text{ and } C(1,u) = u \quad (8)$$

iii. For every $(u_1,u_2), (v_1,v_2) \in [0,1] \times [0,1]$ with $u_1 \le v_1$ and $u_2 \le v_2$

$$C(v_1,v_2) - C(v_1,v_2) - C(u_1,v_2) + C(u_1,u_2) \ge 0 \quad (9)$$

Any function that has the above properties are copula functions. C is a multivariate CDF on $[0,1]^k$ with its marginals uniform on $[0,1]$. The common notation for a copula is $C(U_1, U_2, \dots, U_k)$.

3.2.3 Clayton's copula

The Clayton copula is usually attributed to Joe Clayton (1978), but its origin can actually be traced back at least to [11] The Clayton copula is a left-tailed 'extreme value' copula, exhibiting greater dependence in the lower tail than in the upper tail.

This copula relies upon a single parameter θ with support in $[0,\infty)$, and defined for $\boldsymbol{u} \in [0,1]^2$ as

$$C(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta} \quad (10)$$

The dependence between the observations increases as the value of θ increases, with θ→ 0 + implying independence and θ→ ∞ implying perfect dependence. The Clayton copula can also be extended to negative dependence structures, i.e. those with negative linear correlation.

4. Data Analysis

Extracts from secondary data (website of the Ghana Stock Exchange) was extracted and will be analyzed and discussed, using selected banks on the Stock Market with their lognormally-distributed daily closing stock returns over the period under study and corresponding graphs (descriptive statistics) will be used to show how stocks moved daily over the years.

The EGH underwent a major merger and acquisition in the year 2011. Four years after the merger, it did not trade on the GSE because it had to get herself established and back on her feet. It eventually bounced back onto the market in January 2016. Stock returns for 2016 were not used for our research because a stock return for EGH in the year 2016 was deficient with 61 days as compared to SCB. Our data for both EGH and SCB spanned over the period of January 3rd, 2017 to December 31st, 2019, that is a 3-year data was used for the analysis.

Table 1: Preliminary Results

Descriptive Statistics		
	EGH	SCB
Mean	8.10	21.93
Median	7.60	20.75
Mode	7.00	19.00
Std. Deviation	1.38	6.07
Variance	1.90	36.86
Skewness	1.68	0.77
Kurtosis	1.73	0.22
Range	5.50	27.79
Minimum	6.50	12.21
Maximum	12.00	40.00

The table above contains the summary of basic descriptive statistics for our data. The means for each dataset are located to the right of the medians. Our data depicts positive skewness, i.e., it is skewed to the right. In most cases, for data to be skewed to the right, the mean will be greater than the median. Again, if the data is positively skewed, the Pearson's coefficient is positive. From our dataset, the Pearson's coefficient was **0.77,** depicting a positively skewed or a right tailed dataset.

Again, we can see positive values for the kurtosis, meaning both datasets are asymmetric and skewed to the right. This is consistent with the fact that the skewness for both datasets are positive as stated earlier. Given the above statistics, we have enough evidence to rule out normality from our dataset, especially from the results of skewness and kurtosis which are both not equal to zero.

4.1 Correlation / Dependency Results

Correlation (dependency) is based on daily returns, hence the relationship/dependency on daily returns are compared over a given period that is for 3 years. Evaluating dependence between the two input variables, we have the following;

Correlation type	Coef.	p-value	Sign. 5%
Kendall rank	0.3834	0.0000	Yes
Spearman's rank-order	0.5133	0.0000	Yes
Pearson product-moment	0.7694	0.0000	Yes

4.2 Model Design and Simulation Result

Pearson Correlation (ρ): Determining the Pearson's dependency between the two stocks where X denotes EGH stock and Y denotes SCB stock, we have a correlation matrix;

$$\rho_{XY} = \begin{array}{c|cc} & X & Y \\ \hline X & 1.00 & 0.77 \\ Y & 0.77 & 1.00 \end{array}$$

Looking at the Pearson's Correlation Coefficient, we find that EGH has a perfectly positive correlation with itself and so does SCB, that is, $\rho_{XX} = \rho_{YY} = 1$. Correlation between EGH and SCB signifies a stronger linear correlation, closer to +1. That is, $\rho_{XY} = 0.77$ and $\rho_{YX} = 0.77$. EGH and SCB have a strong relationship with each other. Knowing the value of the correlation coefficient as an investor helps diversify or de-risk his portfolio of assets.

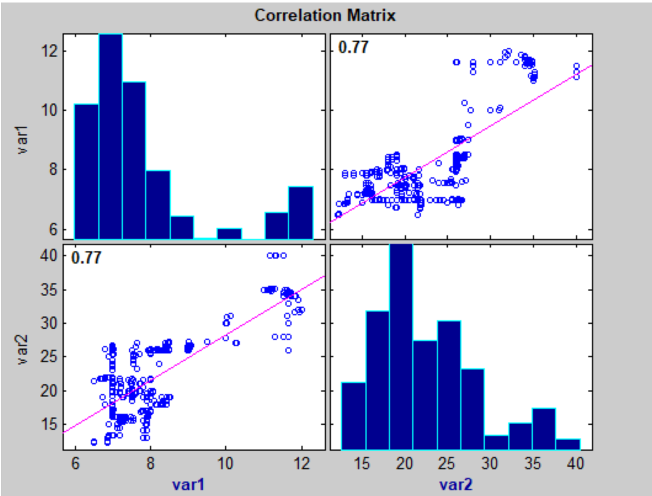

Figure 2: Pearson Correlation Plot

The correlation scatter-histogram plot shows the relationship between each data point. From the graph, we can see initial closely related points from the point of origin to the next three points on both axes, a positive association between points on the graph. They later scatter to the extreme right since its skewed to the right, but still closely related.

Most of the points fell close to the line, which indicates a strong positive relationship between the variables. Since we defined Pearson Correlation Coefficient and seen its properties, it is not convenient for fitting copulas to our data, since it depends on the univariate marginal distributions as well as the copula. Again, the Pearson's allows us to determine the linear trend.

The rank correlation can remedy this situation, since they depend only on copulas and they in turn determine the monotonic relationship between both datasets. Again, looking at the frequency distribution of data from the graph, we can see a positively skewed (skewness = 1.68 & 0.77) and with kurtosis (1.73 & 0.22), the normal distribution would be ruled out and for correlation ranks **Spearman's rho** and **Kendall's tau** to be introduced.

***Spearman's Rank Correlation Coefficient* (ρ_s):** We have Spearman's Coefficient(ρ_s) $= \mathbf{0.51}$

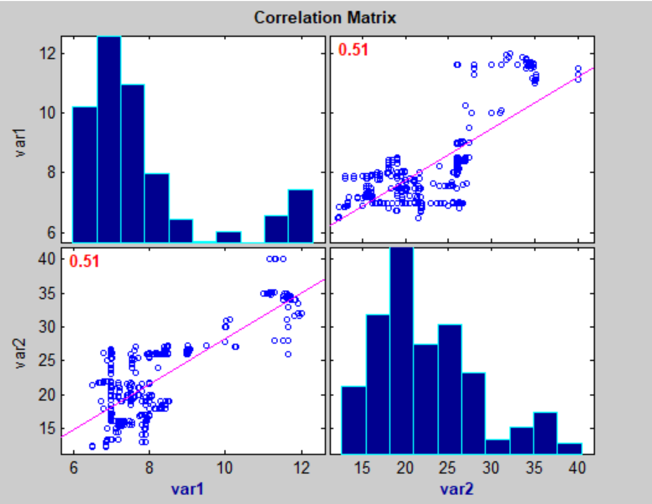

Figure 3: Spearman's Rank Correlation Plot

From the graph, we can see that both variables tend to increase together signifying a positive relationship. The red line represents an upward trend of the correlation. We can see a monotonic relationship between the datasets, in that, the variables we see the variables actually moving in the same direction. The points are seen falling close to the line, which indicates a moderately positive relationship between the variables.

Kendall's Rank Correlation Coefficient* (τ):** A ***tau of ***0.38*** was realized indicating a positive dependence. From the graph, we can see that both variables tend to increase together signifying a positive relationship. The red line represents an upward trend of the correlation. We can see a monotonic relationship between the datasets, in that, we see the variables actually moving in the same direction. The points are seen falling close to the line at the initial points and dispersed further away from the line getting to the end.

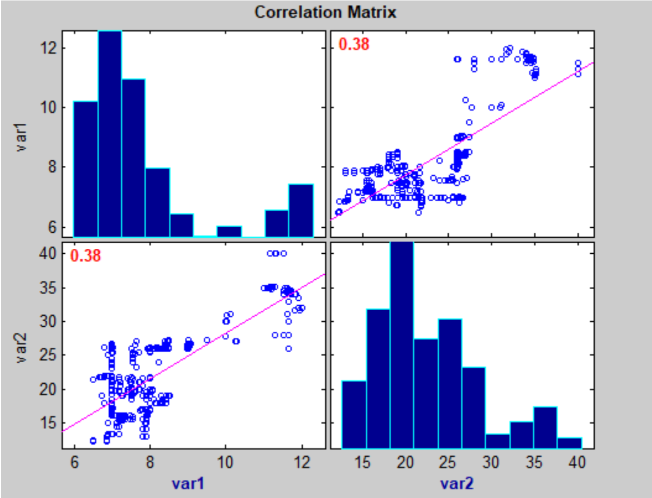

Figure 4: Kendall's Rank Correlation Plot

Hypothesis Testing

Conducting a hypothesis test to test for the significance of the relationship between the datasets, taking at 5% significance level, we have;

For Pearson's Correlation Coefficient,

H_0: $\rho_{XY} = 0$, H_1: $\rho_{XY} \neq 0$, ρ_{XY} = 0.77

p-value = 1.4400e-146

Since p<0.05, we reject H_0 and conclude that there's a significant relationship/dependence between both variables.

For Spearman's Rank Correlation Coefficient,

H_0: $\rho_s = 0$, H_1: $\rho_s \neq 0$, rho = 0.51

p-value = 2.9778e-51

Since p<0.05, we reject H_0 and conclude that there's a significant relationship/dependence between both variables

For Kendall's Rank Correlation Coefficient,

H_0: $\tau = 0$, H_1: $\tau \neq 0$,τ = 0.38

p-value = 7.7858e-53

Since p<0.05, we reject H_0 and conclude that there's a significant relationship/dependence between both variables

Clayton Copula

Copula Joint Probability Distributions

Using the Max-Likelihood technique, the following estimated parameters for Clayton Copulas were obtained;

Table 2: Clayton Copula

Copula Name	RMSE	NSE	p-value	Par#1-Local
Clayton	1.1618	0.9795	0.0020	0.8594

Parameter of Clayton copula is θ and per our table, $\theta = \mathbf{0.8594}$. For $\theta > 0$, the Clayton copula has lower dependence in the tails. For $\theta \to \infty$, the coefficient converges to 1. This is because; the Clayton copula tends to comonotonicity as $\theta \to \infty$. Clayton copula has left-tail dependence. That is, as you get further to the left tails, the variables become more correlated. According to the NSE, the Clayton's copula provides a best fit to our dataset with $NSE = \mathbf{0.9795}$. An NSE value closer to 1 is associated to a perfect fit and is selected as best fit according to the Maximum-Likelihood Estimation technique.

We first saw some features of our dataset using the descriptive statistics. Then we investigated the interdependency / co-movement of our variables 1 and 2 (*EGH* and *SCB*) over the three (3) year period through the different correlation coefficient techniques, all of which displayed significant dependence amongst the two variables. We select the marginal distributions and parameters based on the maximum-likelihood estimation technique Joint probability isolines derived for the Clayton copula is displayed in figure 1. The figure displays the fitted distributions (red lines) compared to the observed (blue dots). Marginal cumulative distributions of both variables in each copula case are seen in figures 2 and 3. The joint probability isolines (Figures 1) are color coded with joint density levels with blue representing lower densities and red representing higher densities. Joint probabilities are renormalized to [0,1] range for better visualizing purposes. Blue dots are observed pairs of both variables and red lines denote fitted distributions. A log-logistic distribution is selected to fit both variables. The Chi-Squared tests for both variables at 5% significance level also confirm our visual inspection that the fitted distributions are acceptable.

Financial markets tend to display tail dependence, especially lower tail dependence. A basic feature displayed from the graphs is the cluster of the blue dots close to the origin (lower left) than at the upper right end. In asset or investment management, we are interested in whether the drops of one or more stocks influence the behavior of the other stocks in the portfolio. In particular, when there's an event where two extreme events occur simultaneously, it means diversification breaks down just when it is needed the most. Copulas in this case showed in which part of the distributions the association or dependency is strongest. Understanding the dependence or correlation structure will enable investors to balance the risk and return of their portfolio and when to hedge their portfolios.

To summarize, our investigation of the dependence structure of our variables under study (*EGH* and *SCB*) sheds light on our objectives in the introduction.

5. Summary, Conclusion and Recommendation

This chapter outlines the summary of findings made in the previous chapter, with conclusions and recommendations for improvements on the choices of the methods used in determining the dependency structure between the two variables (stocks, namely ***EGH*** and ***SCB).***

5.1 Summary

We deduced from our study a positive dependency / correlation structure that exists between the two stocks. From our analysis, the following outcomes were derived;

1) From the initial descriptive statistics derived, we saw the means greater than the medians, i.e., located at the right of the medians depicting a positively skewed dataset. Positive values were derived for kurtosis, meaning both datasets are asymmetric and skewed to the right and t from further analysis, the normal distribution was ruled out.
2) Using the Pearson's Correlation method, we found a strong positive relationship ($\mathbf{i.e.}, \rho = \mathbf{0.77}$) between the variables and from the rank methods of correlation, the Spearman's rho gave us a correlation of $\mathbf{0.51}$ and the Kendall's tau also gave a correlation of $\mathbf{0.38,}$ both depicting a monotonically positive relationship/co-movement between the two variables.
3) Testing for significance on the results of the methods of correlation, we concluded that there existed a significant relationship/dependence between both variables.
4) Employing the Clayton Copula in the analysis, a powerful visualization of the dependence structure amongst both stocks was derived.

5.2 Conclusion

As stated in earlier, risk is very important in every organization. Actuaries are of the view that "*risk is opportunity".* Investing in the financial markets is a big risk to take, in that, you don't know when there will be a downward or upward trend in the performance of the investment and whether prices of different assets or stocks exhibit dependence. This paper has explained in details copulas and provided elementary techniques on their use and how they worked for the dataset, showing positive dependence structures (with the Clayton Copula method employed) amongst the two stocks. We deduced from our diagrams that the marginal distribution copula type displayed different properties and this increased our abilities to find a copula that best fitted our dataset adequately. At a more basic level, one's perspective in dependency can dramatically change according to how information is presented. For example, our main concern was finding the co-movement between the stock returns of the two banks and conditions such as inflation rates, interest rates, exchange rates, etc. (basically, micro and macroeconomic factors) and other factors which may not only be consistent but also influence the financial records impact their performance.

We discovered and showed that the methods used are simple and powerful visualization tool that enabled us detect the dependence structure amongst both stocks. A clear assessment of these dependencies should help in the design of better risk measurement tools in diversification. Understanding the dependence structure will enable investors to properly balance the risk of the investment. Risk may be shared and balanced as a way to mitigate it, allowing investors to adopt more strategic investment decisions.

5.3 Recommendation

Based on the findings of this research, we recommend the following;

1) This paper used data of the stock returns from two companies, all in the same line of business. Both companies were banks and might be exposed to virtually similar threats and this influenced the results. And per the results derived, we saw a common trend in all the

methods employed, that is, a positive dependence/ relationship exists between them. We recommend stock returns from other companies which are not in the same line of business. Doing this will reduce the exposure to risks by holding a diversified portfolio.

2) We recommend the use of stochastic processes or time series models since most stocks are time sensitive in nature
3) The data was not sufficient enough for the nature of the tails to be detected. We recommend further research in this area that will consider relatively longer periods than the three (3) years used in this study.

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