Expression for the Multiplication of HCF and LCM in Geometric Progression

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Abstract: ***With response to Chirag's formula about the relation between Highest Common Factor (HCF), Least Common multiple (LCM) and common ratio of any Geometric Progression (GP) having finite number of terms, in this current paper there is a new theorem have been introducing by the author which is based on the multiplication of terms of a geometric progression which is looking like as a method of summation in Athematic Progression (A.P).***

Keywords: Chirag's formula, Highest Common Factor (HCF), Least Common multiple (LCM), Geometric Progression (GP), Athematic Progression (A.P).

1. Introduction

With reference to Chirag's formula [1] for odd number terms

$$\frac{HCF \cdot LCM}{r} = a \cdot (term\ occurring\ at\ (n-1)^{th})\ place)$$

Where r is common ration of given GP having finite number of terms so, let consider a geometric progression a, ar, ar^2, ar^3, ar^4 here, 5 no. of terms which is an odd no.

Then $T_{(n-1)}=ar^3$ and same as for any G.P $T_{(n-1)}= ar^{(n-2)}$ and $T_n=ar^{(n-1)}$. After it come back on Chirag's formula [1]

$$HCF \cdot LCM = ar \cdot (term\ occurring\ at\ (n-1)^{th})\ place)$$

$$HCF \cdot LCM = 2^{nd}\ term \cdot (term\ occurring\ at\ (n-1)^{th})\ place)$$

Although if $T_{(n-1)}= ar^{(n-2)}$ then

$$HCF \cdot LCM = ar \cdot ar^{(n-2)}$$

$$HCF \cdot LCM = a \cdot ar^{(n-1)}$$

$$HCF \cdot LCM = 1st\ term \cdot last\ term$$

With this we can state a theorem which is-

Theorem

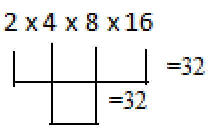
1) If T_1, T_2, $T_{3,} \ldots T_n$ are the terms of GP and $n \in 1, 3, 5..$ Then

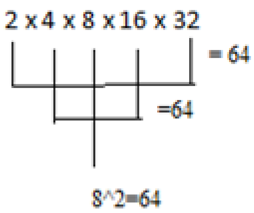
$$T_1T_n = T_2T_{(n-1)} = \cdots\cdots = \left(T_{\left(\frac{n+1}{2}\right)}\right)^2 = \text{LCM} \cdot \text{HCF}$$

2) If T_1, T_2, $T_{3,} \ldots T_n$ are the terms of GP and $n \in 2, 4, 6 \ldots$ Then

$$T_1T_n = T_2T_{(n-1)} = \cdots\cdots = T_{\left(\frac{n}{2}\right)}T_{\left(\frac{n}{2}+1\right)} = \text{LCM} \cdot \text{HCF}$$

Example here 2, 4, 8, 16, 32 is odd termed GP and 2, 4, 8, 16 is even termed GP





It is looking like as a summation method of AP which is gauss's method for it check [2] by using an example we can understand it well so, let an AP- 2, 4, 6, 8 here 2+8=10 and 4+6=10 this is method and it is well known that in many complex problem of series this method helps a lot to solve problem in small period of time same as this method of GP also helpful.

2. Benefits

This method is beneficial in solving complex series in less duration of time respectively.

References

[1] Chirag Gupta-relation between Highest Common Factor (HCF), Least Common multiple (LCM) and common ratio of any Geometric Progression (GP) having finite number of terms, p-887-888, Vol. 9 issue 2, February International Journal of Science and Research ISSN: 2319-7064.

[2] Sk Goyal (Algebra) p.213 Ch-3 (Sequence and Series) 2019, ISBN- 973-93-13191-88-9

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