A Comment on S Nahar & Md. Abdul Alim (2017): A New Statistical Averaging Method to Solve Multi-Objective Linear Programming Problem

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Abstract: ***Samsun Nahar & Md. Abdul Alim proposed new statistical averaging techniques to solve Multi-Objective Linear Programming Problems. The solution of the numerical example by new harmonic averaging technique to solve linear programming problem thus obtained from multi-objective linear programming problem claimed by Samsun Nahar & Md. Abdul Alim is not optimal. The correct optimal solution of the numerical example is given here.***

Keywords: Linear Programming Problems, New statistical averaging method, Arithmetic average, Geometric average, Harmonic average

1. Introduction

The research article proposed by the authors Samsun Nahar & Md. Abdul Alim (2017), they claimed that new statistical averaging techniques provide much better optimized value of objective function as compared to other techniques viz. Chandra Sen's technique and statistical averaging techniques in the case of multi-objective linear programming problems. They converted a multi-objective linear programming problem into a single linear programming problem by new statistical averaging techniques (new arithmetic, new geometric and new harmonic). Then the solution of the linear programming problem is recovered by traditional simplex method. The authors also cited an example to illustrate the developed algorithm.

We noticed the error in the paper at mathematical illustration section. We noted that in the numerical example; the optimal solution is found in the case of applying new harmonic averaging technique as $x_1 = 4$, $x_2 = 3$ with maximum Z = 9.8593. We believe that the solution Z = 9.8593 given in the numerical example is not an optimal solution. We suggest that with maximum Z = 9.9164 would be an optimal solution in this case. For the validation of our suggested answer, here we are solving the linear programming problem thus obtained by applying new harmonic averaging technique to multi-objective linear programming problem taken by the authors in their research article.

2. The Solution

The numerical example taken by Samsun Nahar & Md. Abdul Alim (2017) is:

$$\begin{aligned} &\text{Maximize} && Z_1 = x_1 + 2\,x_2 \\ &\text{Maximize} && Z_2 = x_1 + 0\,x_2 \\ &\text{Minimize} && Z_3 = -2x_1 - 3\,x_2 \\ &\text{Minimize} && Z_4 = 0\,x_1 - x_2 \\ &\text{Subject to,} && 6x_1 + 8\,x_2 \le 48 \\ & && x_1 + x_2 \ge 3 \\ & && x_1 + 0\,x_2 \le 4 \\ & && 0x_1 + x_2 \le 3 \\ &\text{and} && x_1, x_2 \ge 0 \end{aligned}$$

The above multi-objective linear programming problem can be converted to a single objective linear programming problem from new harmonic averaging technique proposed by Samsun Nahar& Md. Abdul Alim as follows :

$$\text{Max. } Z = \frac{\sum_1^r Z_i - \sum_{r+1}^s Z_i}{m}$$

Where m is the harmonic mean of m1 and m2 i.e., $m = \dfrac{2}{\frac{1}{m_1}+\frac{1}{m_2}}$

Firstly, we have to find the value of each linear program associated with the given constraints by any method (Graphical, in the case of having two decision variables /Simplex method or by any method available in the literature). Here we get optimal point as $x_1 = 4$ and$x_2 = 3$ for each of the linear program given above. The values of each of the objective function at optimal point $x_1 = 4$ and$x_2 = 3$ are as follows:

$Z_1 = 10$, $Z_2 = 4$, $Z_3 = -17$, $Z_4 = -3$

Now, m_1 is the minimum of absolute objective function values among all the linear programs which are to be maximized and m_2 is the minimum of absolute objective function values among all the linear programs which are to be minimized in the given multi-objective linear programming problem. Here m_1 = minimum {10, 4} = 4 and m_2= minimum {17,3}= 3.

To apply new harmonic averaging technique, we calculate the value of m as follows:

$$m = \frac{2}{\frac{1}{m_1}+\frac{1}{m_2}} = \quad = \frac{2}{\frac{1}{4}+\frac{1}{3}} = 3.4285$$

Now, the reduced linear programming problem from the technique of new harmonic averaging technique as follows:

$$\text{Max. } Z = \frac{(2x_1+2x_2+2x_1+4x_2)}{3.4285} = \frac{(4x_1+6x_2)}{3.4285}$$

$$= (1.1666)x_1 + (1.7500)x_2$$

Subject to, $6x_1 + 8\,x_2 \le 48$

$x_1 + x_2 \ge 3$

$x_1 + 0\,x_2 \le 4$

$0x_1 + x_2 \le 3$

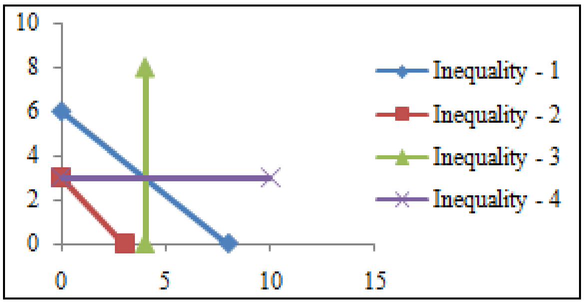
and $x_1, x_2 \ge 0$

We are verifying our suggestion by solving the above linear programming problem through various methods given below:

1) Graphical Method
2) Simplex Method
3) AHA simplex algorithm
4) Gauss elimination technique
5) Modified Fourier elimination technique
6) The technique adopted by the authors.

2.1 Graphical Method

S.No.	Coordinates of point	Value of objective function
1.	(3,0)	Z = 3.4998
2.	(4,0)	Z = 4.6664
3.	(4,3)	Z = 9.9164
4.	(0,3)	Z = 5.2500



It can be observed that the optimal solution by graphical method is $x_1 = 4$, $x_2 = 3$ and the value of corresponding objective function is 9.9164.

2.2 Simplex Method

Max. Z = $(1.1666)x_1 + (1.7500)x_2 + 0\,S_1 + 0\,S_2 + 0\,S_3 + 0\,S_4$ - MA

Subject to, $6x_1 + 8\,x_2 + S_1 = 48$

$x_1 + x_2 - S_2 + A = 3$

$x_1 + 0\,x_2 \quad + S_3 = 4$

$0x_1 + x_2 \quad + S_4 = 3$

and $x_1, x_2, S_1, S_2, S_3, S_4 \ge 0$

The final table of Simplex method is given below:

		C_j	1.1666	1.7500	0	0	0	0
C_B	X_B	b	x_1	x_2	S_1	S_2	S_3	S_4
0	S_1	0	0	0	1	0	-6	1
1.7500	x_2	3	0	1	0	0	0	1
1.1666	x_1	4	1	0	0	0	1	0
0	S_2	4	0	0	0	1	1	1
Net Evaluation Row			0	0	0	0	0	1.7500

This is an optimal solution as all the entries of net evaluation row is either positive or zero. Optimal solution is $x_1 = 4, x_2 = 3$ and the value of objective function at this point is Max. Z = 9.9164.

2.3 AHA simplex algorithm

Max. Z = $(1.1666)x_1 + (1.7500)x_2$

Subject to, $6x_1 + 8\,x_2 \le 48$

$x_1 + x_2 - x_3 \le 3$

$x_1 + 0\,x_2 \le 4$

$0x_1 + x_2 \le 3$

and $x_1, x_2 \ge 0$

Final AHA simplex table for the above linear programming problem is as follows:

x_1	x_2	x_3		b_i
0	0	0	$\le$	9.9164
0	0	0	$\le$	0
1	0	0	$\le$	4
0	0	1	$\le$	4
0	1	0	$\le$	3

Now, it can be observed that all the coefficients of x_j in the objective inequality is either zero or positive. Therefore, this is an optimal solution. The optimal solution occurs at $x_1 = 4$, $x_2 = 3$ with Maximum Z = 9.9164.

2.4 Gauss Elimination Technique

Max.

Z - $(1.1666)x_1$ - $(1.7500)x_2 \le 0$

Subject to $6x_1 + 8\,x_2 \le 48$

$-x_1 - x_2 \le -3$

$x_1 + 0\,x_2 \le 4$

$0x_1 + x_2 \le 3$

$-x_1 \le 0$

$-x_2 \le 0$

After first stage of elimination, we get

$-1.000514315\,x_2 + 5.143151037\,Z \le 48$

$.500085719\,x_2 - .857191839\,Z \le -3$

$-1.500085719\,x_2 + .85719839\,Z \le 4$

$1.500085719\,x_2 - .85719839\,Z \le 0$

$x_2 \le 3$

$-x_2 \le 0$

After second stage of elimination, we have

$Z \le 12.2507$

$Z \ge 9.9164$

$Z \le 10.4993$

$Z \le 9.9164$

$Z \ge 9.3328$

It is obvious that Max. value of Z is 9.9164 which satisfies all the above inequalities. Hence, max. Z = 9.9164. Now, we can find the values of remaining variables by back substitution. The values of x_1 and x_2 are 4 and 3 respectively.

2.5 Modifed Fourier Elimination Technique

Max. Z - $(1.1666)x_1$ - $(1.7500)x_2 \le 0$

Subject to $6x_1 + 8\,x_2 \le 48$

$-x_1 - x_2 \le -3$

$x_1 + 0\,x_2 \le 4$

$0x_1 + x_2 \le 3$

$-x_1 \le 0$

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$$-x_2 \leq 0$$

After eliminating x_1 , the above inequalities reduce to

$$-1.1672x_2 + 6\,Z \leq 55.9968$$
$$x_2 \leq 15$$
$$x_2 \leq 3$$
$$x_2 \leq 6$$
$$-x_2 \leq 0$$

After eliminating x_2 , the inequalities reduce to

$$Z \leq 12.2508$$
$$Z \leq 9.9164$$
$$Z \leq 10.5$$
$$0 \leq 15$$
$$0 \leq 3$$
$$0 \leq 6$$

Out of these, Z = 9.9164 is the only value which satisfies all the inequalities altogether. By putting this value of Z and with the help of back substitution, one can get the values of remaining variables as x_1 = 4 and x_2 = 3.

2.6 AHA simplex algorithm solution for the example adopted by S Nahar & Md. Abdul Alim

Max. Z = $(1.1599)x_1 + (1.7399)x_2$

Subject to $6x_1 + 8\,x_2 \leq 48$

$$x_1 + x_2 - x_3 \leq 3$$
$$x_1 + 0\,x_2 \leq 4$$
$$0x_1 + x_2 \leq 3$$

and $x_1, x_2 \geq 0$

Final AHA simplex table for the above linear programming problem is as follows:

x_1 0	x_2 0	x_3 0	 $\leq$	b_i 9.8593
0	0	0	$\leq$	0
1	0	0	$\leq$	4
0	0	1	$\leq$	4
0	1	0	$\leq$	3

Now, it can be observed that all the coefficients of x_j in the objective inequality is either zero or positive. Therefore, this is an optimal solution. The optimal solution occurs at x_1 = 4, x_2 = 3 with Maximum Z = 9.8593.

3. Conclusion

By all these methods, we obtained the optimal solution as $x_1 = 4$ and $x_2 = 3$ with Max. Z = 9.9164. Hence earlier solution given by Samsun Nahar & Md. Abdul Alim (2015) with maximum Z = 9.8593 is not optimal. A tabular presentation is given to make a clear view of all the techniques used to distinguish the value of objective function.

Technique	Graphical technique	Simplex technique	AHA Simplex technique	Gauss elimination technique	Modified Fourier elimination technique	By S Nahar & Md. A Alim (2017)
Optimal Point	$x_1 = 4$, $x_2 = 3$	$x_1 = 4$, $x_2 = 3$	$x_1 = 4$, $x_2 = 3$	$x_1 = 4$, $x_2 = 3$	$x_1 = 4$, $x_2 = 3$	$x_1 = 4$, $x_2 = 3$
Value -objective function	9.9164	9.9164	9.9164	9.9164	9.9164	9.8593

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